

Week 26

IGCSE Mathematics

Homework bundle for this week

本周作业合集

exercises.lol

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1. What is the probability of an impossible event?

2. The probability of rolling a six is $\frac{1}{6}$. How many sixes are expected in 300 rolls?

3. For two independent events both happening (AND), you:
☐ multiply the probabilities
☐ add the probabilities
☐ subtract them
☐ take the larger one
4. Of 18 French students, 7 also study German. $P(\text{German} \mid \text{French}) = \frac{7}{a}$. What is a ?

5. A bag has 3 red and 5 blue counters. $P(\text{not red}) = \frac{a}{8}$. What is a ?

6. Relative frequency is calculated as:
☐ $(\text{times it happened}) \div (\text{total trials})$
☐ $(\text{total trials}) \div (\text{times it happened})$
☐ $\text{times it happened} \times \text{total trials}$
☐ always $\frac{1}{2}$
7. On the 6 by 6 grid of two dice (36 cells), how many cells give a total of 7?

8. Conditional probability is the probability of an event:
☐ given that another event has already happened
☐ that is impossible
☐ on a scale from 1 to 10
☐ multiplied by 100
9. A certain event has probability 100.
☐ True ☐ False

10. Relative frequency becomes a better estimate as the number of trials increases.

☐ True ☐ False

1. **0**

Impossible events have probability 0; certain events have probability 1.

2. **50**

$$(1/6) \times 300 = 50.$$

3. **multiply the probabilities**

$P(A \text{ and } B) = P(A) \times P(B)$ for independent events.

4. **18**

Given they are French, the total to divide by is the 18 French students.

5. **5**

$$P(\text{red}) = 3/8, \text{ so } P(\text{not red}) = 1 - 3/8 = 5/8; a = 5.$$

6. **(times it happened) $\div$ (total trials)**

Relative frequency = successes $\div$ number of trials.

7. **6**

$$1+6, 2+5, 3+4, 4+3, 5+2, 6+1 \text{ — six cells, so } P(7) = 6/36 = 1/6.$$

8. **given that another event has already happened**

It is the probability of B given that A has occurred.

9. **False**

A certain event has probability 1 (or 100%), not 100. Probability is between 0 and 1.

10. **True**

More trials $\rightarrow$ the relative frequency converges to the true probability.

E8.1 Introduction to probability

Name: _____ Class: _____ Date: _____

Total: 9 marks

KEY WORDS probability 概率 • probability scale 概率标度 • outcome 结果 • expected frequency 期望频数
• tree diagrams 树状图

Objective

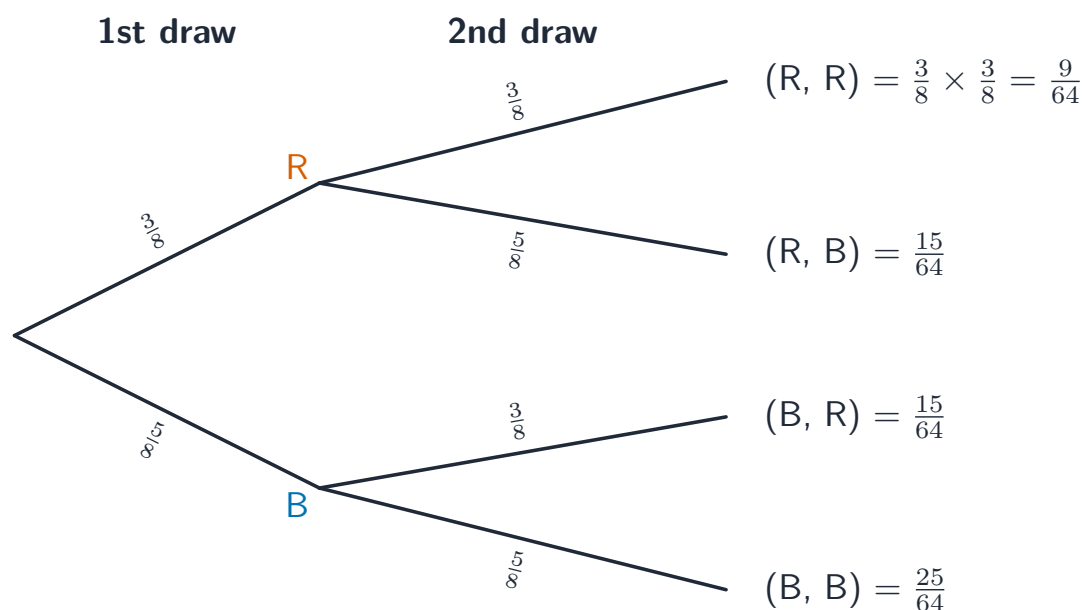
Build the skills to answer Extended exam questions on **probability** 概率 — the **probability scale** 概率标度, $P(A')$, **expected frequency** 期望频数, and **tree diagrams** 树状图.

You must be able to:

- find a probability and use $P(A') = 1 - P(A)$
- find an expected frequency
- use a tree diagram for combined events (with and without replacement)

1 Worked examples

■ Probability and tree diagrams



Probability runs from 0 (impossible) to 1 (certain)

		die 2					
+		1	2	3	4	5	6
die 1	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

the six highlighted cells all give a total of 7

A sample space lists all equally likely outcomes

$$P(\text{event}) = \frac{\text{favourable}}{\text{total}} \quad P(A') = 1 - P(A) \quad \text{expected} = P \times n$$

- **Worked (bag: 3 red, 5 blue):** $P(\text{not red}) = 1 - \frac{3}{8} = \frac{5}{8}$.
- **Tree (with replacement):** $P(\text{red, red}) = \frac{3}{8} \times \frac{3}{8} = \frac{9}{64}$.

Answer shapes: **M1** a correct method, **A1** the correct answer.

2 Practice

2.1 A bag has 4 red and 6 green counters. Find the probability of drawing a red. [1]

2.2 The probability of rain is 0.3. Find the probability of no rain. [1]

2.3 The probability of a six is $\frac{1}{6}$. Find the expected number of sixes in 240 rolls. [2]

3 Exam-style questions

3.1 A bag has 3 red and 5 blue counters. Two are drawn **without replacement**. Find the probability that both are red. [3]

3.2 The probability that a train is late is 0.15. On 200 days, find the expected number of late days. [2]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $\frac{4}{10} = \frac{2}{5}$.

2.2 $1 - 0.3 = 0.7$.

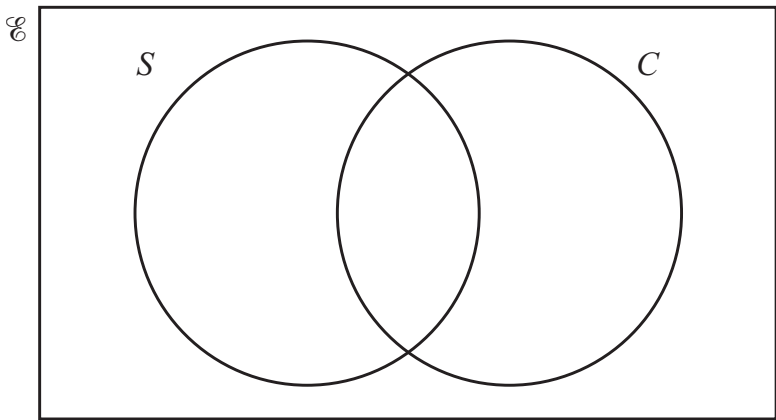
2.3 $\frac{1}{6} \times 240 = 40$.

3.1 first red $\frac{3}{8}$; after a red is taken, 2 reds out of 7 remain $\rightarrow \frac{2}{7}$. $P(\text{red, red}) = \frac{3}{8} \times \frac{2}{7} = \frac{6}{56} = \frac{3}{28}$.

3.2 $0.15 \times 200 = 30$ days.

12 A fitness club has 100 members.

- 60 swim (S).
- 70 cycle (C).
- 25 do not swim or cycle.



(a) Complete the Venn diagram.

[3]

(b) One member of the fitness club is chosen at random.

For this member, find

(i) $P(C)$

..... [1]

(ii) $P(S \cap C)$

..... [1]

(iii) $P(S \cup C')$.

..... [1]

25 Mahir picks one number at random from the numbers 5, 10 and 15. He then picks one number at random from the numbers 4, 5 and 6. He adds the two numbers.

The sample space diagram shows some of the possible outcomes.

		First number			
		+	5	10	15
Second number	4			14	19
	5			15	20
	6			16	21

- (a) Complete the sample space diagram. [1]
- (b) Given that the total of the two numbers is odd, find the probability that one of the numbers added is 15.

..... [2]

26 Write as a single fraction in its simplest form.

(a) $\frac{mp}{25y} \times \frac{15}{m}$

..... [2]

(b) $\frac{3}{2x-5} + \frac{4}{x-3}$

4

1

2

3

4

5

6

7

Samira picks one of these cards at random and replaces it.

(a) Find the probability that she picks an odd number.

..... [1]

(b) Samira repeats this 35 times.

Calculate the number of times Samira is expected to pick an odd number.

..... [1]

E8.4 Conditional probability

Name: _____ Class: _____ Date: _____

Total: 11 marks

KEY WORDS conditional probability 条件概率 • probability 概率 • tree diagram 树状图 • without replacement 无放回

• venn diagram 维恩图

Objective

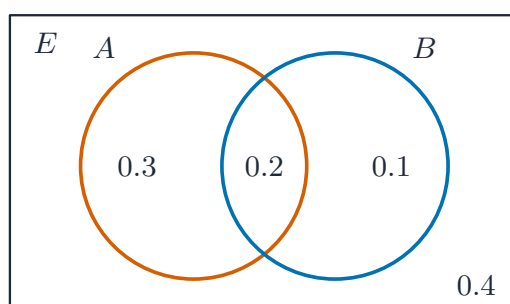
Build the skills to answer Extended exam questions on **conditional probability** 条件概率 — the probability of one event **given that** another has happened, read from a Venn diagram, a two-way table or a tree diagram.

You must be able to:

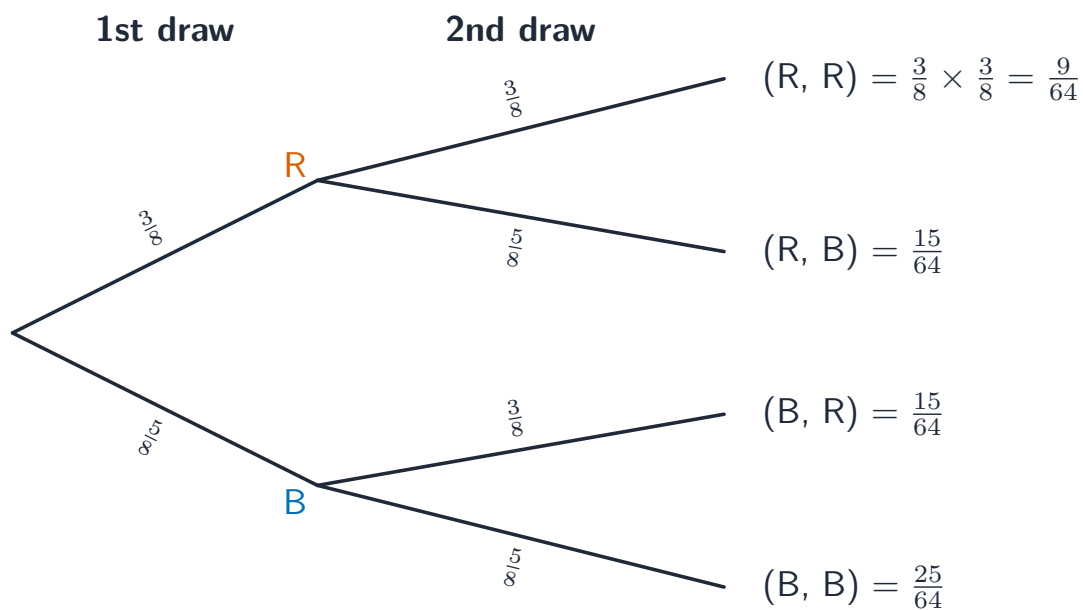
- find a conditional probability from a Venn diagram or table
- restrict attention to the part that matches the condition
- combine with without-replacement reasoning

1 Worked examples

■ Conditional probability



“Given that” means look only at the matching part



A tree diagram handles conditional probability

- **Worked:** 30 students, 18 study French, of whom 7 also study German.
- A French student is picked $\rightarrow P(\text{German} \mid \text{French}) = \frac{7}{18}$ (look only at the 18 French students).

Answer shapes: **M1** a correct method, **A1** the correct answer.

2 Practice

2.1 Of 20 people, 12 own a dog and, of those, 5 also own a cat. A dog owner is chosen. Find the probability they also own a cat. [2]

2.2 In a Venn diagram, $P(A) = 0.5$ and $P(A \cap B) = 0.2$. Find $P(B \mid A)$. [2]

2.3 A two-way table shows 40 students; 24 walk to school and 9 of those are girls. A walker is chosen. Find the probability the walker is a girl. [2]

3 Exam-style questions

3.1 A box has 5 red and 3 blue pens. Two are taken **without replacement**.

(a) Find the probability that the first is red and the second is blue. [2]

(b) Find the probability that both are the same colour. [3]

Solutions

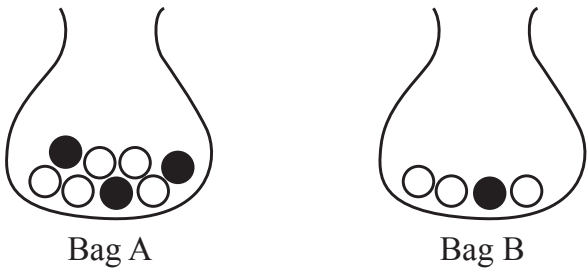
Marking: **M1** a correct method, **A1** the correct answer.

2.1 look only at the 12 dog owners: $\frac{5}{12}$.

2.2 $P(B | A) = \frac{P(A \cap B)}{P(A)} = \frac{0.2}{0.5} = 0.4$.

2.3 look only at the 24 walkers: $\frac{9}{24} = \frac{3}{8}$.

3.1 (a) $\frac{5}{8} \times \frac{3}{7} = \frac{15}{56}$. (b) $P(\text{red, red}) = \frac{5}{8} \times \frac{4}{7} = \frac{20}{56}$; $P(\text{blue, blue}) = \frac{3}{8} \times \frac{2}{7} = \frac{6}{56}$; total
 $= \frac{26}{56} = \frac{13}{28}$.



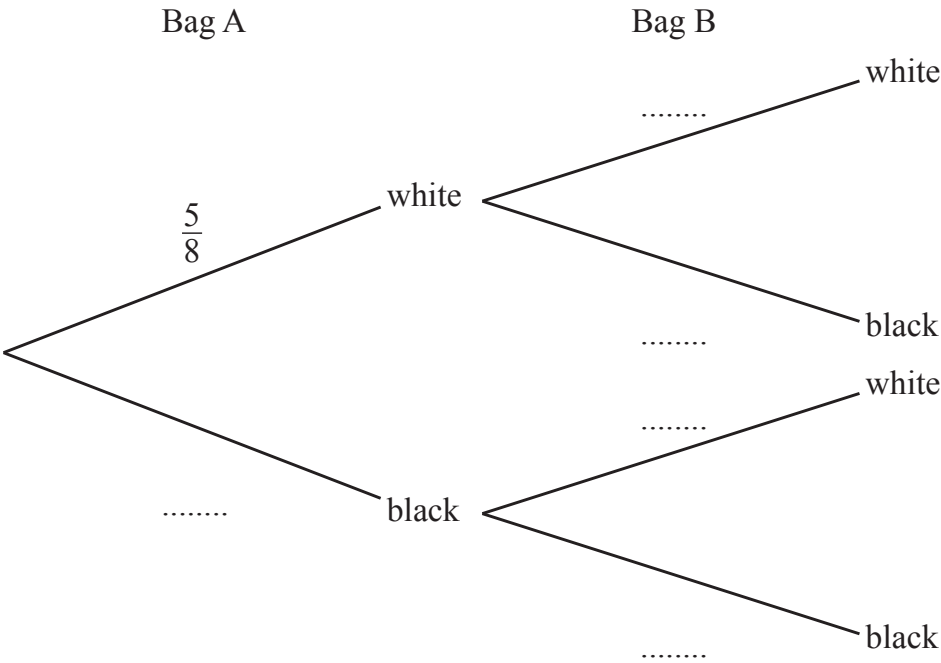
Bag A contains 5 white balls and 3 black balls.
Bag B contains 3 white balls and 1 black ball.

- (a) Two balls are picked at random from bag B without replacement.
- Find the probability that both balls are black.

..... [1]

- (b) The balls are replaced into bag B.
Kyle picks a ball at random from each bag.

- (i) Complete the tree diagram.



[2]

- (ii) Find the probability that the two balls are the same colour.

- (c) The balls are replaced into their bags.
Jo picks a ball at random from bag A and places it into bag B.
She then picks a ball at random from bag B.

Find the probability that she picks a black ball from bag B.

..... [3]

11 Sarah rolls a fair 6-sided dice twice.

Find the probability she rolls a number greater than 4 both times.

..... [2]

12 (a) Write down the value of 73^0 .

..... [1]

(b) Find the value of $\frac{7}{3^{-2}}$.

..... [2]

(c) Write 27×81^2 in the form 3^n .

..... [2]

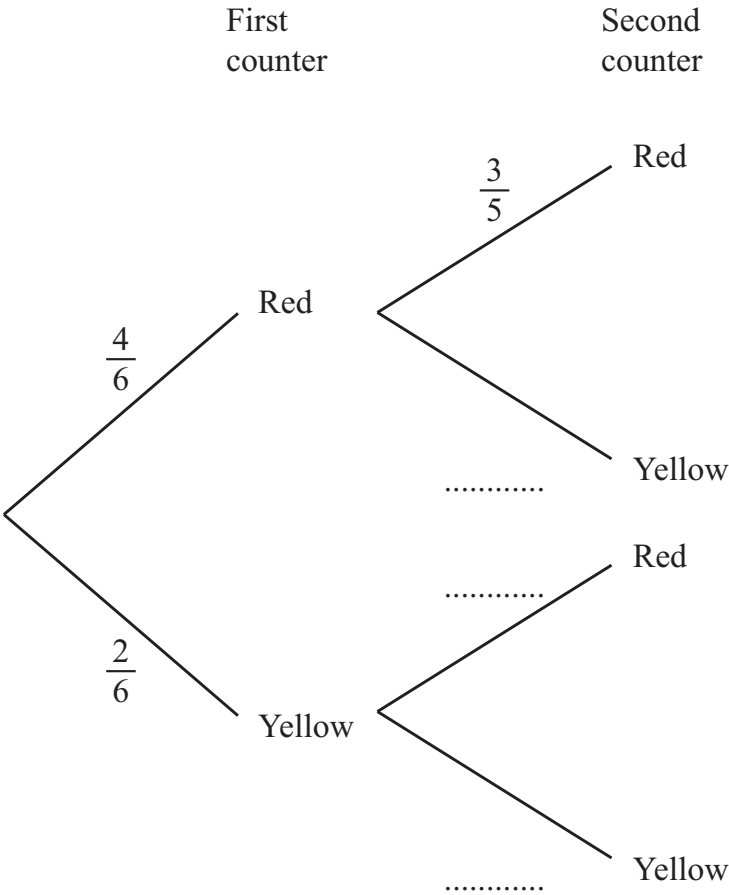
17 (a) A bag contains 6 red marbles, 3 green marbles and 1 blue marble.
Two marbles are picked at random from the bag **with replacement**.

Find the probability that both marbles are green.

..... [2]

- (b) Another bag contains 4 red counters and 2 yellow counters.
Two counters are picked at random from this bag **without replacement**.

(i) Complete the tree diagram.



[2]

- (ii) Find the probability that one of the two counters is yellow.