Probability

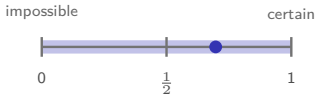
IGCSE Mathematics ▪ Topic 8

IGCSE Mathematics



What we will cover

1. The probability scale
2. Relative & expected frequency
3. Combined events
4. Sample space diagrams
5. Venn diagrams
6. Tree diagrams



the probability scale

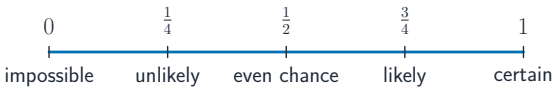
Parts marked (Extended) are tested only on the Extended paper; everything else is for both levels.

1 Basics

The probability scale

Probability runs from 0 (impossible) to 1 (certain), with $\frac{1}{2}$ an even chance.

For equally likely **outcomes** 结果,
$\mathrm{P}(\text{event}) = \dfrac{\text{favourable outcomes}}{\text{total outcomes}}$,
and the probability it does not happen is $\mathrm{P}(A') = 1 - \mathrm{P}(A)$.



Relative and expected frequency

relative frequency 相对频率

$$\frac{\text{times it happened}}{\text{total trials}}$$

better with more trials

expected frequency 期望频数

P $\times$ n; e.g. $\frac{1}{6} \times 300 = 50$ sixes

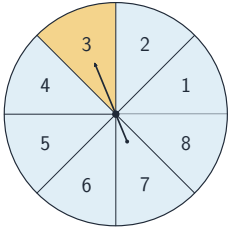
Combined events

AND – multiply

both happen, when **independent** 独立事件

OR – add

either happens, when **mutually exclusive** 互斥



Eight identical sectors, so every number is **equally likely**.

$$\mathrm{P}(3) = \frac{\text{favourable outcomes}}{\text{total outcomes}} = \frac{1}{8}$$

A fair spinner: eight identical sectors, so each number is an equally likely outcome.

The two rules, and the three diagrams

the AND rule	for independent events, $\mathrm{P}(A \text{ and } B) = \mathrm{P}(A) \times \mathrm{P}(B)$
the OR rule	for mutually exclusive events, $\mathrm{P}(A \text{ or } B) = \mathrm{P}(A) + \mathrm{P}(B)$
a sample space diagram	a grid of all equally likely outcomes – best for two dice or two spinners
a Venn diagram	best when events overlap, and for reading conditional probabilities
a tree diagram	best for events in sequence: multiply along a branch, add between branches
random 随机	every outcome has an equal chance; a **biased** 有偏 spinner or die does not

> Multiply along a tree branch, add between branches. Those two moves solve almost every tree question.

Tree diagrams, with and without replacement

a tree diagram	best for events in sequence – two draws, or two stages
on a tree	multiply along a branch, add between branches
with replacement	the second stage's probabilities are the same as the first's
without replacement	they change – the denominator falls by one, and so does the count of whatever was drawn

Without replacement, change both numbers on the second branches: 3 red out of 5, then 2 red out of 4, so P(red, red) $= \frac{3}{5} \times \frac{2}{4} = \frac{3}{10}$.

2 Diagrams

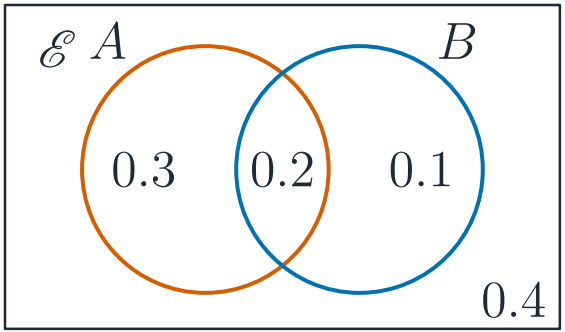
Sample space diagrams

	die 2					
die 1	**1**	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

the six highlighted cells all give a total of 7

A **sample space diagram** 样本空间图 lists every outcome in a grid. Two dice give 36 equally likely cells; count the ones you want.

Venn diagrams



A **Venn diagram** 维恩图 sorts outcomes into sets.

- $\mathrm{P}(A \cap B)$ – in both
- $\mathrm{P}(A \cup B)$ – in either

Worked example – conditional probability (Extended)

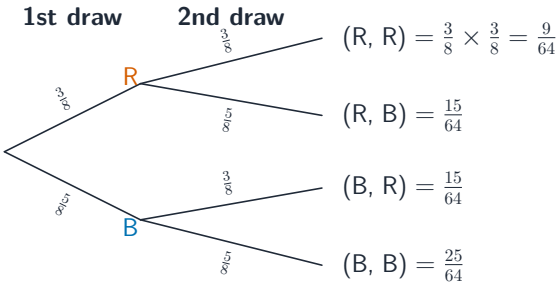
In a class of 30, 18 study French and, of those, 7 also study German. A French student is picked. Find the probability they study German.

- **conditional probability** 条件概率: the probability of one event given that another has happened
- look only at the 18 French students
- $\mathrm{P}(\text{German} \mid \text{French}) = \dfrac{7}{18}$

The condition replaces the denominator: you are no longer choosing from 30 but from 18. Circle the region you are inside before writing any fraction.

Tree diagrams

On a **tree diagram** 树状图, multiply along branches & add the paths you want. Two reds (with replacement): $\frac{3}{8} \times \frac{3}{8} = \frac{9}{64}$.



From impossible to certain



Dice: the probability scale runs from impossible (0) to certain (1).

Exam tips

- Every probability lies between 0 and 1, and the probabilities of all the outcomes add to 1.
- For "**and**" (both events) *multiply* the probabilities; for "**or**" (either event) *add* them. On a tree diagram, multiply along the branches.
- Watch for **without replacement**: the second probability changes because one item has already been removed.
- Expected frequency $=$ probability $\times$ number of trials.

3 Recap

Worked example – probability of 'not'

A bag holds 3 red and 5 blue counters. Find the probability of *not* drawing red.

- Total counters $= 3 + 5 = 8$.
- $P(\text{red}) = \dfrac{3}{8}$
- $P(\text{not red}) = 1 - P(\text{red}) = 1 - \dfrac{3}{8} = \dfrac{5}{8}$
- Check: there are 5 blue out of 8 – the same answer, as it must be.

$P(\text{not } A) = 1 - P(A)$. All probabilities lie between 0 and 1: a negative answer or one above 1 means you have made a slip.

Topic 8 – key takeaways

1 Scale

0 to 1; $\mathrm{P}(A') = 1 - \mathrm{P}(A)$

2 Expected

P $\times$ n over n trials

3 Combined

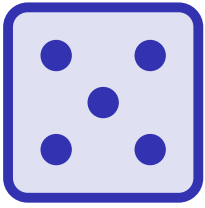
AND multiply, OR add

4 Trees

multiply along branches, add the paths

Check yourself

1. What does probability 0 mean?
2. If $\mathrm{P}(A) = 0.3$, what is $\mathrm{P}(A')$?
3. Do you add or multiply for AND?
4. What do you do along a tree branch?



Answers **1.** impossible **2.** 0.7 **3.** multiply **4.** multiply the probabilities