

Algebra & Graphs

IGCSE Mathematics ■ Topic 2

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What we will cover

1 Simplify, expand & factorise

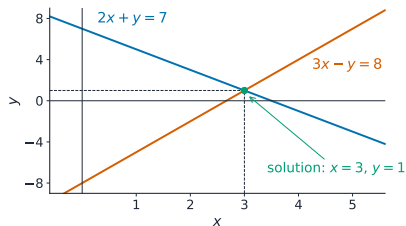
2 Solving equations

3 Quadratics & inequalities

4 Sequences

5 Graph shapes

6 Travel graphs



solving by graph

Simplify and expand

	x	2
3	$3x$	6

$$3(x + 2) = 3x + 6$$



quadratic bridge

Factorising

common factor

$$\begin{aligned} &9x^2 + 15xy \\ &= 3x(3x + 5y) \end{aligned}$$

**difference
of squares**

$$\begin{aligned} &a^2 - b^2 \\ &= (a+b)(a-b) \end{aligned}$$

quadratic

split the middle
term, then group

Factorise 因式分解 is the reverse of expanding: take out the common factor first.

Solving equations

$$2x + 3 = 11 \text{ (start)}$$

$$2x = 8 \quad \text{subtract 3 from both sides}$$

$$x = 4 \quad \text{divide both sides by 2}$$

Do the **same** to both sides until the **unknown** 未知数 is alone.

Simultaneous 联立方程

$$2x + y = 7, \quad 3x - y = 8$$

$$\text{add: } 5x = 15, \text{ so } x = 3, y = 1$$

Quadratic equations

factorise

$$(x + 2)(x + 3) = 0$$

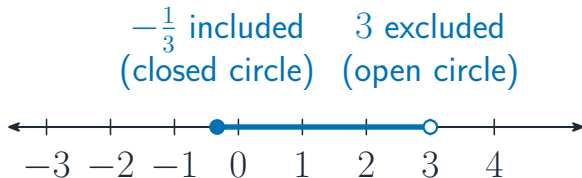
**complete
the square**

$$(x + 3)^2 = 8$$

formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Inequalities



Solve like an equation, but **reverse** the sign if you $\times$ or $\div$ by a negative. Number line: **closed** circle includes the value, **open** excludes it.

Sequences

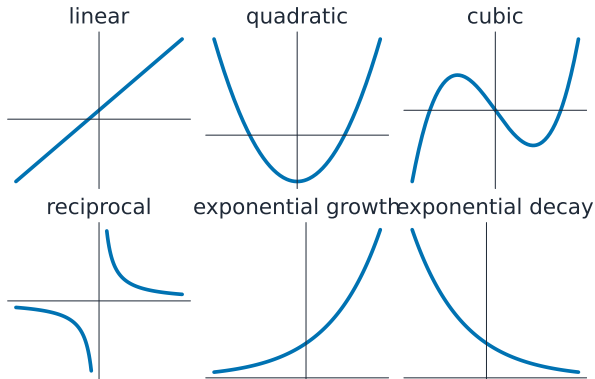


Find the n th term from how the terms change.

- linear $2, 5, 8, 11 \dots \rightarrow 3n - 1$
- quadratic: $2 \times$ the 2nd difference
 $= n^2$ part

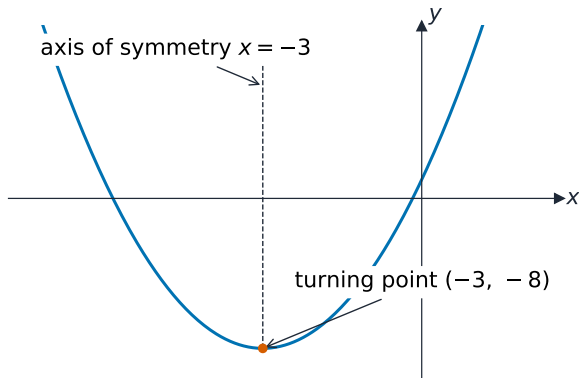
$$2, 5, 10, 17 \dots \rightarrow n^2 + 1.$$

Graph shapes



Know each shape: **linear** 一次, **quadratic** 二次 (parabola), cubic, **reciprocal** 反比例 & **exponential** 指数. **Roots** 根 are where the graph crosses the x-axis ($y = 0$).

Completing the square

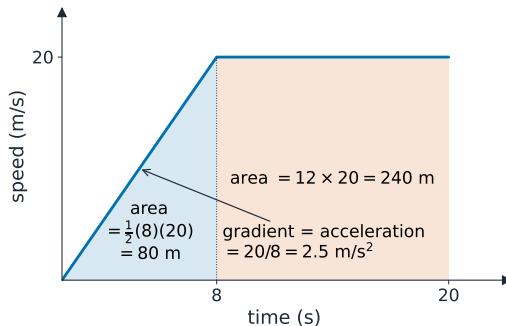
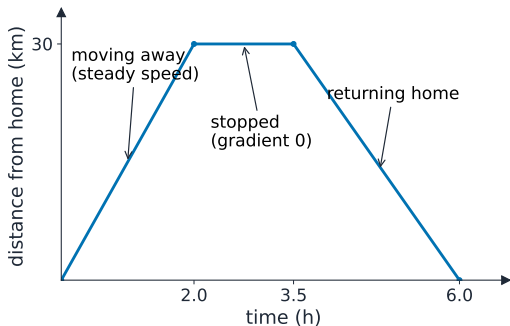


Writing $y = (x + p)^2 + q$ shows the **turning point** 顶点 at $(-p, q)$.

$$x^2 + 6x + 1 = (x + 3)^2 - 8$$

So the lowest point is $(-3, -8)$.

Travel graphs



Distance-time: the **gradient** 斜率 is the speed (flat = stopped). Speed-time: gradient = **acceleration** 加速度, area = distance.

Differentiation (Extended)

Differentiation 微分 finds the gradient of a curve. If $y = ax^n$:

$$\frac{dy}{dx} = a n x^{n-1}$$

differentiate a sum term by term.

A **turning point** 驻点 is where the gradient is **zero** – set $dy/dx = 0$ and solve.

Turning point of $y = x^2 - 6x + 5$

$$\frac{dy}{dx} = 2x - 6 = 0 \Rightarrow x = 3$$

$$y = 9 - 18 + 5 = -4, \text{ so } (3, -4)$$

Functions (Extended)

A **function** 函数 turns each input into one output, $f(x)$.

- **inverse** 反函数 f^{-1} **undoes** it: swap x and y , solve
- **composite** 复合函数 $gf(x)$ means do f **first**, then g

$$f(x) = 2x, g(x) = x + 3$$

$$gf(x) = g(2x) = 2x + 3$$

$$fg(x) = f(x + 3) = 2x + 6$$

Order matters: $gf \neq fg$.

Direct and inverse proportion (Extended)

- **direct** 正比例: $y = kx$ – one is a fixed multiple of the other
- **inverse** 反比例: $y = k/x$ – one rises as the other falls

Always **find k first** from the given pair.

$y \propto x$; $y = 12$ when $x = 3$. Find y at $x = 7$.

$$12 = k \times 3 \Rightarrow k = 4, \text{ so } y = 4x$$

$$y = 4 \times 7 = 28$$

Topic 2 – key takeaways

1

Expand

multiply out; factorise reverses it

2

Quadratics

factorise, complete the square, or formula

3

Inequalities

flip the sign when dividing by a negative

4

Graphs

gradient = rate; area under speed–time = distance

Check yourself

- 1 Expand $(x + 2)(x - 4)$.
- 2 Factorise $9x^2 - 16$.
- 3 When must you reverse an inequality sign?
- 4 What does the area under a speed–time graph give?



Answers 1. $x^2 - 2x - 8$ 2. $(3x + 4)(3x - 4)$ 3. when $\times$ or $\div$ by a negative 4. the distance travelled