

Algebra & Graphs

IGCSE Mathematics ■ Topic 2

IGCSE Mathematics



Algebra & Graphs

What we will cover

- 1 Simplify, expand & factorise
- 2 Solving equations
- 3 Quadratics & inequalities
- 4 Sequences
- 5 Graph shapes
- 6 Travel graphs

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Manipulating algebra

Simplify and expand

$$\begin{array}{cc} & x & 2 \\ 3 & \boxed{3x} & \boxed{6} \end{array} \quad 3(x+2) = 3x + 6$$

In **algebra** 代数 we use letters to stand for numbers.

A letter whose value can change is a **variable** 变量.

Factorising

common factor

$$\begin{aligned} 9x^2 + 15xy \\ = 3x(3x + 5y) \end{aligned}$$

difference of squares

$$\begin{aligned} a^2 - b^2 \\ = (a+b)(a-b) \end{aligned}$$

quadratic

split the middle term, then group

Factorise 因式分解 is the reverse of expanding: take out the common factor first.

Solving equations

$$\begin{aligned} 2x + 3 &= 11 \quad (\text{start}) \\ 2x &= 8 && \text{subtract 3 from both sides} \\ x &= 4 && \text{divide both sides by 2} \end{aligned}$$

Do the **same** to both sides until the **unknown** 未知数 is alone.

Simultaneous 联立方程

$$\begin{aligned} 2x + y &= 7, \quad 3x - y = 8 \\ \text{add: } 5x &= 15, \text{ so } x = 3, y = 1 \end{aligned}$$

2 Quadratics and more

Algebra & Graphs

└ Quadratics and more

Quadratic equations

factorise

$$(x+2)(x+3) = 0$$

**complete
the square**

$$(x+3)^2 = 8$$

formula

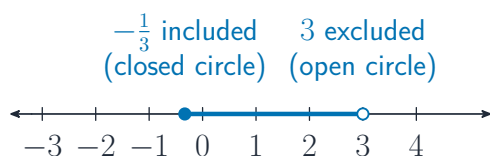
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Algebra & Graphs

└ Quadratics and more

Inequalities

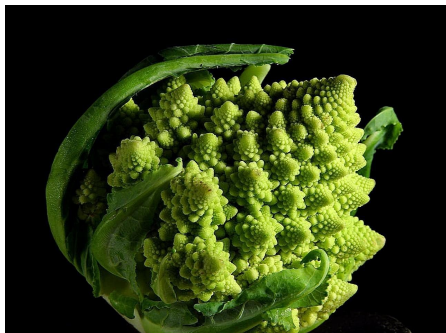
Solve like an equation, but **reverse** the sign if you $\times$ or $\div$ by a negative. Number line: **closed** circle includes the value, **open** excludes it.



An **inequality** 不等式 uses $<$, $>$, $\leq$ or $\geq$.

Solve it like an equation, but **reverse the sign** if you multiply or divide by a negative number.

Sequences



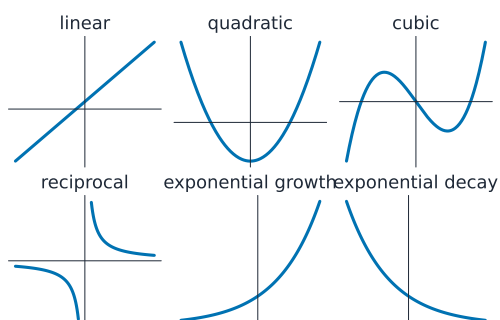
Find the n th term from how the terms change.

- linear $2, 5, 8, 11 \dots \rightarrow 3n - 1$
- quadratic: $2 \times$ the 2nd difference
= n^2 part

$2, 5, 10, 17 \dots \rightarrow n^2 + 1$.

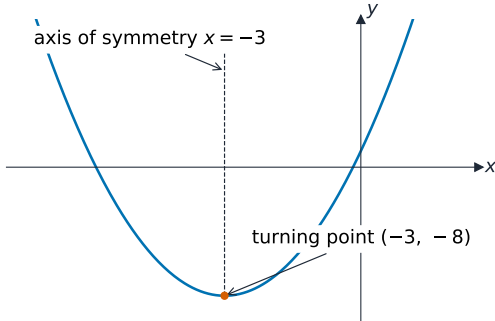
3 Graphs

Graph shapes



Know each shape: **linear** 一次, **quadratic** 二次 (parabola), cubic, **reciprocal** 反比例 & **exponential** 指数. **Roots** 根 are where the graph crosses the x -axis ($y = 0$).

Completing the square

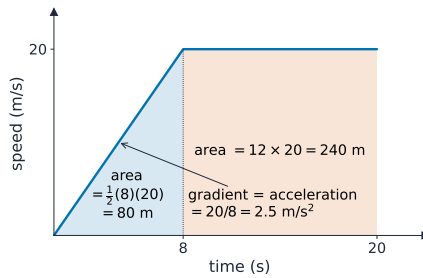
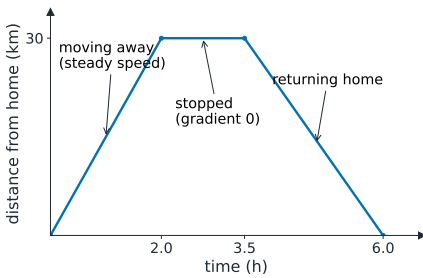


Writing $y = (x + p)^2 + q$ shows the **turning point** 顶点 at $(-p, q)$.

$$x^2 + 6x + 1 = (x + 3)^2 - 8$$

So the lowest point is $(-3, -8)$.

Travel graphs



Distance-time: the **gradient** 斜率 is the speed (flat = stopped). Speed-time: gradient = **acceleration** 加速度, area = distance.

Differentiation (Extended)

Differentiation 微分 finds the gradient of a curve. If $y = ax^n$:

$$\frac{dy}{dx} = a n x^{n-1}$$

differentiate a sum term by term.

A **turning point** 驻点 is where the gradient is **zero** – set $dy/dx = 0$ and solve.

Turning point of $y = x^2 - 6x + 5$

$$\frac{dy}{dx} = 2x - 6 = 0 \Rightarrow x = 3$$

$$y = 9 - 18 + 5 = -4, \text{ so } (3, -4)$$

Functions (Extended)

A **function** 函数 turns each input into one output, $f(x)$.

- **inverse** 反函数 f^{-1} undoes it: swap x and y , solve
- **composite** 复合函数 $gf(x)$ means do f first, then g

$$f(x) = 2x, g(x) = x + 3$$

$$gf(x) = g(2x) = 2x + 3$$

$$fg(x) = f(x + 3) = 2x + 6$$

Order matters: $gf \neq fg$.

Direct and inverse proportion (Extended)

- **direct** 正比例: $y = kx$ – one is a fixed multiple of the other
- **inverse** 反比例: $y = k/x$ – one rises as the other falls

Always find k first from the given pair.

$y \propto x$; $y = 12$ when $x = 3$. Find y at $x = 7$.

$$12 = k \times 3 \Rightarrow k = 4, \text{ so } y = 4x$$

$$y = 4 \times 7 = 28$$

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Recap

Topic 2 – key takeaways

1 Expand

multiply out; factorise reverses it

2 Quadratics

factorise, complete the square, or formula

3 Inequalities

flip the sign when dividing by a negative

4 Graphs

gradient = rate; area under speed–time = distance

Check yourself

1 Expand $(x + 2)(x - 4)$.

2 Factorise $9x^2 - 16$.

3 When must you reverse an inequality sign?

4 What does the area under a speed–time graph give?



Answers 1. $x^2 - 2x - 8$ 2. $(3x + 4)(3x - 4)$ 3. when $\times$ or $\div$ by a negative 4. the distance travelled