

Week 22

IGCSE Mathematics

Homework bundle for this week

本周作业合集

exercises.lol

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1. A right-angled triangle has short sides 3 cm and 4 cm. Find the hypotenuse (cm).

2. In a right-angled triangle the hypotenuse is 10 cm and the angle is 30° . The opposite side = $10 \times \sin 30^\circ$. Find it (cm).

3. What is the exact value of $\sin 30^\circ$? (as a decimal)

4. A triangle has $b = 7$, $c = 8$ and the angle between them $A = 40^\circ$. Using $a^2 = 49 + 64 - 2(7)(8)\cos 40^\circ \approx 27.2$, find a (1 dp).

5. A box has a base 6 cm by 8 cm. Find the length of the base diagonal (cm).

6. The area of any triangle (two sides a , b with angle C between them) is:
☐ $\frac{1}{2}ab \sin C$
☐ $ab \cos C$
☐ $\frac{1}{2}ab$
☐ $a^2 + b^2$
7. The hypotenuse of a right-angled triangle is:
☐ the longest side, opposite the right angle
☐ the shortest side
☐ next to the right angle
☐ always 5 cm
8. You know all three sides and want an angle. Which rule do you use?
☐ cosine rule
☐ sine rule
☐ Pythagoras only
☐ SOH-CAH-TOA

9. In a 3-D problem you look for a right-angled triangle inside the solid to work with.

☐ True ☐ False

10. A triangle with sides 6, 8, 10 is right-angled.

☐ True ☐ False

IGCSE Mathematics

1. **5**

$$\sqrt{3^2 + 4^2} = \sqrt{25} = 5 \text{ cm.}$$

2. **5**

$$10 \times \sin 30^\circ = 10 \times 0.5 = 5 \text{ cm.}$$

3. **0.5**

$$\sin 30^\circ = 1/2 = 0.5.$$

4. **5.2**

$$a = \sqrt{27.2} \approx 5.2 \text{ cm.}$$

5. **10**

$$\sqrt{6^2 + 8^2} = \sqrt{100} = 10 \text{ cm.}$$

6. **$\frac{1}{2}ab \sin C$**

Area = $\frac{1}{2}ab \sin C$ uses two sides and the included angle.

7. **the longest side, opposite the right angle**

The hypotenuse is the longest side and lies opposite the 90° angle.

8. **cosine rule**

With three sides (or two sides and the included angle), use the cosine rule.

9. **True**

The whole 3-D method is to reduce it to a flat right-angled triangle.

10. **True**

$$6^2 + 8^2 = 36 + 64 = 100 = 10^2. \text{ It is a multiple of the 3-4-5 triple.}$$

E6.1 Pythagoras' theorem

Name: _____ Class: _____ Date: _____

Total: 11 marks

KEY WORDS triangle 三角形 • right-angled triangle 直角三角形 • hypotenuse 斜边 • pythagoras' theorem 勾股定理
• opposite 对边

Objective

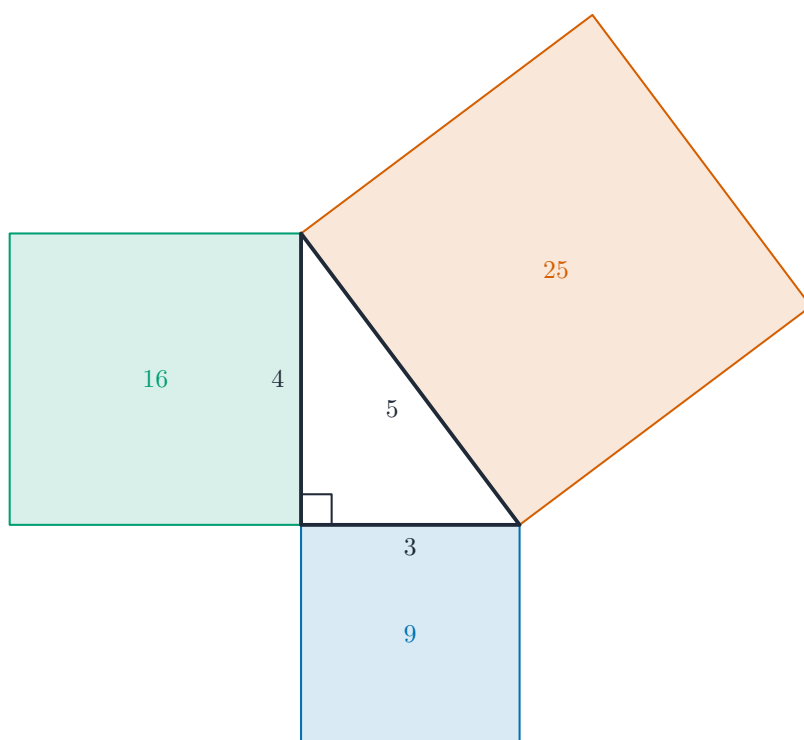
Build the skills to answer Extended exam questions on **Pythagoras' theorem** 勾股定理 — finding a missing side of a **right-angled triangle** 直角三角形.

You must be able to:

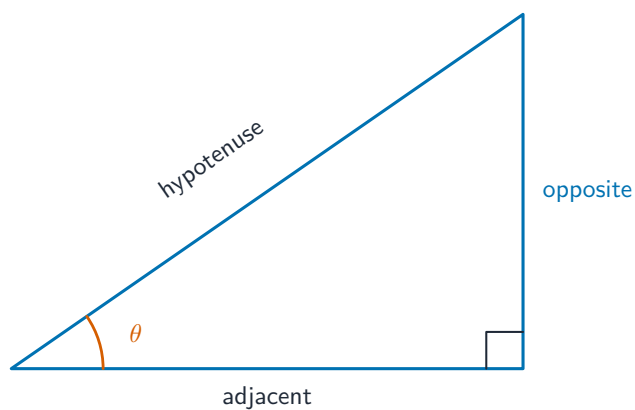
- use $a^2 + b^2 = c^2$ to find the hypotenuse
- rearrange to find a shorter side
- apply Pythagoras to a practical problem

1 Worked examples

- Pythagoras' theorem



The hypotenuse is the longest side, opposite the right angle



The same right triangle is used in trigonometry

$$a^2 + b^2 = c^2$$

- **Find hypotenuse:** legs 3 and 4 $\rightarrow c = \sqrt{9 + 16} = 5$.
- **Find a short side:** hyp 13, leg 5 $\rightarrow \text{other} = \sqrt{169 - 25} = \sqrt{144} = 12$.

Answer shapes: M1 a correct method, A1 the correct answer.

2 Practice

2.1 A right-angled triangle has legs 6 cm and 8 cm. Find the hypotenuse. [2]

2.2 A right-angled triangle has hypotenuse 17 cm and one leg 8 cm. Find the other leg. [2]

2.3 A right-angled triangle has legs 5 cm and 12 cm. Find the hypotenuse. [2]

3 Exam-style questions

3.1 A ladder of length 5 m leans against a wall. Its foot is 1.4 m from the wall. Find how high up the wall it reaches, correct to 2 decimal places. [3]

3.2 A rectangle is 9 cm by 12 cm. Find the length of its diagonal. [2]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $c = \sqrt{6^2 + 8^2} = \sqrt{100} = 10 \text{ cm.}$

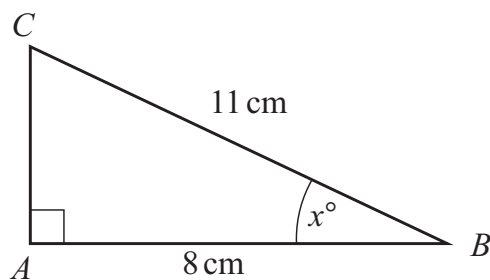
2.2 other leg $= \sqrt{17^2 - 8^2} = \sqrt{289 - 64} = \sqrt{225} = 15 \text{ cm.}$

2.3 $c = \sqrt{5^2 + 12^2} = \sqrt{169} = 13 \text{ cm.}$

3.1 height $= \sqrt{5^2 - 1.4^2} = \sqrt{25 - 1.96} = \sqrt{23.04} = 4.80 \text{ m.}$

3.2 diagonal $= \sqrt{9^2 + 12^2} = \sqrt{81 + 144} = \sqrt{225} = 15 \text{ cm.}$

10

NOT TO
SCALE

The diagram shows a right-angled triangle ABC .

- (a) Work out the exact length of AC .

..... cm [3]

- (b) $\cos x = k$

Write down the value of k .

$k =$ [1]

E6.2 Right-angled triangles

Name: _____ Class: _____ Date: _____

Total: 12 marks

KEY WORDS adjacent 邻边 • sine 正弦 • right-angled triangles 直角三角形 • cosine 余弦 • triangle 三角形

Objective

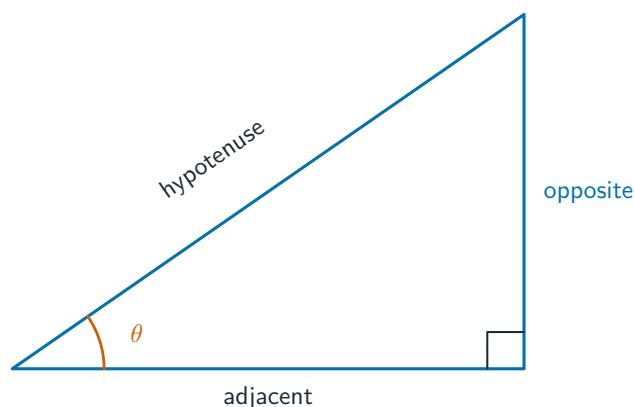
Build the skills to answer Extended exam questions on **right-angled triangles** 直角三角形 — the **sine, cosine and tangent** 正弦、余弦、正切 ratios (SOH-CAH-TOA), to find a side or an angle.

You must be able to:

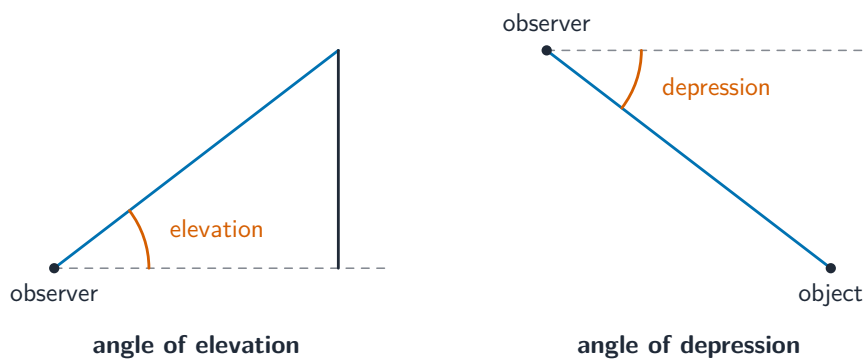
- label opposite, adjacent and hypotenuse
- use sin, cos, tan to find a missing side
- use the inverse ratios to find a missing angle

1 Worked examples

■ SOH-CAH-TOA



Give angle answers to 1 decimal place



Angles of elevation and depression apply right-triangle trig

$$\sin \theta = \frac{\text{opp}}{\text{hyp}} \quad \cos \theta = \frac{\text{adj}}{\text{hyp}} \quad \tan \theta = \frac{\text{opp}}{\text{adj}}$$

- **Find a side:** hyp 10, angle $30^\circ \rightarrow \text{opp} = 10 \sin 30^\circ = 5$.
- **Find an angle:** opp 4, adj 3 $\rightarrow \tan \theta = \frac{4}{3} \rightarrow \theta = \tan^{-1} \frac{4}{3} = 53.1^\circ$.

Answer shapes: **M1** a correct method, **A1** the correct answer.

2 Practice

2.1 A right-angled triangle has hypotenuse 12 cm and angle 40° . Find the opposite side. [2]

2.2 A right-angled triangle has opposite 5 cm and hypotenuse 10 cm. Find the angle. [2]

2.3 A right-angled triangle has adjacent 8 cm and angle 35° . Find the opposite side. [2]

3 Exam-style questions

3.1 In a right-angled triangle, the side opposite an angle is 7 cm and the adjacent side is 10 cm. Find the angle. [2]

3.2 A right-angled triangle has hypotenuse 15 cm and an angle of 52° . Find the side adjacent to the angle. [2]

3.3 A ramp rises 1.2 m over a horizontal distance of 5 m. Find the angle the ramp makes with the horizontal. [2]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $\text{opp} = 12 \times \sin 40^\circ = 12 \times 0.643 = 7.71 \text{ cm}.$

2.2 $\sin \theta = \frac{5}{10} = 0.5 \rightarrow \theta = 30^\circ.$

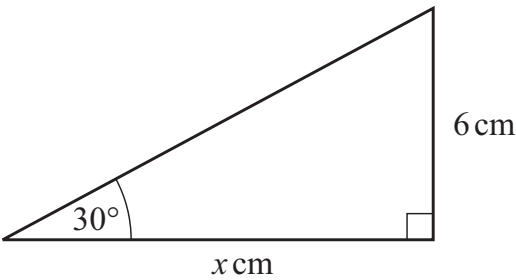
2.3 $\text{opp} = 8 \times \tan 35^\circ = 8 \times 0.700 = 5.60 \text{ cm}.$

3.1 $\tan \theta = \frac{7}{10} = 0.7 \rightarrow \theta = \tan^{-1}(0.7) = 35.0^\circ.$

3.2 $\text{adj} = 15 \times \cos 52^\circ = 15 \times 0.616 = 9.23 \text{ cm}.$

3.3 $\tan \theta = \frac{1.2}{5} = 0.24 \rightarrow \theta = \tan^{-1}(0.24) = 13.5^\circ.$

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NOT TO
SCALE

Find the exact value of x .

$x = \dots\dots\dots [4]$

E6.4 Trigonometric functions

Name: _____ Class: _____ Date: _____

Total: 11 marks

KEY WORDS sine and cosine rules 正弦和余弦定理 • exact values 精确值 • trigonometric equation 三角方程
 • trigonometric functions 三角函数 • sine 正弦

Objective

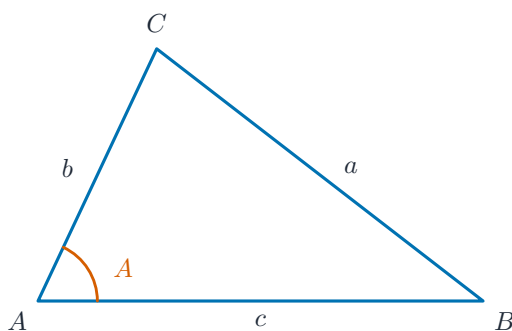
Build the skills to answer Extended exam questions on **trigonometric functions** 三角函数 — **exact values** 精确值, solving **trigonometric equations** 三角方程, and the **sine and cosine rules** 正弦和余弦定理.

You must be able to:

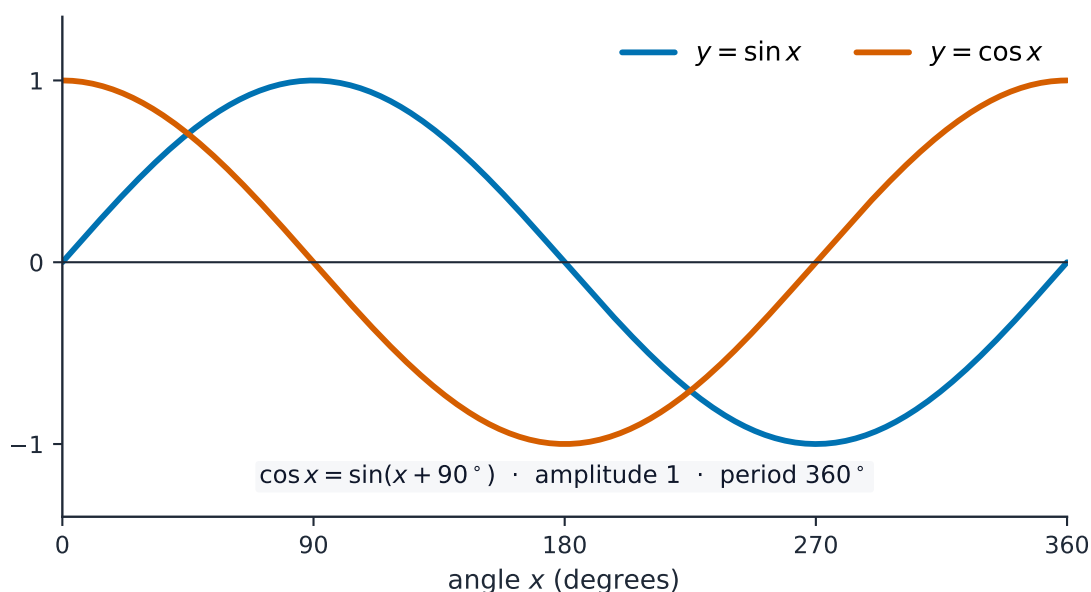
- recall exact values of $\sin$, $\cos$, $\tan$ for $0-90^\circ$
- use the cosine rule and the sine rule in any triangle
- find the area of a triangle with $\frac{1}{2}ab \sin C$

1 Worked examples

■ The sine and cosine rules



For any triangle, not just right-angled ones



The sine and cosine function graphs

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad a^2 = b^2 + c^2 - 2bc \cos A \quad \text{area} = \frac{1}{2}ab \sin C$$

- **Cosine rule (two sides + included angle):** $b = 7$, $c = 8$, $A = 40^\circ \rightarrow a^2 = 49 + 64 - 112 \cos 40^\circ = 27.2 \rightarrow a = 5.2$ cm.
- **Exact:** $\sin 60^\circ = \frac{\sqrt{3}}{2}$; solve $\sin x = \frac{\sqrt{3}}{2}$ ($0-360^\circ$) $\rightarrow x = 60^\circ$ or 120° .

Answer shapes: **M1** a correct method, **A1** the correct answer.

2 Practice

2.1 Write down the exact value of $\cos 30^\circ$. [1]

2.2 Solve $\sin x = \frac{1}{2}$ for $0^\circ \leq x \leq 360^\circ$. [2]

2.3 Find the area of a triangle with sides $a = 6$ cm, $b = 9$ cm and included angle $C = 50^\circ$. [2]

3 Exam-style questions

3.1 A triangle has $b = 5$ cm, $c = 7$ cm and the angle between them $A = 60^\circ$. Find side a . [3]

3.2 In a triangle, $a = 8$ cm, angle $A = 40^\circ$ and angle $B = 75^\circ$. Use the sine rule to find side b . [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $\cos 30^\circ = \frac{\sqrt{3}}{2}.$

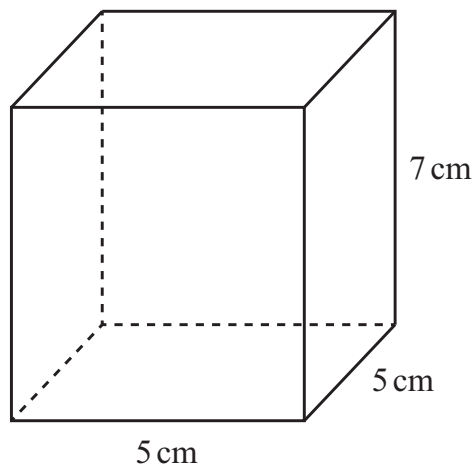
2.2 $x = 30^\circ$ or $x = 180 - 30 = 150^\circ.$

2.3 $\text{area} = \frac{1}{2}(6)(9) \sin 50^\circ = 27 \times 0.766 = 20.7 \text{ cm}^2.$

3.1 $a^2 = 5^2 + 7^2 - 2(5)(7) \cos 60^\circ = 25 + 49 - 70(0.5) = 39; a = \sqrt{39} = 6.24 \text{ cm}.$

3.2 $\frac{b}{\sin 75^\circ} = \frac{8}{\sin 40^\circ} \rightarrow b = \frac{8 \sin 75^\circ}{\sin 40^\circ} = \frac{8 \times 0.966}{0.643} = 12.0 \text{ cm}.$

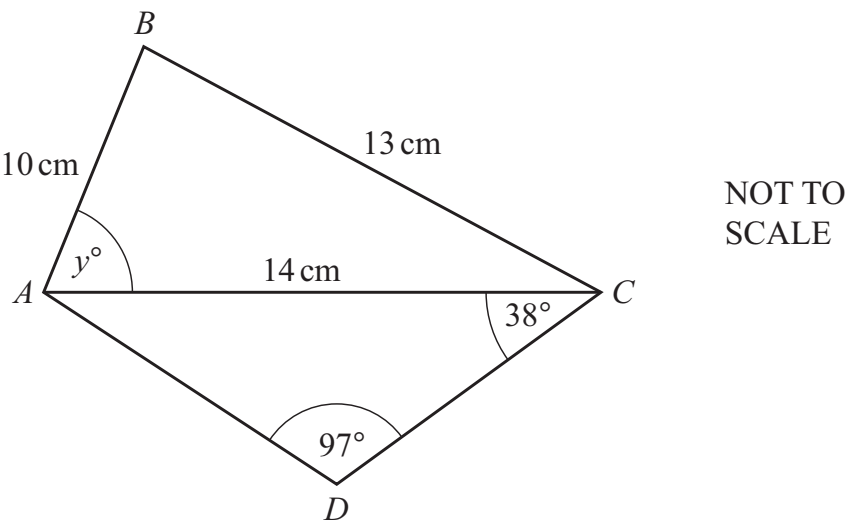
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NOT TO
SCALE

The diagram shows a box in the shape of a cuboid.
Mala has a straight rod of length 10 cm.

Show that this rod does **not** fit completely inside the box.

[3]

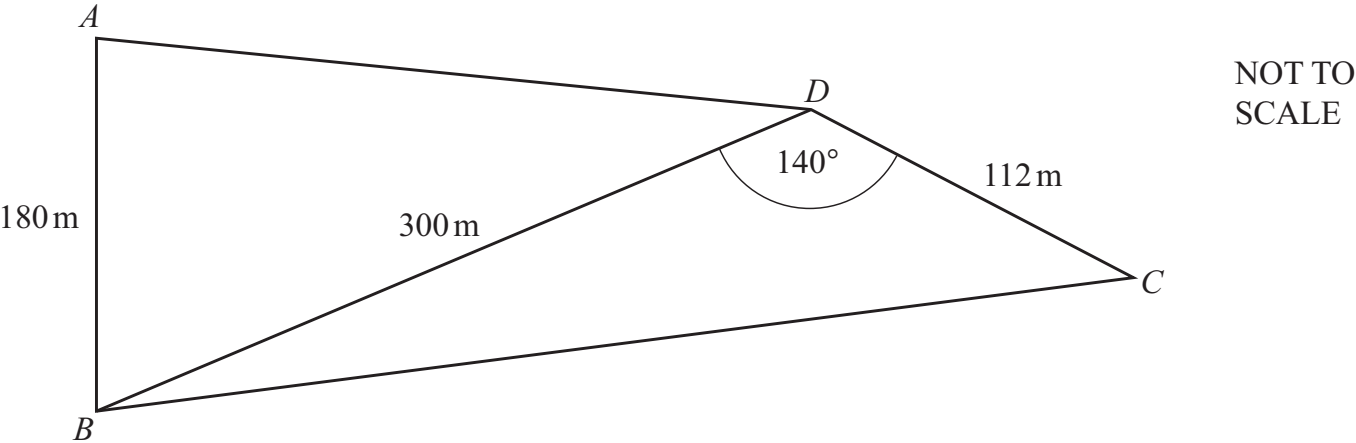


(a) Calculate the value of y .

$y = \dots\dots\dots [3]$

(b) Calculate BD .

15



The diagram shows a field, $ABCD$, in the shape of a quadrilateral.
 BD is a straight path across the field.

(a) Calculate BC .

$BC = \dots\dots\dots \text{ m}$ [3]

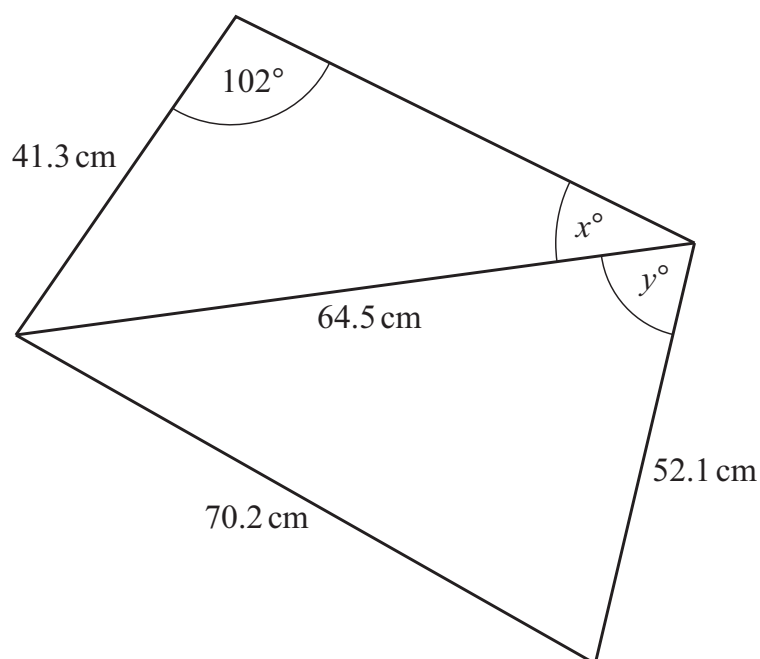
(b) Calculate angle DBC .

Angle $DBC = \dots\dots\dots$ [3]

(c) The total area of the field, $ABCD$, is $35\,900\text{ m}^2$.

Work out the length of the shortest distance from D to AB .

18

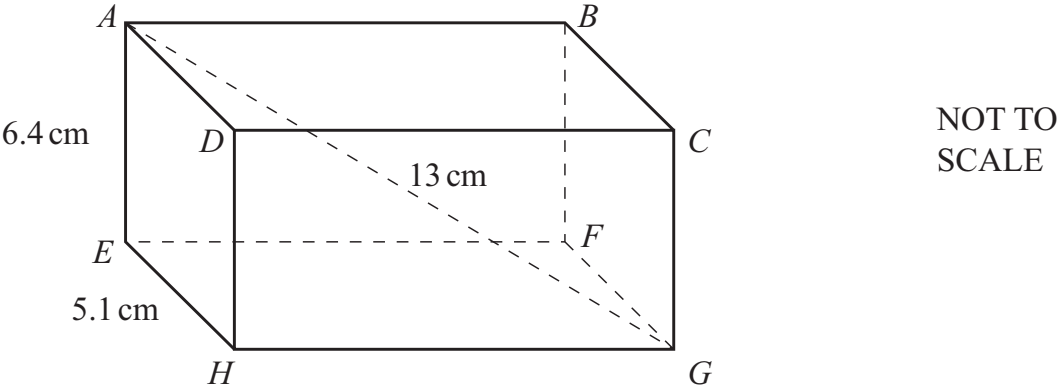
NOT TO
SCALE

(a) Calculate the value of x .

$x = \dots\dots\dots$ [3]

(b) Calculate the value of y .

$y = \dots\dots\dots$ [3]



The diagram shows a cuboid $ABCDEFGH$.
 $AE = 6.4\text{ cm}$, $EH = 5.1\text{ cm}$ and $AG = 13\text{ cm}$.

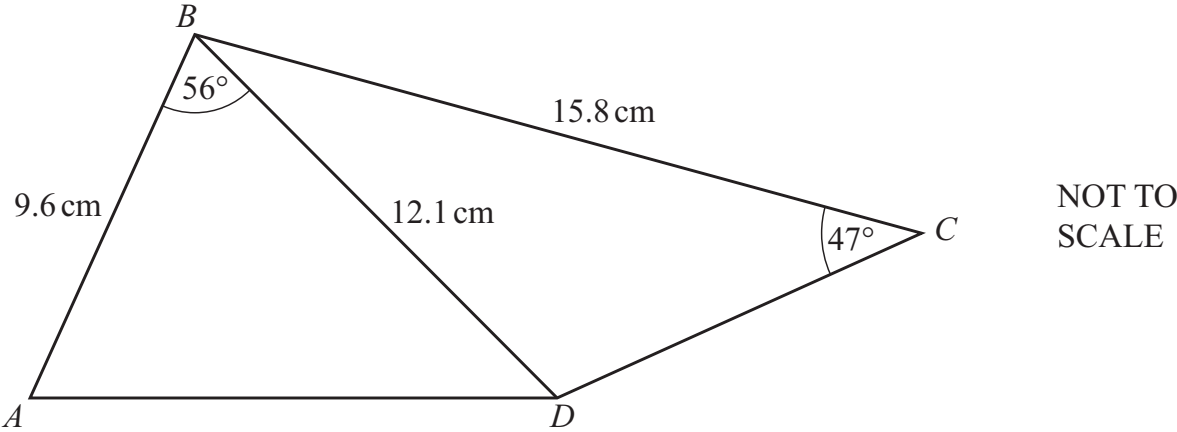
(a) Calculate EF .

$EF = \dots\dots\dots\text{ cm}$ [3]

(b) Calculate the angle between the line AG and the base $EFGH$ of the cuboid.

$\dots\dots\dots$ [3]

17



The diagram shows a quadrilateral $ABCD$.

(a) Calculate AD .

$AD = \dots\dots\dots\text{ cm}$ [3]

(b) Calculate the obtuse angle BDC .

Angle $BDC = \dots\dots\dots$ [4]

(c) Calculate the area of the quadrilateral.

$\dots\dots\dots\text{ cm}^2$ [3]