Week 24

IGCSE Mathematics

Homework bundle for this week

本周作业合集

exercises.lol

Contents 目录

IGCSE Mathematics Name: ______________ Score: ______

1. Reflect the point (3, 2) in the y-axis. The image is (a, 2). What is a?

   ____

2. a = (3, 1) and b = (2, −4). The top number of a + b is?

   ____

3. If a and b are the position vectors of A and B, then the vector AB is:

   - ☐ b − a
   - ☐ a − b
   - ☐ a + b
   - ☐ ½(a + b)

4. Translate (5, 3) by the vector (−2, 4). The image is (3, b). What is b?

   ____

5. a = (3, 1) and b = (2, −4). The bottom number of a + b is?

   ____

6. M is the midpoint of AB. The position vector OM is:

   - ☐ ½(a + b)
   - ☐ a + b
   - ☐ b − a
   - ☐ ½(b − a)

7. Which transformation changes the size of a shape?

   - ☐ enlargement
   - ☐ reflection
   - ☐ rotation
   - ☐ translation

8. Find the magnitude (length) of the vector (3, 4).

   ____

9. Two vectors are parallel if one is a scalar multiple of the other.

   ☐ True ☐ False

10. The magnitude of the vector (3, 4) is $3 + 4 = 7$.

☐ True ☐ False

7. Transformations and vectors

IGCSE Mathematics

1. **-3**

   Reflecting in the y-axis flips the sign of x: $a = -3$.

2. **5**

   $3 + 2 = 5$.

3. **b − a**

   AB = (position of B) − (position of A) = b − a.

4. **7**

   Move 4 up: $3 + 4 = 7$.

5. **-3**

   $1 + (-4) = -3$.

6. **½(a + b)**

   OM = a + ½(b − a) = ½(a + b).

7. **enlargement**

   Only enlargement changes size; the other three keep it congruent.

8. **5**

   $\sqrt{(3^2 + 4^2)} = \sqrt{25} = 5$.

9. **True**

   Parallel vectors point the same (or opposite) way, so one is k times the other.

10. **False**

    Magnitude uses Pythagoras: $\sqrt{(3^2 + 4^2)} = 5$, not the sum $3 + 4 = 7$.

IGCSE MATHEMATICS • EXERCISE SHEET

E7.1 Transformations

Name: ________ Class: ______ Date: _______ **Total: 26 marks**

KEY WORDS reflection 反射 • translation 平移 • rotation 旋转 • centre 中心 • scale factor 比例因子

Objective

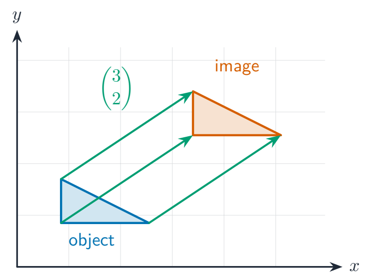
Build the skills to answer Extended exam questions on **transformations** 变换 — **reflection, rotation, enlargement and translation** 反射、旋转、放大、平移 — and describing them fully.

You must be able to:

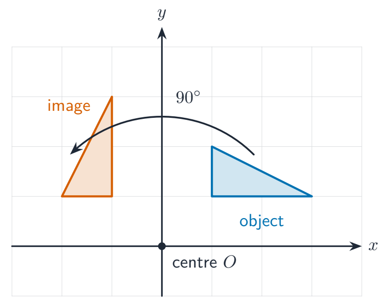
- find the image of a point under each transformation
- describe a transformation with all the required details
- use a column vector for a translation

1 Worked examples

The four transformations



Describe fully: name the type + all its details



Rotation is one of the four transformations

- **Reflection** 反射: give the mirror line. $(3, 2)$ in the y-axis $\to (-3, 2)$.
- **Rotation** 旋转: give centre, angle, direction. $(3, 1)$, $90°$ anticlockwise about $O \to (-1, 3)$.
- **Enlargement** 放大: give centre and scale factor k. $(1, 2)$ from O, $k = 2 \to (2, 4)$.
- **Translation** 平移: give the column vector. $(5, 3)$ by $\begin{pmatrix}-2\\4\end{pmatrix} \to (3, 7)$.

Answer shapes: M1 a correct method, **A1** the correct answer.

Describing a transformation fully

"Describe fully" means give the **name** and every piece of information that fixes it. Miss one and the marks go.

Transformation	What must also be given
reflection	the equation of the mirror line, for example $y = x$ or $x = 2$
rotation	the angle, the direction (clockwise or anticlockwise) and the centre
translation	the column vector $\begin{pmatrix}a\\b\end{pmatrix}$
enlargement	the scale factor and the centre of enlargement

Only **one** transformation may be named: writing "a reflection and then a translation" scores nothing.

How to identify it. If the shape is the same size and the same way round, it is a translation or a rotation; if it is the same size but reversed, it is a reflection; if it is a different size, it is an enlargement. For a rotation, join a point to its image and to a candidate centre and check the angle. For an enlargement, the centre is where the lines through matching pairs of vertices meet, and the scale factor is $\dfrac{\text{image length}}{\text{object length}}$.

Worked example. A has vertices $(1, 2)$, $(4, 2)$, $(1, 4)$ and B has $(2, 4)$, $(8, 4)$, $(2, 8)$. Describe the transformation fully.

1. The image is bigger, so it is an enlargement.
2. Scale factor: the base goes from 3 to 6, so it is 2.
3. Centre: each image coordinate is exactly twice the object coordinate, so the centre is the origin $(0, 0)$.
4. Answer: an enlargement, scale factor 2, centre $(0, 0)$.

A **negative scale factor** puts the image on the other side of the centre and turns it upside down; a **fractional scale factor** makes it smaller.

2 Practice

2.1 Reflect the point $(4, 3)$ in the x-axis. [1]

2.2 Translate the point $(2, 1)$ by the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$. [1]

2.3 Enlarge the point $(2, 3)$ from the origin by scale factor 2. [1]

2.4 Describe fully the transformation that maps $(3, 2)$ onto $(-3, 2)$. [2]

2.5 State the two things you must give to describe a rotation fully, besides its name. [2]

3 Exam-style questions

3.1 The point $(4, 1)$ is rotated $90°$ anticlockwise about the origin. Find the image. [2]

3.2 A quadrilateral has vertices $(2, 1)$, $(5, 1)$, $(5, 3)$ and $(2, 3)$. After one transformation its image has vertices $(6, 3)$, $(15, 3)$, $(15, 9)$ and $(6, 9)$. State the name of this transformation and give the two further pieces of information needed to describe it fully. [3]

3.3 A shape is enlarged from the origin by scale factor $\frac{1}{2}$. The point $(6, 4)$ is on the shape. Find its image. [2]

3.4 Triangle P has vertices $(2,1)$, $(6,1)$ and $(2,3)$.

(a) Triangle P is reflected in the line $x = 1$ to give triangle Q. Write the coordinates of the vertices of Q. [3]

(b) Triangle P is enlarged with centre $(0,0)$ and scale factor $\frac{3}{2}$ to give triangle R. Write the coordinates of the vertices of R. [3]

(c) Triangle P is rotated 90° clockwise about the origin to give triangle S. Write the coordinates of the vertices of S. [3]

(d) Triangle T has vertices $(-2,-1)$, $(-6,-1)$ and $(-2,-3)$. Describe fully the single transformation that maps P onto T. [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $(4,-3)$.

2.2 $(2+3,\ 1-2) = (5,-1)$.

2.3 $(2\times 2,\ 3\times 2) = (4,6)$.

2.4 A reflection in the y-axis, the line $x = 0$.

2.5 The angle of turn and the direction, and the centre of rotation.

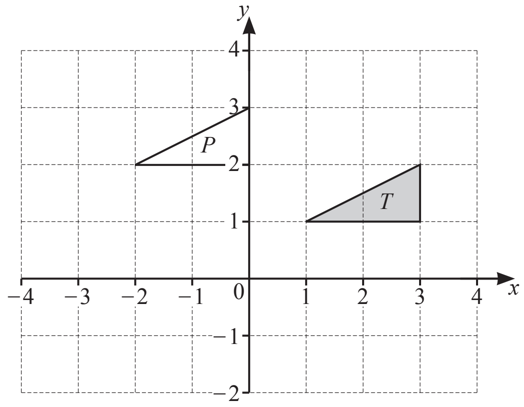
3.1 the rule is $(x,y) \to (-y,x)$, so $(4,1) \to (-1,4)$.

3.2 An enlargement, scale factor 3, centre $(0,0)$. (Each image coordinate is three times the object coordinate, so the centre is the origin.)

3.3 $\left(6\times\frac{1}{2},\ 4\times\frac{1}{2}\right) = (3,2)$.

3.4 (a) Reflecting in $x = 1$ sends (x,y) to $(2-x,y)$; $(0,1)$, $(-4,1)$ and $(0,3)$.
(b) Multiply each coordinate by $\frac{3}{2}$; $(3,1.5)$, $(9,1.5)$ and $(3,4.5)$.
(c) A 90° clockwise rotation about the origin sends (x,y) to $(y,-x)$; $(1,-2)$, $(1,-6)$ and $(3,-2)$.
(d) A rotation of 180° about the origin $(0,0)$. (An enlargement of scale factor -1, centre the origin, is also accepted.)

5



(a) Describe fully the **single** transformation that maps triangle T onto triangle P.

....

.... [2]

(b) Draw the image of triangle T after an enlargement of scale factor 2, centre (3, 3). [2]

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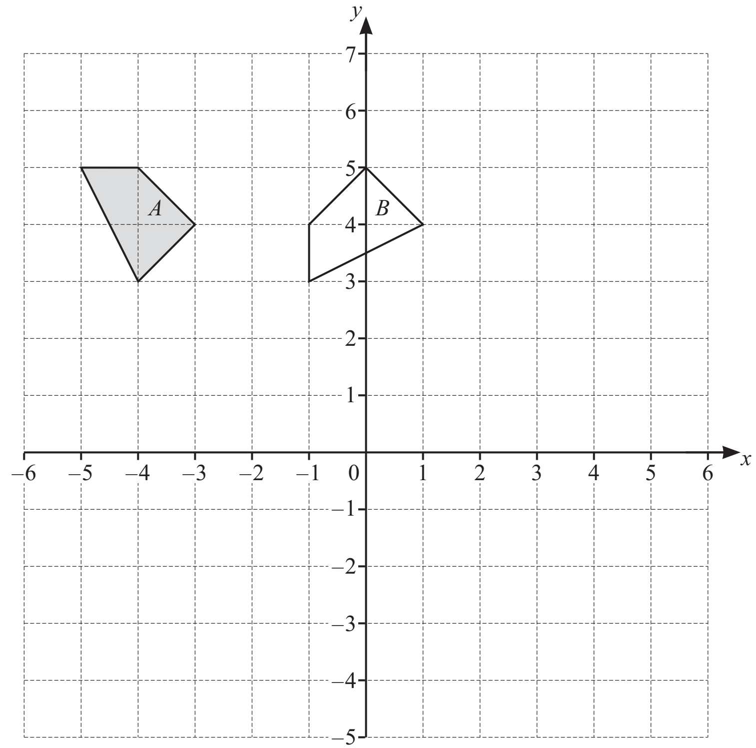

(a) On the diagram, draw the image of

(i) shape A after a translation by the vector $\begin{pmatrix} 1 \\ -7 \end{pmatrix}$ [2]

(ii) shape A after a reflection in the line $y = x + 1$. [3]

(b) Describe fully the **single** transformation that maps shape A onto shape B.

....

.... [3]

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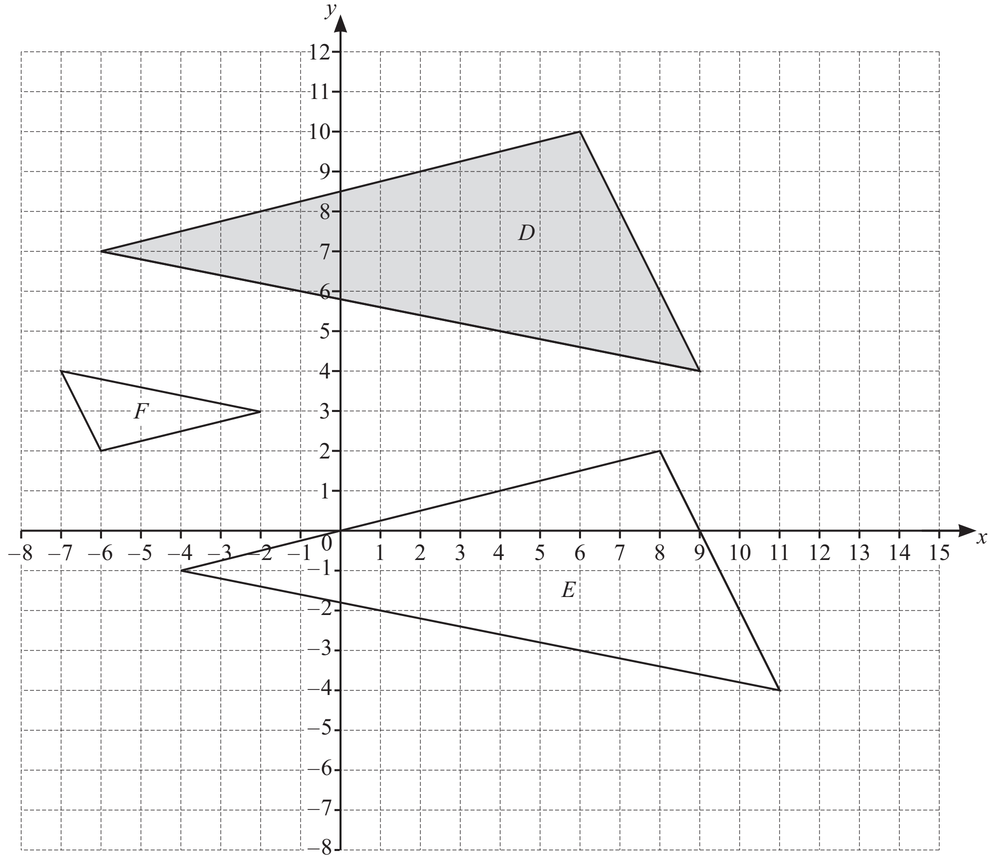


(a) Describe fully the **single** transformation that maps triangle D onto triangle E.

....

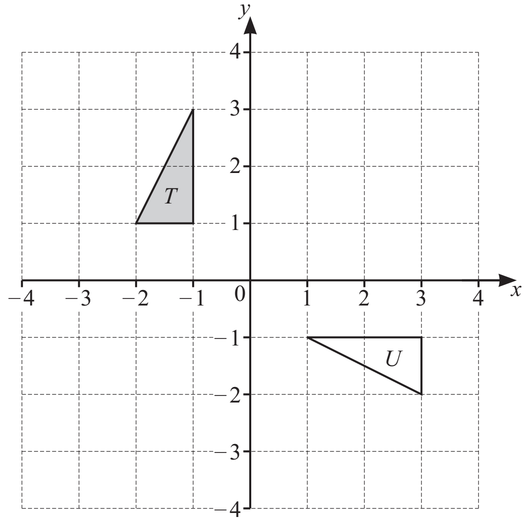
.... [2]

(b) Describe fully the **single** transformation that maps triangle D onto triangle F.

....

.... [3]

5



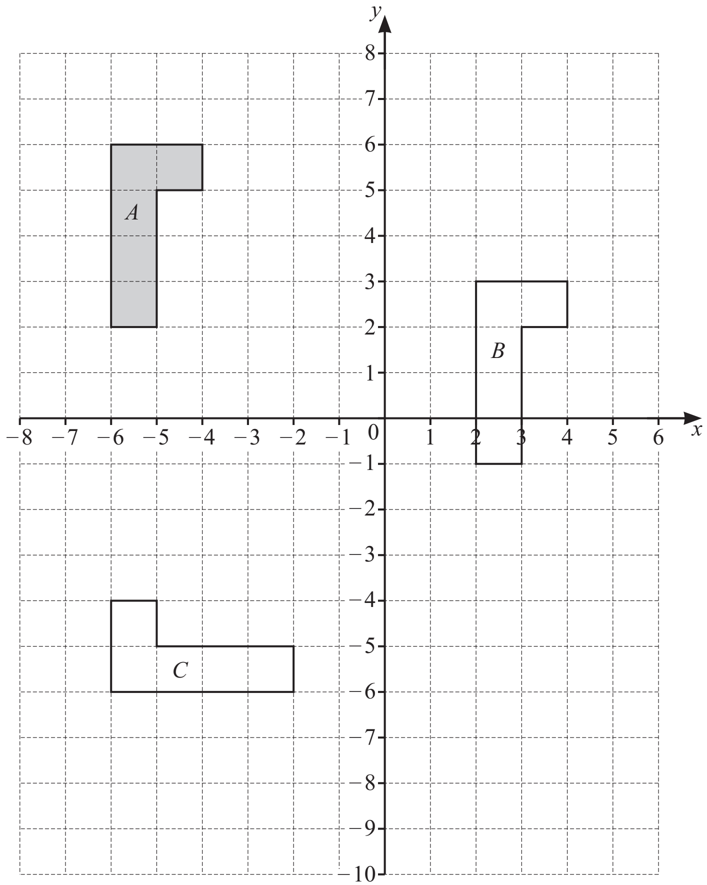
(a) Translate triangle T by the vector $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$. [1]

(b) Describe fully the **single** transformation that maps triangle T onto triangle U.

....

.... [3]

4



(a) Describe fully the **single** transformation that maps

(i) shape A onto shape B

....

.... [2]

(ii) shape A onto shape C.

....

.... [3]

(b) On the grid, draw the image of shape A after a reflection in the line $x = -2$. [2]

IGCSE MATHEMATICS • EXERCISE SHEET

E7.2 Vectors in two dimensions

Name: ____________ Class: ________ Date: __________ **Total: 21 marks**

KEY WORDS magnitude 模 • column vectors 列向量 • position vectors 位置向量 • vectors in two dimensions 二维向量 • scalar 标量

Objective

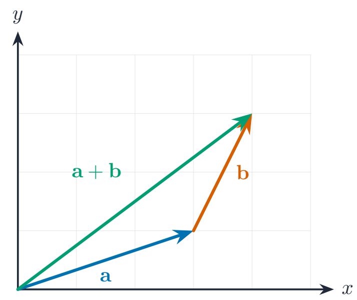
Build the skills to answer Extended exam questions on **vectors in two dimensions** 二维向量 — adding, subtracting and scaling **column vectors** 列向量, the **magnitude** 模, and **position vectors** 位置向量.

You must be able to:

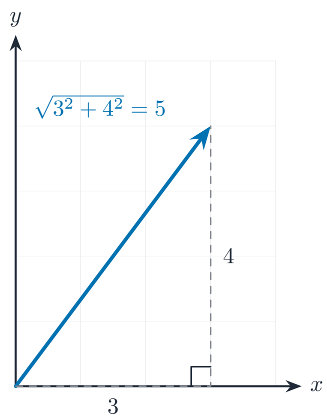
- add, subtract and multiply column vectors by a scalar
- find the magnitude of a vector
- use $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$

1 Worked examples

■ Working with vectors



Work on the top and bottom numbers separately



The magnitude (length) of a vector

- **Add/scale:** $\mathbf{a}=\begin{pmatrix}3\\1\end{pmatrix}$, $\mathbf{b}=\begin{pmatrix}2\\-4\end{pmatrix}\rightarrow\mathbf{a}+\mathbf{b}=\begin{pmatrix}5\\-3\end{pmatrix}$; $3\mathbf{a}=\begin{pmatrix}9\\3\end{pmatrix}$.
- **Magnitude 模:** $\left|\begin{pmatrix}x\\y\end{pmatrix}\right|=\sqrt{x^2+y^2}\rightarrow\left|\begin{pmatrix}3\\4\end{pmatrix}\right|=5$.
- **Position vectors:** $\overrightarrow{AB}=\mathbf{b}-\mathbf{a}$.

Answer shapes: M1 a correct method, **A1** the correct answer.

2 Practice

2.1 $\mathbf{a}=\begin{pmatrix}4\\2\end{pmatrix}$ and $\mathbf{b}=\begin{pmatrix}1\\-3\end{pmatrix}$. Find $\mathbf{a}+\mathbf{b}$. [1]

2.2 Find $2\mathbf{a}$ for $\mathbf{a}=\begin{pmatrix}3\\-1\end{pmatrix}$. [1]

2.3 Find the magnitude of $\begin{pmatrix}6\\8\end{pmatrix}$. [2]

3 Exam-style questions

3.1 $\mathbf{a}=\begin{pmatrix}5\\2\end{pmatrix}$ and $\mathbf{b}=\begin{pmatrix}1\\6\end{pmatrix}$. Find $\mathbf{a}-\mathbf{b}$. [2]

3.2 The position vector of A is $\begin{pmatrix}1\\3\end{pmatrix}$ and of B is $\begin{pmatrix}7\\11\end{pmatrix}$. Find $\overrightarrow{AB}$ and its magnitude. [3]

3.3 $\mathbf{a} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$.

(a) Work out $2\mathbf{a} - \mathbf{b}$ as a column vector. [2]

(b) Find $|\mathbf{a}|$, giving your answer in surd form and to 2 decimal places. [3]

(c) $OABC$ is a parallelogram with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{b}$. Write $\overrightarrow{AC}$ and $\overrightarrow{OB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [3]

(d) M is the midpoint of AB. Show that $\overrightarrow{OM} = \mathbf{a} + \frac{1}{2}\mathbf{b}$, and hence write $\overrightarrow{OM}$ as a column vector. [4]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $\begin{pmatrix} 4+1 \\ 2-3 \end{pmatrix} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$.

2.2 $\begin{pmatrix} 6 \\ -2 \end{pmatrix}$.

2.3 $\sqrt{6^2 + 8^2} = \sqrt{100} = 10$.

3.1 $\begin{pmatrix} 5-1 \\ 2-6 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \end{pmatrix}$.

3.2 $\overrightarrow{AB} = \mathbf{b} - \mathbf{a} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}$; magnitude $= \sqrt{6^2 + 8^2} = 10$.

3.3 (a) $2\mathbf{a} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$; $2\mathbf{a} - \mathbf{b} = \begin{pmatrix} 6-(-2) \\ -2-4 \end{pmatrix} = \begin{pmatrix} 8 \\ -6 \end{pmatrix}$.

(b) $|\mathbf{a}| = \sqrt{3^2 + (-1)^2} = \sqrt{10} = 3.16$ to 2 d.p..

(c) $\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC} = -\mathbf{a} + \mathbf{b}$; in a parallelogram $\overrightarrow{OB} = \mathbf{a} + \mathbf{b}$.

(d) $\overrightarrow{AB} = \overrightarrow{OC} = \mathbf{b}$, so $\overrightarrow{AM} = \frac{1}{2}\mathbf{b}$; $\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM} = \mathbf{a} + \frac{1}{2}\mathbf{b}$; $= \begin{pmatrix} 3 \\ -1 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$.

8 $\overrightarrow{DE} = \begin{pmatrix} \text{[illegible]} \\ -4 \end{pmatrix}$

(a) Find $5\overrightarrow{DE}$.

$\begin{pmatrix} \quad \\ \quad \end{pmatrix}$ [1]

(b) Find $|\overrightarrow{DE}|$.

.... [2]

(c) D is the point (–2, –3).

Find the coordinates of the point E.

(...................... , ...) [2]

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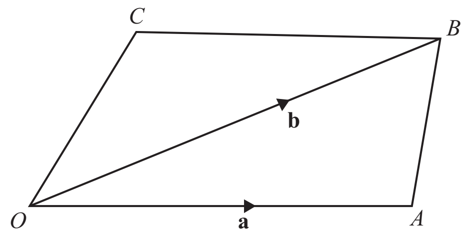

NOT TO SCALE

In the diagram, OA is parallel to CB.
$OA : CB = 4 : 3$
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

(a) Find $\overrightarrow{AB}$ in terms of **a** and **b**.

$\overrightarrow{AB} =$ .... [1]

(b) M is the midpoint of OC.

Find $\overrightarrow{AM}$ in terms of **a** and **b**.
Give your answer in its simplest form.

$\overrightarrow{AM} =$ .... [3]

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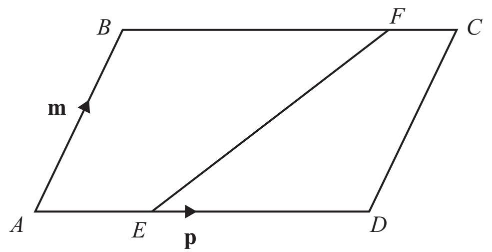

NOT TO SCALE

$ABCD$ is a parallelogram.
$\overrightarrow{AB} = \mathbf{m}$ and $\overrightarrow{AD} = \mathbf{p}$.
F is a point on BC and $BF = 4FC$.
E is a point on AD and $AE : ED = 1 : 2$.

(a) Find $\overrightarrow{EF}$, in terms of $\mathbf{m}$ and $\mathbf{p}$, in its simplest form.

.... [3]

(b) EF and DC are extended to meet at the point G.

Find $\overrightarrow{CG}$, in terms of $\mathbf{m}$ and/or $\mathbf{p}$, in its simplest form.

8 $\mathbf{m} = \begin{pmatrix} \text{[illegible]} \\ 5 \end{pmatrix}$ $\mathbf{n} = \begin{pmatrix} \text{[illegible]} \\ -3 \end{pmatrix}$

(a) Find $2\mathbf{m} - \mathbf{n}$.

$\begin{pmatrix} \\ \end{pmatrix}$ [2]

(b) The vector $\begin{pmatrix} 5 \\ \sqrt{y} \end{pmatrix}$ has a magnitude of 7.

Find the value of y.

$y =$ .... [2]

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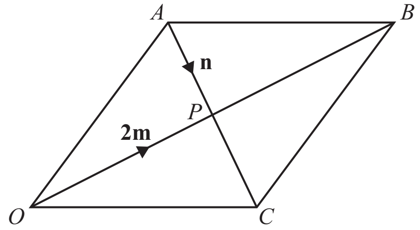

NOT TO SCALE

OABC is a rhombus and *O* is the origin.
The diagonals of the rhombus intersect at *P*.
$\overrightarrow{OP} = 2\mathbf{m}$ and $\overrightarrow{AP} = \mathbf{n}$.

(a) Find, in terms of **m** and **n**, in its simplest form

(i) $\overrightarrow{OA}$

$\overrightarrow{OA} =$ .... [1]

(ii) $\overrightarrow{OC}$.

$\overrightarrow{OC} =$ .... [1]

(b) *D* is the point such that $\overrightarrow{AD} = 10\mathbf{m} - 3\mathbf{n}$.

Show that *OADC* is a trapezium.

[3]

9 *A* is the point $(3, -1)$.

$\overrightarrow{AB} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$

(a) $\overrightarrow{AC} = 2\overrightarrow{AB}$

Find the coordinates of the point *C*.

(.................. , ...) [2]

(b) The length of *AB* is $k\sqrt{5}$.

Find the value of *k*.

$k =$ .... [2]

(c) *P* is a point on *AB*.

$AP : PB = 1 : 3$

Find the position vector of *P*.

$\begin{pmatrix} \quad \\ \quad \end{pmatrix}$ [2]