IGCSE Mathematics

A mark scheme writes "M1 A1" and leaves the reasoning to you. These are the same problems written the way you should write them in the exam: every step visible, the method marks earned before the answer, and a note on where the marks actually sit. Work each one on paper before reading the solution. 官方答案只写 M1 A1; 这里把每一步都写出来, 并标明分数在哪里。

Number

A jacket costs $84 after a 20% discount. Find the original price. [3]

After a 20% discount the price is 80% of the original, so the multiplier is 0.8. Therefore original $\times 0.8 = 84$, giving original $= 84 \div 0.8 = \$105$.

The method mark is for dividing by the MULTIPLIER 0.8. Adding 20% to $84 gives $100.80 and is wrong – a percentage change is taken from the original, not from the new price.

Check by working forwards: 20% of 105 is 21, and $105 - 21 = 84$. One line, and it catches the commonest error in the paper.

Write 0.00408 in standard form, and calculate $(4.08 \times 10^{-3}) \times (2 \times 10^{5})$, giving your answer in standard form. [3]

$0.00408 = 4.08 \times 10^{-3}$. Multiplying: $(4.08 \times 2) \times 10^{-3+5} = 8.16 \times 10^{2}$.

Standard form needs the first number to satisfy $1 \le a < 10$. Writing 81.6×10^{1} is arithmetically correct but loses the accuracy mark.

Add the indices when multiplying; subtract when dividing. Multiplying the indices is the standard slip.

$p = 4.6$ and $q = 2.1$, both correct to 1 decimal place. Find the upper bound of $\frac{p}{q}$. [2]

The bounds are $4.55 \le p < 4.65$ and $2.05 \le q < 2.15$. A quotient is largest when the numerator is largest and the denominator is smallest, so the upper bound is $\frac{4.65}{2.05} = 2.268\ldots = 2.27$ (3 s.f.).

The mark is for choosing the RIGHT pair of bounds. For a quotient it is max ÷ min; for a difference it is max − min. Using upper ÷ upper is the usual error.

Bounds are half of the last place: 1 d.p. gives ±0.05.

Algebra

Solve $\frac{3}{x-1} + \frac{2}{x+2} = 1$, giving your answers to 2 decimal places. [6]

Multiply throughout by $(x-1)(x+2)$: $3(x+2) + 2(x-1) = (x-1)(x+2)$. Expanding: $3x + 6 + 2x - 2 = x^2 + x - 2$, so $5x + 4 = x^2 + x - 2$ and $x^2 - 4x - 6 = 0$. By the formula, $x = \frac{4 \pm \sqrt{16+24}}{2} = \frac{4 \pm \sqrt{40}}{2}$, giving $x = 5.16$ or $x = -1.16$ (2 d.p.).

Multiplying EVERY term by the common denominator is the first method mark. Multiplying only the fractions and forgetting the 1 on the right is the commonest loss.

Two decimal places means the formula, not factorising – check the discriminant is not a perfect square before hunting for factors.

Both roots are wanted. Giving only the positive one loses the final accuracy mark.

Make r the subject of $V = \frac{1}{3}\pi r^2 h$. [3]

Multiply by 3: $3V = \pi r^2 h$. Divide by πh: $r^2 = \frac{3V}{\pi h}$. Take the positive square root, since r is a length: $r = \sqrt{\frac{3V}{\pi h}}$.

Clear the fraction first, then divide by everything multiplying r^2, then square-root LAST. Square-rooting early is where the method mark is lost.

Saying the root is positive because r is a length is worth writing; it costs one clause.

Solve the simultaneous equations $y = x^2 - 3x + 4$ and $y = 2x - 2$. [5]

Equating: $x^2 - 3x + 4 = 2x - 2$, so $x^2 - 5x + 6 = 0$, which factorises as $(x-2)(x-3) = 0$. Hence $x = 2$ or $x = 3$. Substituting into the LINEAR equation: when $x = 2$, $y = 2$; when $x = 3$, $y = 4$.

Substitute back into the SIMPLER equation. Using the quadratic doubles the arithmetic and the chance of an error.

Pair the answers: $(2, 2)$ and $(3, 4)$. A list of four separate values without pairing loses a mark.

Geometry and mensuration

A sector of a circle has radius 8 cm and angle 135°. Find its area and its perimeter. [5]

Area $= \frac{135}{360} \times \pi \times 8^2 = \frac{3}{8} \times 64\pi = 24\pi = 75.4$ cm^2 (3 s.f.). Arc length $= \frac{135}{360} \times 2\pi \times 8 = 6\pi = 18.85$ cm, so the perimeter $= 18.85 + 8 + 8 = 34.8$ cm (3 s.f.).

The perimeter of a sector includes the TWO RADII as well as the arc. Giving the arc alone is the single commonest error on this question.

Keep π exact until the final line; rounding 24π early costs the accuracy mark.

In triangle ABC, $AB = 7$ cm, $BC = 9$ cm and angle $ABC = 52°$. Find AC, and the area of the triangle. [5]

By the cosine rule, $AC^2 = 7^2 + 9^2 - 2(7)(9)\cos 52° = 49 + 81 - 126\cos 52° = 130 - 77.58 = 52.42$, so $AC = 7.24$ cm (3 s.f.). Area $= \frac{1}{2}(7)(9)\sin 52° = 24.8$ cm^2 (3 s.f.).

Use the cosine rule when you know two sides and the angle BETWEEN them. Reaching for the sine rule here is the usual wrong turn.

The area formula $\frac{1}{2}ab\sin C$ needs the INCLUDED angle – the same angle, which is why both parts use 52°.

Do not round AC and then use it for the area; use the exact given values.

Statistics and probability

A bag holds 5 red and 3 blue counters. Two are taken without replacement. Find the probability that they are different colours. [4]

P(red then blue) $= \frac{5}{8} \times \frac{3}{7} = \frac{15}{56}$. P(blue then red) $= \frac{3}{8} \times \frac{5}{7} = \frac{15}{56}$. These are the only ways, and they are mutually exclusive, so P(different) $= \frac{15}{56} + \frac{15}{56} = \frac{30}{56} = \frac{15}{28}$.

Without replacement means the SECOND denominator is 7, not 8. Leaving it as 8 is the commonest error and loses both method marks.

There are TWO orders. Giving only red-then-blue halves the answer.

An alternative is 1 − P(same), which is quicker but needs both same-colour cases.

The table shows the times t minutes taken by 40 students. Estimate the mean, given the grouped data with midpoints 5, 15, 25, 35 and frequencies 6, 14, 12, 8. [4]

Multiply each midpoint by its frequency: $5(6) + 15(14) + 25(12) + 35(8) = 30 + 210 + 300 + 280 = 820$. The mean is $820 \div 40 = 20.5$ minutes.

Use the MIDPOINT of each class, not the class boundary or the class width. That choice is the first method mark.

Divide by the total FREQUENCY (40), never by the number of classes (4). Dividing by 4 gives 205 and should look obviously wrong.

The answer is an ESTIMATE because the original values within each class are unknown – say so if the question asks why.