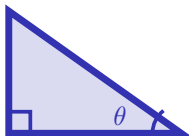


Trigonometry

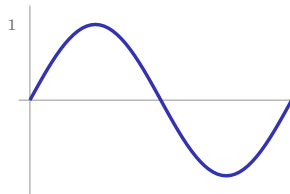
IGCSE Mathematics ■ Topic 6

IGCSE Mathematics



What we will cover

- 1 Pythagoras' theorem
- 2 SOH-CAH-TOA
- 3 Elevation & depression
- 4 Exact values
- 5 Trig graphs
- 6 Sine & cosine rules

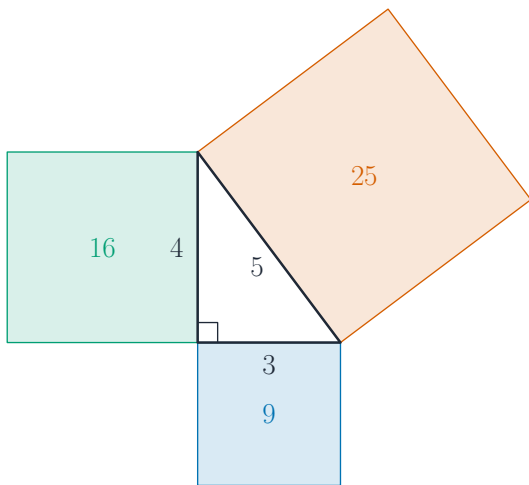


trig graphs

1

Right-angled triangles

Pythagoras' theorem

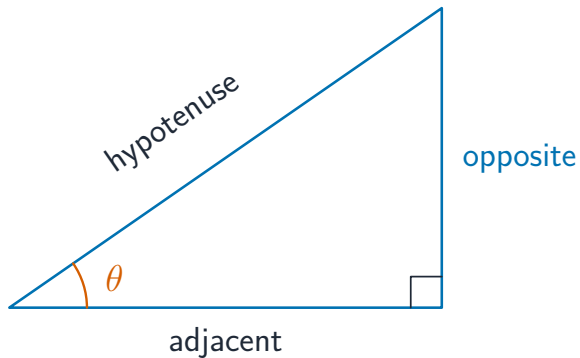


$a^2 + b^2 = c^2$ – use it to find a missing side.

$$b^2 = 13^2 - 5^2 = 169 - 25 = 144,$$

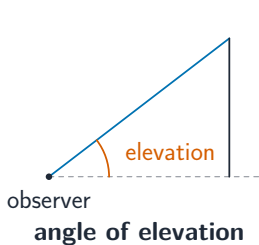
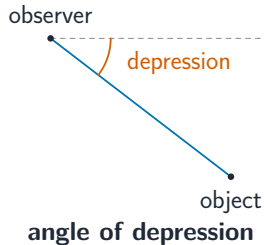
so $b = \sqrt{144} = 12$ cm.

SOH-CAH-TOA



- $y = \sin x$ is a wave starting at 0, peaking at 90° , back to 0 at 180° , down to -1 at 270° .
- $y = \cos x$ is the same wave but starting at 1.
- $y = \tan x$ rises steeply and repeats every 180° .

Elevation and depression



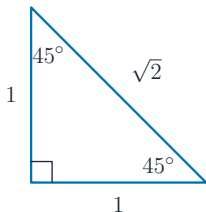
The **angle of elevation** 仰角 looks **up**; the **angle of depression** 俯角 looks **down**.
Tower: $50 \tan 40^\circ = 42.0 \text{ m}$.

2

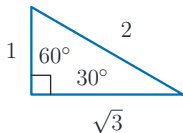
Values and graphs

Exact values

	30°	45°	60°
sin	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
cos	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
tan	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$



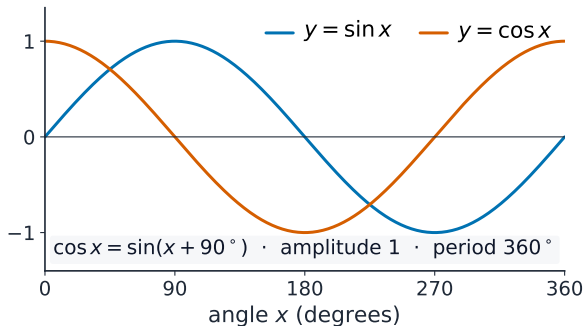
45° triangle



30° – 60° triangle

- $y = \sin x$ is a wave starting at 0, peaking at 90° , back to 0 at 180° , down to -1 at 270° .
- $y = \cos x$ is the same wave but starting at 1.
- $y = \tan x$ rises steeply and repeats every 180° .

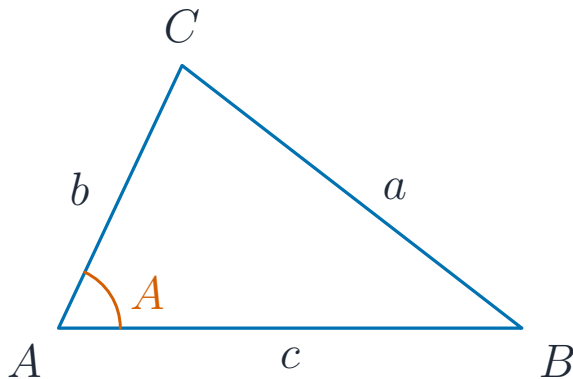
Trig graphs



$y = \sin x$ & $y = \cos x$ are waves between -1 & 1 .

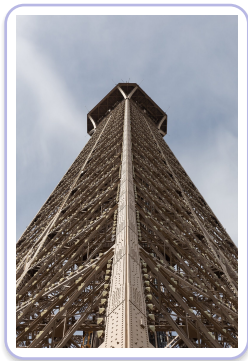
A trig equation has **several** answers in $0-360^\circ$ – use the wave's symmetry. $\sin x = \frac{\sqrt{3}}{2} \Rightarrow x = 60^\circ$ or 120° .

Sine and cosine rules



- $y = \sin x$ is a wave starting at 0, peaking at 90° , back to 0 at 180° , down to -1 at 270° .
- $y = \cos x$ is the same wave but starting at 1.
- $y = \tan x$ rises steeply and repeats every 180° .

Angles of elevation and depression (Extended)



Looking up at a tower involves an angle of elevation.

Graphs and trigonometric equations (Extended)



A Ferris wheel: a point on the rim traces a sine curve as it turns.

The rules, the ambiguous case, and 3D

the sine rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} - \text{for a side and its opposite angle}$$

the cosine rule

$$a^2 = b^2 + c^2 - 2bc \cos A - \text{for two sides and the included angle, or three sides}$$

a worked value

$$a^2 = 7^2 + 8^2 - 2(7)(8) \cos 40^\circ = 113 - 112(0.766) = 27.2, \text{ so } a = 5.2$$

the ambiguous case 歧义情形

the sine rule can give **two** valid angles – one acute and one obtuse – when you are given two sides and a non-included angle

the tangent ratio 正切比

$$\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$$

in 3D

find a **right-angled triangle inside the solid**, then use Pythagoras or trigonometry on that plane

For a 3D question, **draw the triangle on its own** before calculating. Working inside the perspective drawing is where the mistakes come from.

Exam tips

- Use **Pythagoras** ($a^2 + b^2 = c^2$) when there is no angle; use **SOH-CAH-TOA** when an angle is involved.
- The **hypotenuse** is always opposite the right angle. Label the sides (opposite, adjacent, hypotenuse) relative to the angle you are using.
- To find an angle, use the inverse ($\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$), and check your calculator is set to **degrees**.
- For a triangle with no right angle, use the **sine rule** or the **cosine rule** – the cosine rule when you know two sides and the angle between them.

3

Recap

Worked example – Pythagoras

A right-angled triangle has hypotenuse 13 cm and one short side 5 cm. Find the other short side.

- $a^2 + b^2 = c^2$, where c is the **hypotenuse** (opposite the right angle).
- $5^2 + b^2 = 13^2$
- $25 + b^2 = 169 \Rightarrow b^2 = 144$
- $b = \sqrt{144} = 12 \text{ cm}$

Finding a **shorter** side means **subtract**; finding the hypotenuse means **add**. Check: the hypotenuse must be the **longest** side.

Topic 6 – key takeaways

1

Pythagoras

$a^2 + b^2 = c^2$ for a right triangle

2

Ratios

SOH-CAH-TOA; inverse to find an angle

3

Graphs

sine & cosine waves; many solutions

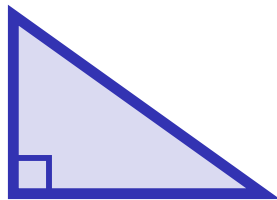
4

Any triangle

the sine rule & the cosine rule

Check yourself

- 1 What is the hypotenuse?
- 2 Which ratio is opp over adj?
- 3 What is $\sin 30^\circ$ exactly?
- 4 Which rule works for a non-right triangle?



Answers 1. the longest side, opposite the right angle 2. tangent 3. $\frac{1}{2}$ 4. the sine or cosine rule