

Week 24

IGCSE Mathematics

Homework bundle for this week

本周作业合集

exercises.lol

Contents 目录

Topic 7 quiz	2
Exercise sheet E7.1	5
Past-paper questions E7.1	9
Exercise sheet E7.2	14
Past-paper questions E7.2	18

1. Reflect the point $(3, 2)$ in the y -axis. The image is $(a, 2)$. What is a ?

2. $a = (3, 1)$ and $b = (2, -4)$. The top number of $a + b$ is?

3. If a and b are the position vectors of A and B , then the vector AB is:

☐ $b - a$

☐ $a - b$

☐ $a + b$

☐ $\frac{1}{2}(a + b)$

4. Translate $(5, 3)$ by the vector $(-2, 4)$. The image is $(3, b)$. What is b ?

5. $a = (3, 1)$ and $b = (2, -4)$. The bottom number of $a + b$ is?

6. M is the midpoint of AB . The position vector OM is:

☐ $\frac{1}{2}(a + b)$

☐ $a + b$

☐ $b - a$

☐ $\frac{1}{2}(b - a)$

7. Which transformation changes the size of a shape?

☐ enlargement

☐ reflection

☐ rotation

☐ translation

8. Find the magnitude (length) of the vector $(3, 4)$.

9. Two vectors are parallel if one is a scalar multiple of the other.

☐ True ☐ False

10. The magnitude of the vector $(3, 4)$ is $3 + 4 = 7$.

☐ True ☐ False

1. **-3**

Reflecting in the y-axis flips the sign of x: $a = -3$.

2. **5**

$$3 + 2 = 5.$$

3. **$b - a$**

$$AB = (\text{position of B}) - (\text{position of A}) = b - a.$$

4. **7**

$$\text{Move 4 up: } 3 + 4 = 7.$$

5. **-3**

$$1 + (-4) = -3.$$

6. **$\frac{1}{2}(a + b)$**

$$OM = a + \frac{1}{2}(b - a) = \frac{1}{2}(a + b).$$

7. **enlargement**

Only enlargement changes size; the other three keep it congruent.

8. **5**

$$\sqrt{3^2 + 4^2} = \sqrt{25} = 5.$$

9. **True**

Parallel vectors point the same (or opposite) way, so one is k times the other.

10. **False**

Magnitude uses Pythagoras: $\sqrt{3^2 + 4^2} = 5$, not the sum $3 + 4 = 7$.

E7.1 Transformations

Name: _____ Class: _____ Date: _____

Total: 9 marks

KEY WORDS reflection 反射 • translation 平移 • rotation 旋转 • centre 中心 • scale factor 比例因子

Objective

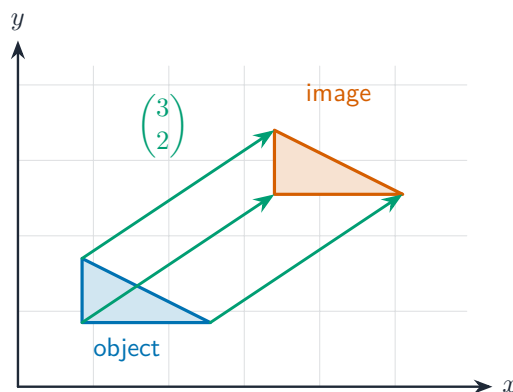
Build the skills to answer Extended exam questions on **transformations** 变换 — **reflection, rotation, enlargement and translation** 反射、旋转、放大、平移 — and describing them fully.

You must be able to:

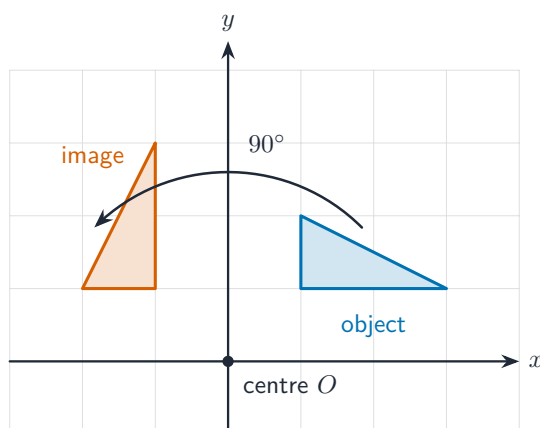
- find the image of a point under each transformation
- describe a transformation with all the required details
- use a column vector for a translation

1 Worked examples

■ The four transformations



Describe fully: name the type + all its details



Rotation is one of the four transformations

- **Reflection** 反射: give the mirror line. $(3, 2)$ in the y -axis $\rightarrow (-3, 2)$.
- **Rotation** 旋转: give centre, angle, direction. $(3, 1)$, 90° anticlockwise about $O \rightarrow (-1, 3)$.
- **Enlargement** 放大: give centre and scale factor k . $(1, 2)$ from O , $k = 2 \rightarrow (2, 4)$.
- **Translation** 平移: give the column vector. $(5, 3)$ by $\begin{pmatrix} -2 \\ 4 \end{pmatrix} \rightarrow (3, 7)$.

Answer shapes: **M1** a correct method, **A1** the correct answer.

2 Practice

2.1 Reflect the point $(4, 3)$ in the x -axis. [1]

2.2 Translate the point $(2, 1)$ by the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$. [1]

2.3 Enlarge the point $(2, 3)$ from the origin by scale factor 2. [1]

3 Exam-style questions

3.1 The point $(4, 1)$ is rotated 90° anticlockwise about the origin. Find the image. [2]

3.2 Describe fully the single transformation that maps the point $(2, 5)$ onto $(2, -5)$. [2]

3.3 A shape is enlarged from the origin by scale factor $\frac{1}{2}$. The point $(6, 4)$ is on the shape. Find its image. [2]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $(4, -3)$.

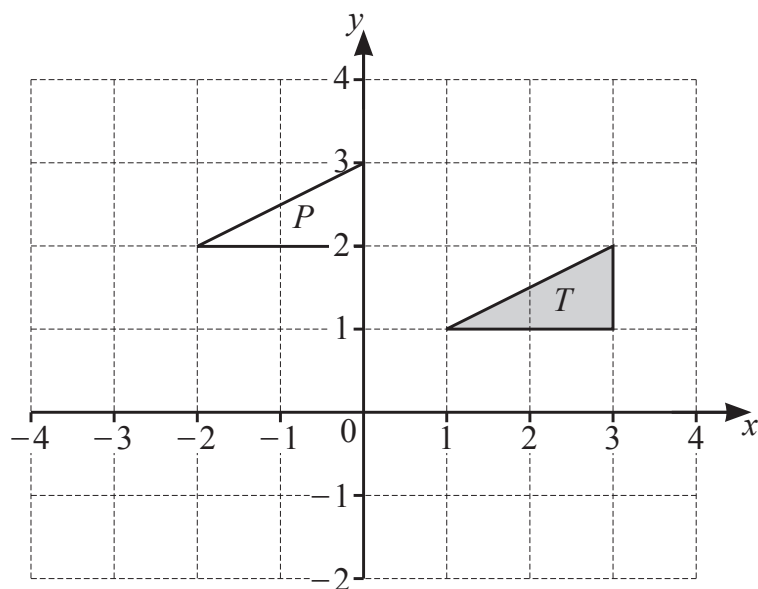
2.2 $(2 + 3, 1 - 2) = (5, -1)$.

2.3 $(2 \times 2, 3 \times 2) = (4, 6)$.

3.1 the rule is $(x, y) \rightarrow (-y, x)$, so $(4, 1) \rightarrow (-1, 4)$.

3.2 a reflection in the x -axis (the y -coordinate changes sign).

3.3 $(6 \times \frac{1}{2}, 4 \times \frac{1}{2}) = (3, 2)$.



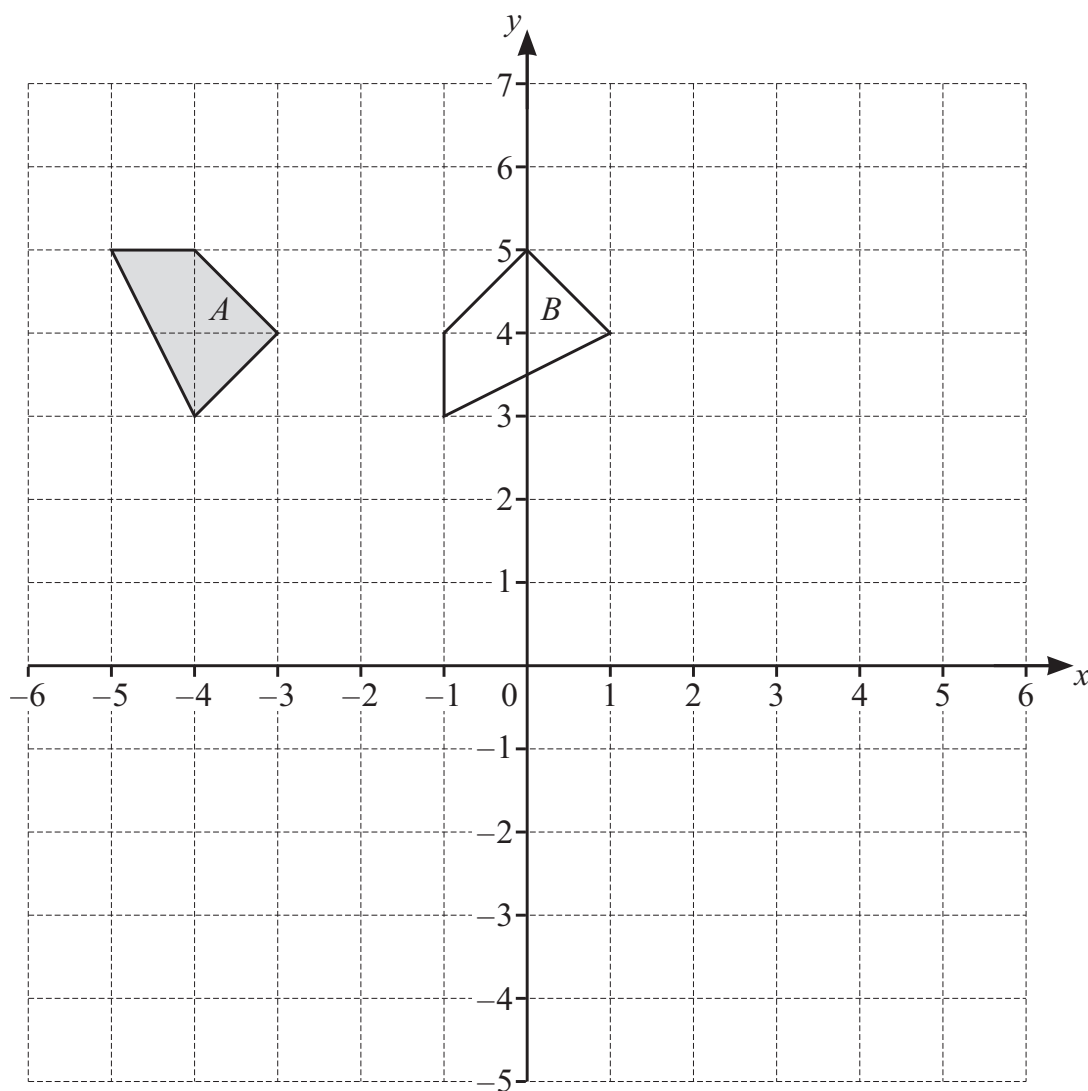
- (a) Describe fully the **single** transformation that maps triangle T onto triangle P .

.....

..... [2]

- (b) Draw the image of triangle T after an enlargement of scale factor 2, centre $(3, 3)$.

[2]



(a) On the diagram, draw the image of

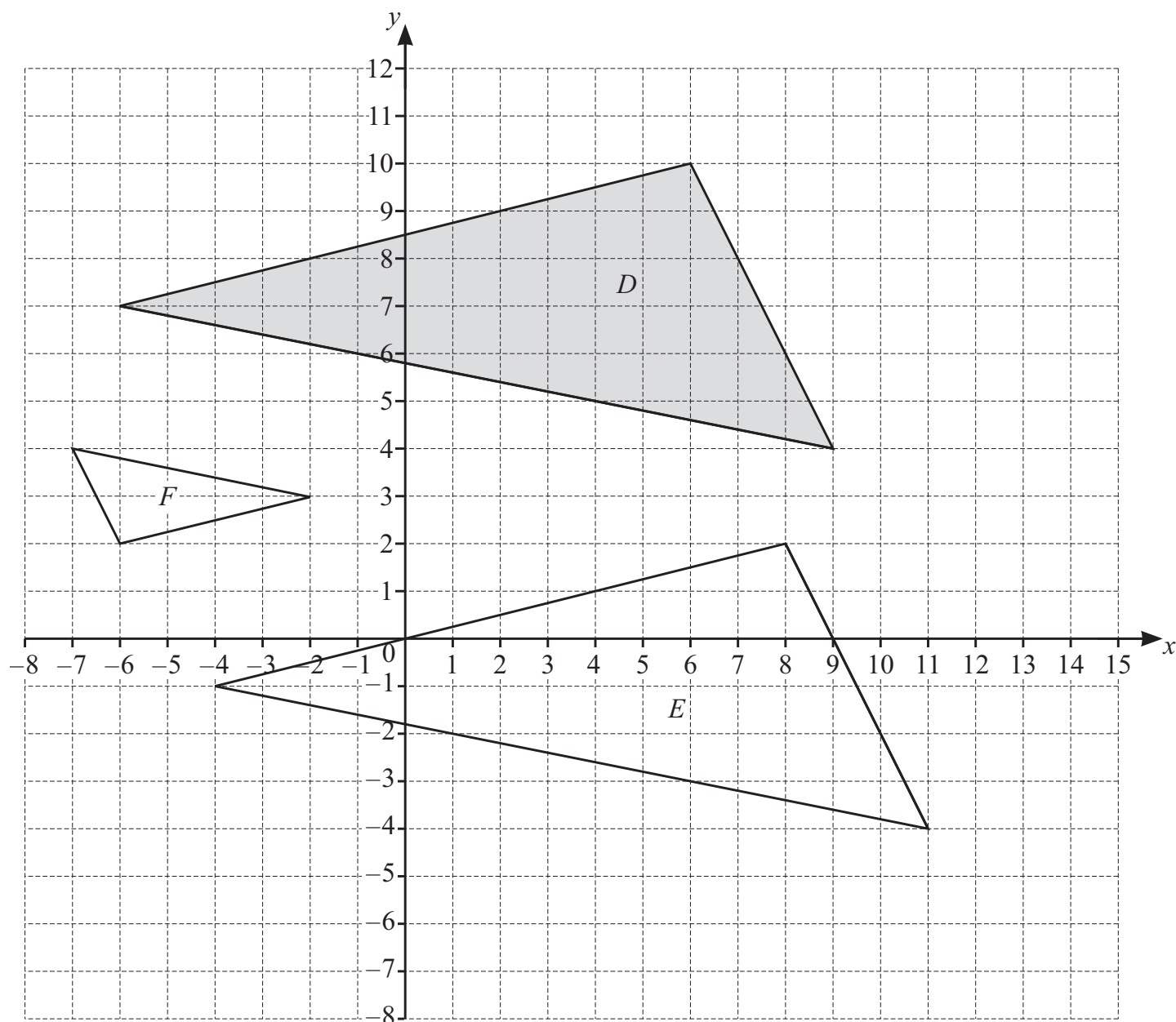
(i) shape A after a translation by the vector $\begin{pmatrix} 1 \\ -7 \end{pmatrix}$ [2]

(ii) shape A after a reflection in the line $y = x + 1$. [3]

(b) Describe fully the **single** transformation that maps shape A onto shape B .

.....

..... [3]

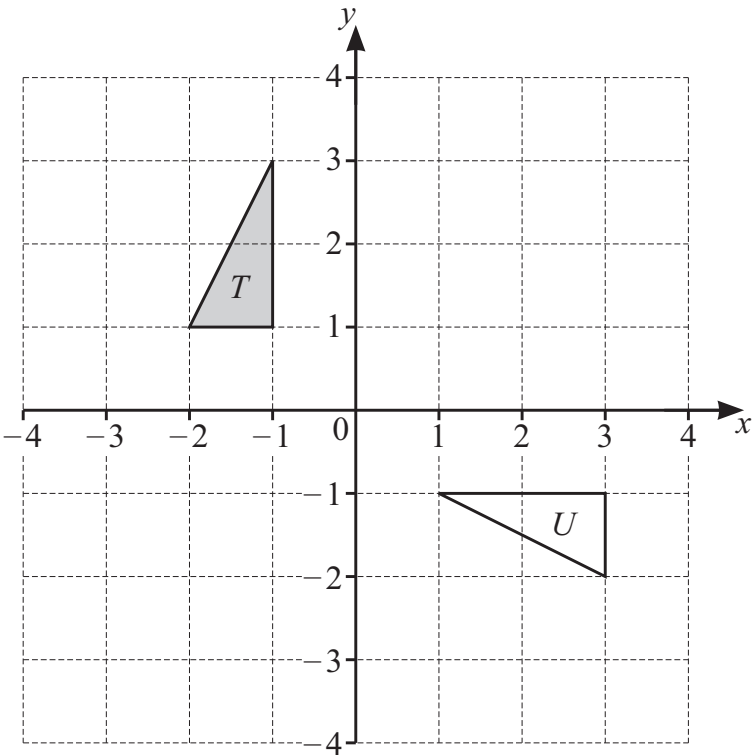


(a) Describe fully the **single** transformation that maps triangle *D* onto triangle *E*.

.....
..... [2]

(b) Describe fully the **single** transformation that maps triangle *D* onto triangle *F*.

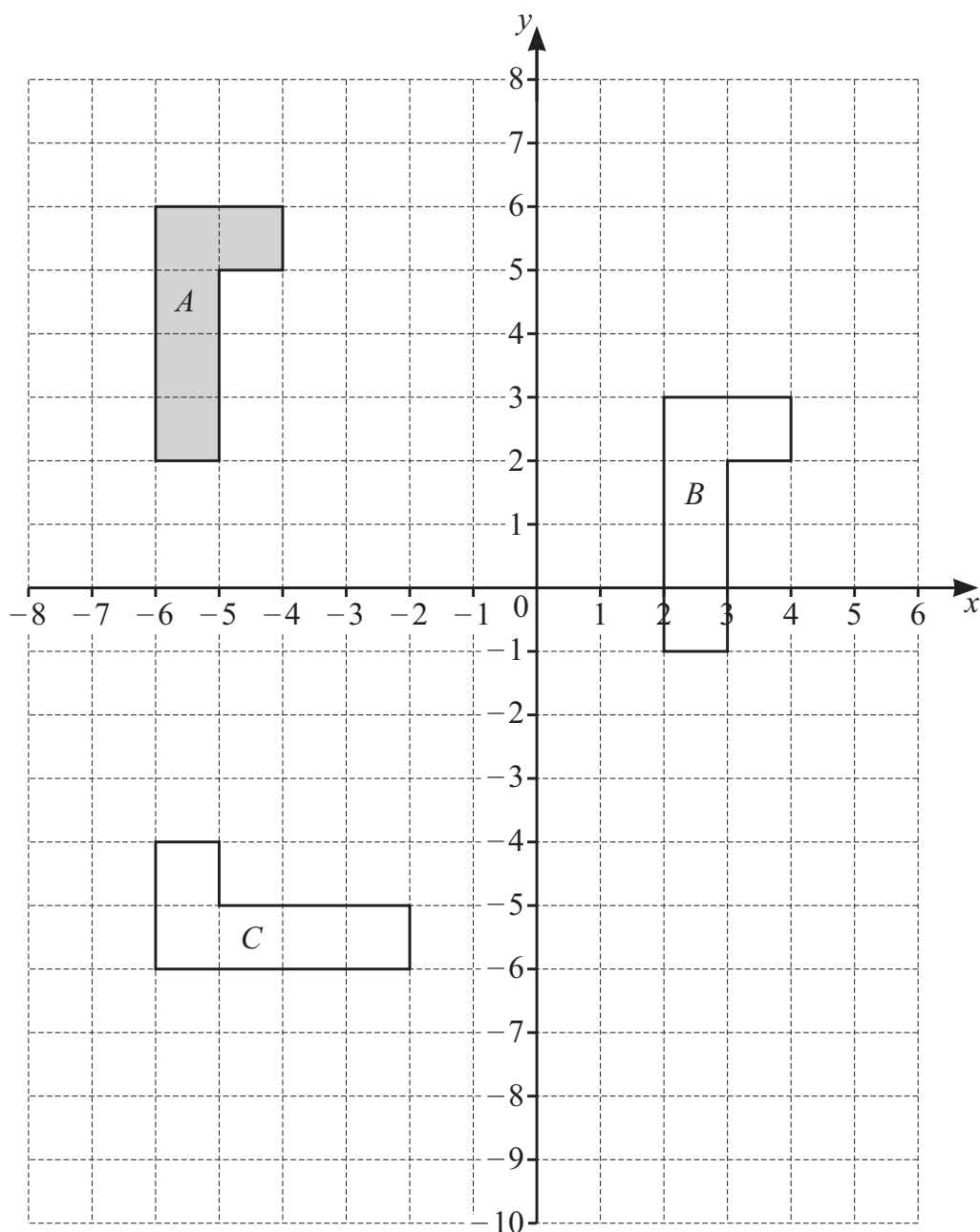
.....
..... [3]



(a) Translate triangle *T* by the vector $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$. [1]

(b) Describe fully the **single** transformation that maps triangle *T* onto triangle *U*.

.....
..... [3]



(a) Describe fully the **single** transformation that maps

(i) shape *A* onto shape *B*

.....
..... [2]

(ii) shape *A* onto shape *C*.

.....
..... [3]

(b) On the grid, draw the image of shape *A* after a reflection in the line $x = -2$. [2]

E7.2 Vectors in two dimensions

Name: _____ Class: _____ Date: _____

Total: 9 marks

KEY WORDS magnitude 模 • column vectors 列向量 • position vectors 位置向量 • vectors in two dimensions 二维向量

- scalar 标量

Objective

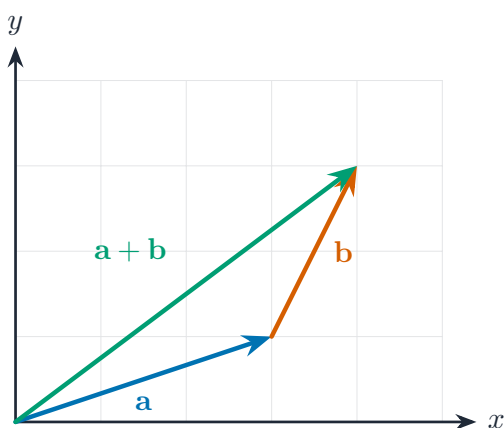
Build the skills to answer Extended exam questions on **vectors in two dimensions** 二维向量 — adding, subtracting and scaling **column vectors** 列向量, the **magnitude** 模, and **position vectors** 位置向量.

You must be able to:

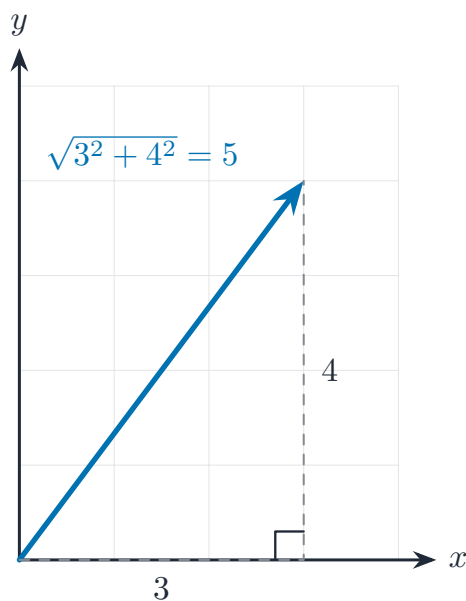
- add, subtract and multiply column vectors by a scalar
- find the magnitude of a vector
- use $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$

1 Worked examples

■ Working with vectors



Work on the top and bottom numbers separately



The magnitude (length) of a vector

- **Add/scale:** $\mathbf{a} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 2 \\ -4 \end{pmatrix} \rightarrow \mathbf{a} + \mathbf{b} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$; $3\mathbf{a} = \begin{pmatrix} 9 \\ 3 \end{pmatrix}$.
- **Magnitude 模:** $\left| \begin{pmatrix} x \\ y \end{pmatrix} \right| = \sqrt{x^2 + y^2} \rightarrow \left| \begin{pmatrix} 3 \\ 4 \end{pmatrix} \right| = 5$.
- **Position vectors:** $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$.

Answer shapes: M1 a correct method, A1 the correct answer.

2 Practice

2.1 $\mathbf{a} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$. Find $\mathbf{a} + \mathbf{b}$. [1]

2.2 Find $2\mathbf{a}$ for $\mathbf{a} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$. [1]

2.3 Find the magnitude of $\begin{pmatrix} 6 \\ 8 \end{pmatrix}$. [2]

3 Exam-style questions

3.1 $\mathbf{a} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$. Find $\mathbf{a} - \mathbf{b}$. [2]

3.2 The position vector of A is $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and of B is $\begin{pmatrix} 7 \\ 11 \end{pmatrix}$. Find $\overrightarrow{AB}$ and its magnitude. [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

$$\mathbf{2.1} \quad \begin{pmatrix} 4+1 \\ 2-3 \end{pmatrix} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}.$$

$$\mathbf{2.2} \quad \begin{pmatrix} 6 \\ -2 \end{pmatrix}.$$

$$\mathbf{2.3} \quad \sqrt{6^2 + 8^2} = \sqrt{100} = 10.$$

$$\mathbf{3.1} \quad \begin{pmatrix} 5-1 \\ 2-6 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \end{pmatrix}.$$

$$\mathbf{3.2} \quad \overrightarrow{AB} = \mathbf{b} - \mathbf{a} = \begin{pmatrix} 6 \\ 8 \end{pmatrix}; \text{ magnitude} = \sqrt{6^2 + 8^2} = 10.$$

8 $DE = \begin{pmatrix} \sim \\ -4 \end{pmatrix}$

(a) Find $5\overrightarrow{DE}$.

[1]

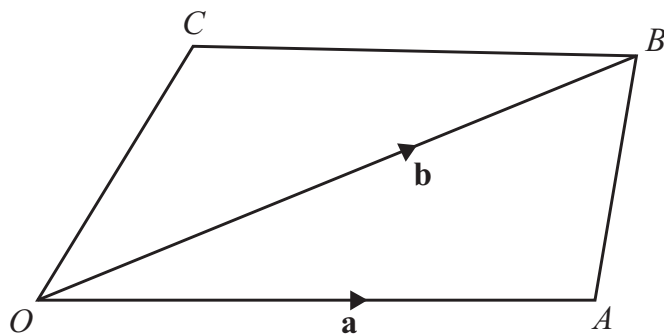
(b) Find $|\overrightarrow{DE}|$.

[2]

(c) D is the point $(-2, -3)$.

Find the coordinates of the point E .

[2]

NOT TO
SCALE

In the diagram, OA is parallel to CB .

$$OA : CB = 4 : 3$$

$$\overrightarrow{OA} = \mathbf{a} \text{ and } \overrightarrow{OB} = \mathbf{b}.$$

(a) Find $\overrightarrow{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

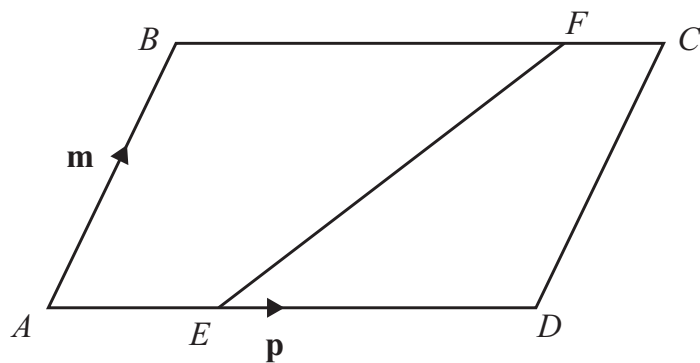
$$\overrightarrow{AB} = \dots\dots\dots [1]$$

(b) M is the midpoint of OC .

Find $\overrightarrow{AM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

Give your answer in its simplest form.

$$\overrightarrow{AM} = \dots\dots\dots [3]$$

NOT TO
SCALE

$ABCD$ is a parallelogram.

$\overrightarrow{AB} = \mathbf{m}$ and $\overrightarrow{AD} = \mathbf{p}$.

F is a point on BC and $BF = 4FC$.

E is a point on AD and $AE : ED = 1 : 2$.

(a) Find $\overrightarrow{EF}$, in terms of $\mathbf{m}$ and $\mathbf{p}$, in its simplest form.

..... [3]

(b) EF and DC are extended to meet at the point G .

Find $\overrightarrow{CG}$, in terms of $\mathbf{m}$ and/or $\mathbf{p}$, in its simplest form.

8 $\mathbf{m} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ $\mathbf{n} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

(a) Find $2\mathbf{m} - \mathbf{n}$.

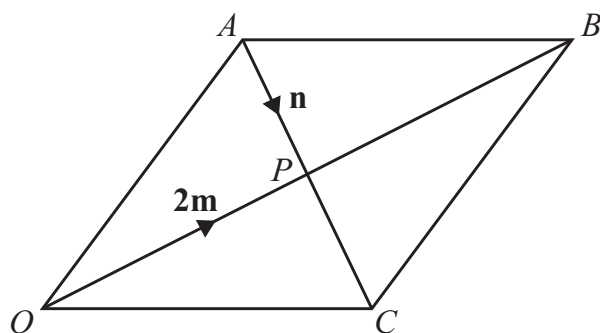
[2]

(b) The vector $\begin{pmatrix} 5 \\ \sqrt{y} \end{pmatrix}$ has a magnitude of 7.
Find the value of y .

$y =$

 [2]

21

NOT TO
SCALE

$OABC$ is a rhombus and O is the origin.
The diagonals of the rhombus intersect at P .
 $\vec{OP} = 2\mathbf{m}$ and $\vec{AP} = \mathbf{n}$.

(a) Find, in terms of $\mathbf{m}$ and $\mathbf{n}$, in its simplest form

(i) $\vec{OA}$

$$\vec{OA} = \dots\dots\dots [1]$$

(ii) $\vec{OC}$.

$$\vec{OC} = \dots\dots\dots [1]$$

(b) D is the point such that $\vec{AD} = 10\mathbf{m} - 3\mathbf{n}$.

Show that $OADC$ is a trapezium.

9 A is the point $(3, -1)$.

$$\overrightarrow{AB} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$$

(a) $\overrightarrow{AC} = 2\overrightarrow{AB}$

Find the coordinates of the point C .

(..... ,) [2]

(b) The length of AB is $k\sqrt{5}$.

Find the value of k .

$k =$ [2]

(c) P is a point on AB .

$$AP : PB = 1 : 3$$

Find the position vector of P .