

Week 26

IGCSE Mathematics

Homework bundle for this week

本周作业合集

exercises.lol

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8. Probability

Starter Quiz · 课前小测

IGCSE Mathematics

Name: _____ Score: _____

- What is the probability of an impossible event?

- The probability of rolling a six is $1/6$. How many sixes are expected in 300 rolls?

- For two independent events both happening (AND), you:
 - ☐ multiply the probabilities
 - ☐ add the probabilities
 - ☐ subtract them
 - ☐ take the larger one
- Of 18 French students, 7 also study German. $P(\text{German} | \text{French}) = 7/a$. What is a ?

- A bag has 3 red and 5 blue counters. $P(\text{not red}) = a/8$. What is a ?

- Relative frequency is calculated as:
 - ☐ $(\text{times it happened}) \div (\text{total trials})$
 - ☐ $(\text{total trials}) \div (\text{times it happened})$
 - ☐ $\text{times it happened} \times \text{total trials}$
 - ☐ always $1/2$
- On the 6 by 6 grid of two dice (36 cells), how many cells give a total of 7?

- Conditional probability is the probability of an event:
 - ☐ given that another event has already happened
 - ☐ that is impossible
 - ☐ on a scale from 1 to 10
 - ☐ multiplied by 100
- A certain event has probability 100.
 - ☐ True ☐ False

10. Relative frequency becomes a better estimate as the number of trials increases.

☐ True ☐ False

8. Probability

IGCSE Mathematics

1. **0**

Impossible events have probability 0; certain events have probability 1.

2. **50**

$(1/6) \times 300 = 50$.

3. **multiply the probabilities**

$P(A \text{ and } B) = P(A) \times P(B)$ for independent events.

4. **18**

Given they are French, the total to divide by is the 18 French students.

5. **5**

$P(\text{red}) = 3/8$, so $P(\text{not red}) = 1 - 3/8 = 5/8$; $a = 5$.

6. **(times it happened) $\div$ (total trials)**

Relative frequency = successes $\div$ number of trials.

7. **6**

1+6, 2+5, 3+4, 4+3, 5+2, 6+1 — six cells, so $P(7) = 6/36 = 1/6$.

8. **given that another event has already happened**

It is the probability of B given that A has occurred.

9. **False**

A certain event has probability 1 (or 100%), not 100. Probability is between 0 and 1.

10. **True**

More trials $\rightarrow$ the relative frequency converges to the true probability.

E8.1 Introduction to probability

Name: _____ Class: _____ Date: _____ **Total: 21 marks**

KEY WORDS probability 概率 • probability scale 概率标度 • outcome 结果
• expected frequency 期望频数 • tree diagrams 树状图

Objective

Build the skills to answer Extended exam questions on **probability** 概率 — the **probability scale** 概率标度, $P(A')$, **expected frequency** 期望频数, and **tree diagrams** 树状图.

You must be able to:

- find a probability and use $P(A') = 1 - P(A)$
- find an expected frequency
- use a tree diagram for combined events (with and without replacement)

1 Worked examples

■ Probability and tree diagrams

1st draw

2nd draw

R

B

$\frac{3}{8} \times \frac{3}{8} = \frac{9}{64}$

$(R, R) = \frac{3}{8} \times \frac{3}{8} = \frac{9}{64}$

$\frac{3}{8} \times \frac{5}{8} = \frac{15}{64}$

$(R, B) = \frac{3}{8} \times \frac{5}{8} = \frac{15}{64}$

$\frac{5}{8} \times \frac{3}{8} = \frac{15}{64}$

$(B, R) = \frac{5}{8} \times \frac{3}{8} = \frac{15}{64}$

$\frac{5}{8} \times \frac{5}{8} = \frac{25}{64}$

$(B, B) = \frac{5}{8} \times \frac{5}{8} = \frac{25}{64}$

Probability runs from 0 (impossible) to 1 (certain)

IGCSE Mathematics • E8.1 Introduction to probability 1

5

		die 2					
		1	2	3	4	5	6
die 1	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

the six highlighted cells all give a total of 7

A sample space lists all equally likely outcomes

$P(\text{event}) = \frac{\text{favourable}}{\text{total}}$ $P(A') = 1 - P(A)$ expected = $P \times n$

- **Worked (bag: 3 red, 5 blue):** $P(\text{not red}) = 1 - \frac{3}{8} = \frac{5}{8}$.
- **Tree (with replacement):** $P(\text{red, red}) = \frac{3}{8} \times \frac{3}{8} = \frac{9}{64}$.

Answer shapes: M1 a correct method, A1 the correct answer.

2 Practice

2.1 A bag has 4 red and 6 green counters. Find the probability of drawing a red. [1]

2.2 The probability of rain is 0.3. Find the probability of no rain. [1]

2.3 The probability of a six is $\frac{1}{6}$. Find the expected number of sixes in 240 rolls. [2]

IGCSE Mathematics • E8.1 Introduction to probability 2

6

3 Exam-style questions

3.1 A bag has 3 red and 5 blue counters. Two are drawn **without replacement**. Find the probability that both are red. [3]

3.2 The probability that a train is late is 0.15. On 200 days, find the expected number of late days. [2]

3.3 A bag holds 5 red, 3 blue and 2 green counters. One counter is taken at random.

(a) Write down $P(\text{red})$ and $P(\text{not red})$, and state the connection between them. [3]

(b) The counter is replaced and the experiment is repeated 250 times. Find the expected number of blue counters. [2]

(c) Two counters are now taken **without** replacement. Draw a tree diagram for “green” and “not green”, and use it to find the probability that exactly one is green. [5]

(d) Explain why the probabilities on the second pair of branches differ from those on the first. [2]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $\frac{4}{10} = \frac{2}{5}$.

2.2 $1 - 0.3 = 0.7$.

2.3 $\frac{1}{6} \times 240 = 40$.

3.1 first red $\frac{3}{8}$; after a red is taken, 2 reds out of 7 remain $\rightarrow \frac{2}{7}$. $P(\text{red, red}) = \frac{3}{8} \times \frac{2}{7} = \frac{6}{56} = \frac{3}{28}$.

3.2 $0.15 \times 200 = 30$ days.

3.3 (a) $P(\text{red}) = \frac{5}{10} = \frac{1}{2}$; $P(\text{not red}) = \frac{5}{10} = \frac{1}{2}$; $P(A') = 1 - P(A)$ — the two must add to 1, because the counter is either red or not.

(b) $P(\text{blue}) = \frac{3}{10}$; expected number $= \frac{3}{10} \times 250 = 75$.

(c) First pair of branches $\frac{2}{10}$ and $\frac{8}{10}$; second pair $\frac{1}{9}$, $\frac{8}{9}$ after green and $\frac{2}{9}$, $\frac{7}{9}$ after not green. Exactly one green $= \left(\frac{2}{10} \times \frac{8}{9}\right) + \left(\frac{8}{10} \times \frac{2}{9}\right) = \frac{16}{90} + \frac{16}{90} = \frac{32}{90} = \frac{16}{45}$.

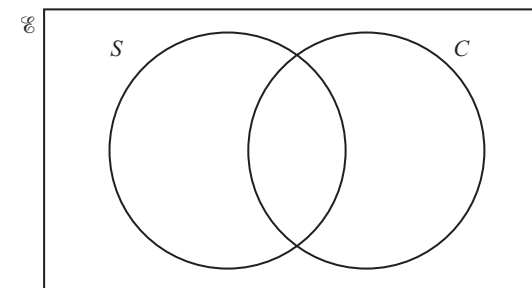
(d) The first counter is not replaced, so only 9 counters remain for the second draw; and the number of greens left depends on what was drawn first, so the two second-stage pairs are different.

12 A fitness club has 100 members.

60 swim (S).

70 cycle (C).

25 do not swim or cycle.



(a) Complete the Venn diagram.

[3]

(b) One member of the fitness club is chosen at random.

For this member, find

(i) $P(C)$

..... [1]

(ii) $P(S \cap C)$

..... [1]

(iii) $P(S \cup C')$.

..... [1]

25 Mahir picks one number at random from the numbers 5, 10 and 15. He then picks one number at random from the numbers 4, 5 and 6. He adds the two numbers.

The sample space diagram shows some of the possible outcomes.

		First number		
+		5	10	15
Second number	4		14	19
	5		15	20
	6		16	21

- (a) Complete the sample space diagram. [1]
- (b) Given that the total of the two numbers is odd, find the probability that one of the numbers added is 15.

..... [2]

26 Write as a single fraction in its simplest form.

(a) $\frac{mp}{25y} \times \frac{15}{m}$

..... [2]

(b) $\frac{3}{2x-5} + \frac{4}{x-3}$

4

1

2

3

4

5

6

7

Samira picks one of these cards at random and replaces it.

- (a) Find the probability that she picks an odd number.

..... [1]

- (b) Samira repeats this 35 times.

Calculate the number of times Samira is expected to pick an odd number.

..... [1]

E8.4 Conditional probability

Name: _____ Class: _____ Date: _____ **Total: 11 marks**

KEY WORDS conditional probability 条件概率 • probability 概率 • tree diagram 树状图
• without replacement 无放回 • venn diagram 维恩图

Objective

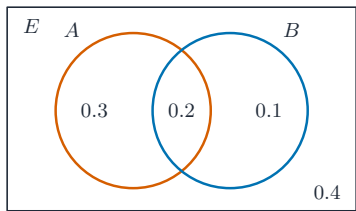
Build the skills to answer Extended exam questions on **conditional probability** 条件概率 — the probability of one event **given that** another has happened, read from a Venn diagram, a two-way table or a tree diagram.

You must be able to:

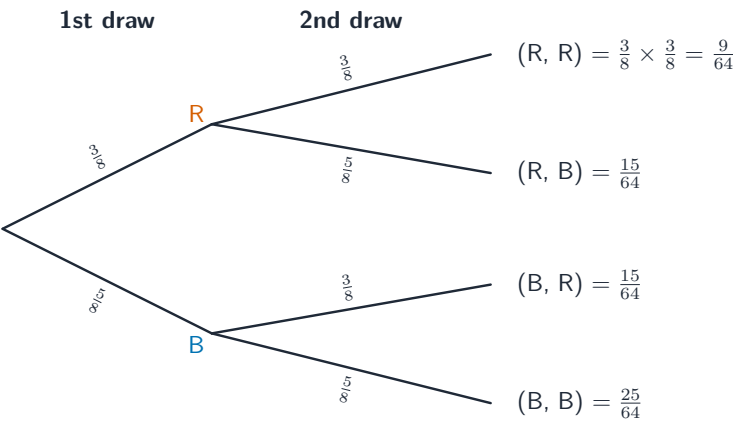
- find a conditional probability from a Venn diagram or table
- restrict attention to the part that matches the condition
- combine with without-replacement reasoning

1 Worked examples

Conditional probability



“Given that” means look only at the matching part



A tree diagram handles conditional probability

- **Worked:** 30 students, 18 study French, of whom 7 also study German.
- A French student is picked $\rightarrow P(\text{German} \mid \text{French}) = \frac{7}{18}$ (look only at the 18 French students).

Answer shapes: M1 a correct method, A1 the correct answer.

2 Practice

2.1 Of 20 people, 12 own a dog and, of those, 5 also own a cat. A dog owner is chosen. Find the probability they also own a cat. [2]

2.2 In a Venn diagram, $P(A) = 0.5$ and $P(A \cap B) = 0.2$. Find $P(B \mid A)$. [2]

2.3 A two-way table shows 40 students; 24 walk to school and 9 of those are girls. A walker is chosen. Find the probability the walker is a girl. [2]

3 Exam-style questions

3.1 A box has 5 red and 3 blue pens. Two are taken **without replacement**.

(a) Find the probability that the first is red and the second is blue. [2]

(b) Find the probability that both are the same colour. [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

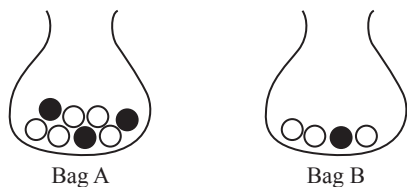
2.1 look only at the 12 dog owners: $\frac{5}{12}$.

2.2 $P(B | A) = \frac{P(A \cap B)}{P(A)} = \frac{0.2}{0.5} = 0.4$.

2.3 look only at the 24 walkers: $\frac{9}{24} = \frac{3}{8}$.

3.1 (a) $\frac{5}{8} \times \frac{3}{7} = \frac{15}{56}$. (b) $P(\text{red, red}) = \frac{5}{8} \times \frac{4}{7} = \frac{20}{56}$; $P(\text{blue, blue}) = \frac{3}{8} \times \frac{2}{7} = \frac{6}{56}$; total $= \frac{26}{56} = \frac{13}{28}$.

20



Bag A contains 5 white balls and 3 black balls.
Bag B contains 3 white balls and 1 black ball.

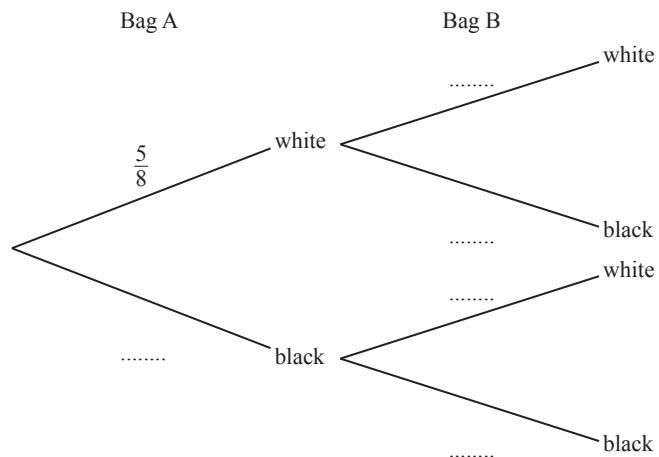
- (a) Two balls are picked at random from bag B without replacement.

Find the probability that both balls are black.

..... [1]

- (b) The balls are replaced into bag B.
Kyle picks a ball at random from each bag.

- (i) Complete the tree diagram.



[2]

- (ii) Find the probability that the two balls are the same colour.

- (c) The balls are replaced into their bags.
Jo picks a ball at random from bag A and places it into bag B.
She then picks a ball at random from bag B.

Find the probability that she picks a black ball from bag B.

..... [3]

11 Sarah rolls a fair 6-sided dice twice.
Find the probability she rolls a number greater than 4 both times.

..... [2]

12 (a) Write down the value of 73^0 .

..... [1]

(b) Find the value of $\frac{7}{3^{-2}}$.

..... [2]

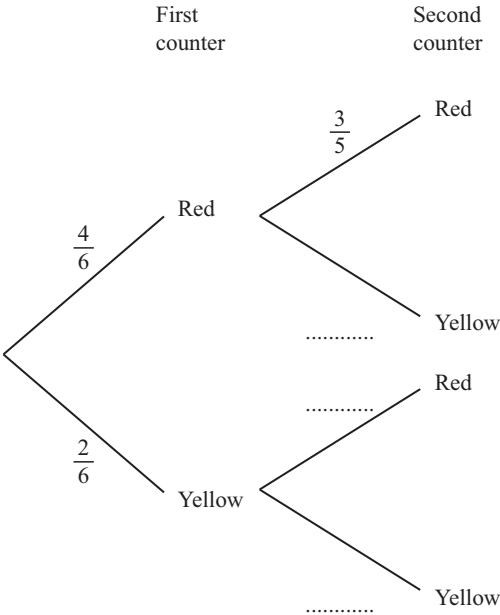
(c) Write 27×81^2 in the form 3^n .

..... [2]

17 (a) A bag contains 6 red marbles, 3 green marbles and 1 blue marble.
Two marbles are picked at random from the bag **with replacement**.
Find the probability that both marbles are green.

..... [2]

(b) Another bag contains 4 red counters and 2 yellow counters.
Two counters are picked at random from this bag **without replacement**.
(i) Complete the tree diagram.



[2]

(ii) Find the probability that one of the two counters is yellow.

..... [3]