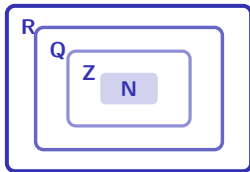


Number

IGCSE Mathematics ■ Topic 1

IGCSE Mathematics



What we will cover

- 1 Types of number
- 2 Primes, HCF & LCM
- 3 Sets & Venn diagrams
- 4 Indices & standard form
- 5 Fractions, decimals & %
- 6 Ratio, rates & bounds

multiply the powers

$$a^m \times a^n = a^{m+n}$$

add:

$$2^{-3} \times 2^4 = 2^1 = 2$$

divide the powers

$$a^m \div a^n = a^{m-n}$$

subtract:

$$2^3 \div 2^4 = 2^{-1} = \frac{1}{2}$$

power of a power

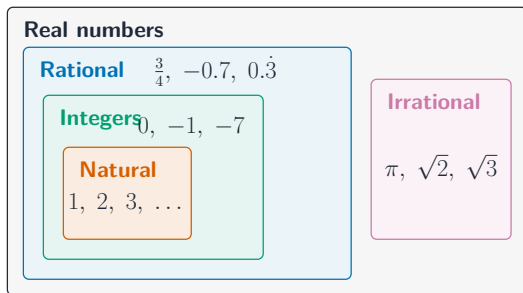
$$(a^m)^n = a^{mn}$$

multiply:

$$(2^3)^2 = 2^6 = 64$$

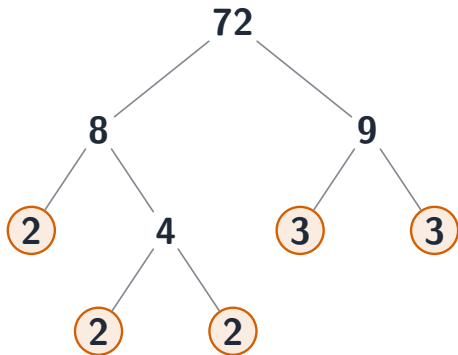
indices and standard form

Types of number



- **natural** 自然数 $1, 2, 3 \dots$;
integers 整数 $\dots, -1, 0, 1 \dots$
- **rational** 有理数 – a fraction $\frac{a}{b}$
- **irrational** 无理数 – e.g. $\pi, \sqrt{2}$
- **prime** 质数, **square** 平方数,
cube 立方数

Primes, HCF and LCM



$$72 = 2^3 \times 3^2$$

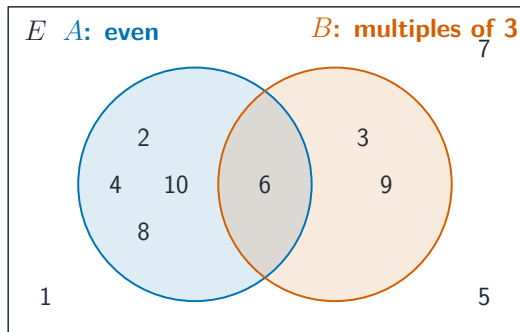
HCF & LCM of 72 and 120

$$72 = 2^3 \times 3^2, \quad 120 = 2^3 \times 3 \times 5$$

$$\text{HCF} = \text{lowest powers} = 2^3 \times 3 = \mathbf{24}$$

$$\text{LCM} = \text{highest powers} = 2^3 \times 3^2 \times 5 = \mathbf{360}$$

Sets and Venn diagrams



A **set** 集合 is a collection of **elements** 元素.

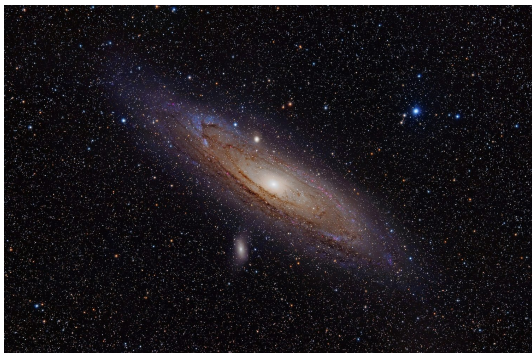
- $A \cap B$ – **intersection** 交集 (in both)
- $A \cup B$ – **union** 并集 (in either)
- A' – **complement** 补集; $\emptyset$ – **empty set** 空集

Indices

$$a^m \times a^n = a^{m+n} \quad a^m \div a^n = a^{m-n} \quad (a^m)^n = a^{mn}$$
$$a^0 = 1 \quad a^{-n} = \frac{1}{a^n} \quad a^{1/n} = \sqrt[n]{a}$$

$$\text{e.g. } 2^{-3} \times 2^4 = 2^1 = 2; \quad 81^{1/2} = 9; \quad 8^{-2/3} = \frac{1}{4}.$$

Standard form



Standard form 科学记数法 $A \times 10^n$,
with $1 \leq A < 10$.

■ $4\,500\,000 = 4.5 \times 10^6$

■ $0.00072 = 7.2 \times 10^{-4}$

$(3 \times 10^5)(2 \times 10^{-2}) = 6 \times 10^3$.

Surds (Extended)

A **surd** 根式 is an irrational root, kept in **exact** form:

$$\sqrt{a} \sqrt{b} = \sqrt{ab}, \quad \sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$

Simplify by taking out the largest **square factor**.

To **rationalise** 分母有理化 $\frac{1}{\sqrt{3}-1}$,
multiply top and bottom by the **conjugate** $\sqrt{3}+1$ – the surd on the bottom disappears.

Simplify

$$\sqrt{20} = \sqrt{4 \times 5} = 2\sqrt{5}$$

$$\sqrt{200} - \sqrt{32} = 10\sqrt{2} - 4\sqrt{2} = 6\sqrt{2}$$

$$\frac{10}{\sqrt{5}} = \frac{10\sqrt{5}}{5} = 2\sqrt{5}$$

Exponential growth and decay (Extended)

A quantity that changes by the **same percent** each period grows or decays **exponentially** 指数变化:

$$\text{value} = P \times (\text{multiplier})^t$$

growth $\times (1 + r)$, decay $\times (1 - r)$.

**A \$20 000 car loses 15% a year.
Value after 4 years?**

multiplier = 0.85

$$20\,000 \times 0.85^4 \approx \$10\,440$$

Raise the multiplier to the **power** t – do not subtract 15% four times.

Rounding and estimation

- **decimal places** 小数位: digits after the point ($3.14159 \rightarrow 3.14$, 2 d.p.)
- **significant figures** 有效数字: from the first non-zero digit
- to **estimate** 估算: round each number to 1 s.f., then work it out

$$\text{Estimate } \frac{41.3}{9.79 \times 0.765}$$

$$\approx \frac{40}{10 \times 0.8} = \frac{40}{8} = 5$$

An estimate **checks** a calculator answer – if they are far apart, you slipped.

Fractions, decimals and percentages



One value, three forms:

- fraction $\rightarrow$ decimal: divide top by bottom
- decimal $\rightarrow$ % : $\times 100$
- simplify by the **HCF** 最大公因数

$$\frac{3}{8} = 0.375 = 37.5\%.$$

Percentages and interest



$\% \text{ change} = \frac{\text{change}}{\text{original}} \times 100$. Use a
multiplier 乘数 (+15% $\rightarrow \times 1.15$).

Compound interest

\$500 at 4% for 3 yr = $500 \times 1.04^3 =$
\$562.43

Ratio, rates and bounds

ratio 比

share \$48 as 3:5
→ \$18 & \$30

speed

$$\frac{\text{distance}}{\text{time}};$$
$$\text{density} = \frac{m}{V}$$

bounds 界

635 m (nearest m):
 $634.5 \leq h < 635.5$

Topic 1 – key takeaways

1 HCF & LCM

lowest vs highest powers of the primes

2 Indices

add powers to multiply, subtract to divide

3 Standard form

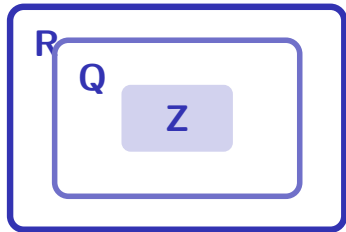
$A \times 10^n$ with $1 \leq A < 10$

4 Percentages

multiplier; compound = $P(1 + r/100)^t$

Check yourself

- 1 Is $\sqrt{2}$ rational or irrational?
- 2 What is $2^3 \times 2^4$?
- 3 Write 0.00072 in standard form.
- 4 Increase \$40 by 25% using a multiplier.



Answers 1. irrational 2. $2^7 = 128$ 3. 7.2×10^{-4} 4. $40 \times 1.25 = \$50$