Week 21

IGCSE Mathematics

Homework bundle for this week

本周作业合集

exercises.lol

Contents 目录

IGCSE Mathematics Name: ______ Score: ______

1. A right-angled triangle has short sides 3 cm and 4 cm. Find the hypotenuse (cm).

   ______

2. In a right-angled triangle the hypotenuse is 10 cm and the angle is 30°. The opposite side = 10 × sin 30°. Find it (cm).

   ______

3. What is the exact value of sin 30°? (as a decimal)

   ______

4. A triangle has b = 7, c = 8 and the angle between them A = 40°. Using $a^2 = 49 + 64 - 2(7)(8)\cos 40° \approx 27.2$, find a (1 dp).

   ______

5. A box has a base 6 cm by 8 cm. Find the length of the base diagonal (cm).

   ______

6. The area of any triangle (two sides a, b with angle C between them) is:

   ☐ ½ab sin C

   ☐ ab cos C

   ☐ ½ab

   ☐ $a^2 + b^2$

7. The hypotenuse of a right-angled triangle is:

   ☐ the longest side, opposite the right angle

   ☐ the shortest side

   ☐ next to the right angle

   ☐ always 5 cm

8. You know all three sides and want an angle. Which rule do you use?

   ☐ cosine rule

   ☐ sine rule

   ☐ Pythagoras only

   ☐ SOH-CAH-TOA

9. In a 3-D problem you look for a right-angled triangle inside the solid to work with.

   ☐ True ☐ False

10. A triangle with sides 6, 8, 10 is right-angled.

   ☐ True ☐ False

IGCSE Mathematics

1. **5**

   $\sqrt{}(3^2 + 4^2) = \sqrt{}25 = 5$ cm.

2. **5**

   $10 \times \sin 30° = 10 \times 0.5 = 5$ cm.

3. **0.5**

   $\sin 30° = 1/2 = 0.5$.

4. **5.2**

   $a = \sqrt{}27.2 \approx 5.2$ cm.

5. **10**

   $\sqrt{}(6^2 + 8^2) = \sqrt{}100 = 10$ cm.

6. **½ab sin C**

   Area = ½ab sin C uses two sides and the included angle.

7. **the longest side, opposite the right angle**

   The hypotenuse is the longest side and lies opposite the 90° angle.

8. **cosine rule**

   With three sides (or two sides and the included angle), use the cosine rule.

9. **True**

   The whole 3-D method is to reduce it to a flat right-angled triangle.

10. **True**

   $6^2 + 8^2 = 36 + 64 = 100 = 10^2$. It is a multiple of the 3-4-5 triple.

IGCSE MATHEMATICS • EXERCISE SHEET

E6.1 Pythagoras' theorem

Name: ______________ Class: __________ Date: ____________ **Total: 11 marks**

KEY WORDS triangle 三角形 • right-angled triangle 直角三角形 • hypotenuse 斜边 • pythagoras' theorem 勾股定理 • opposite 对边

Objective

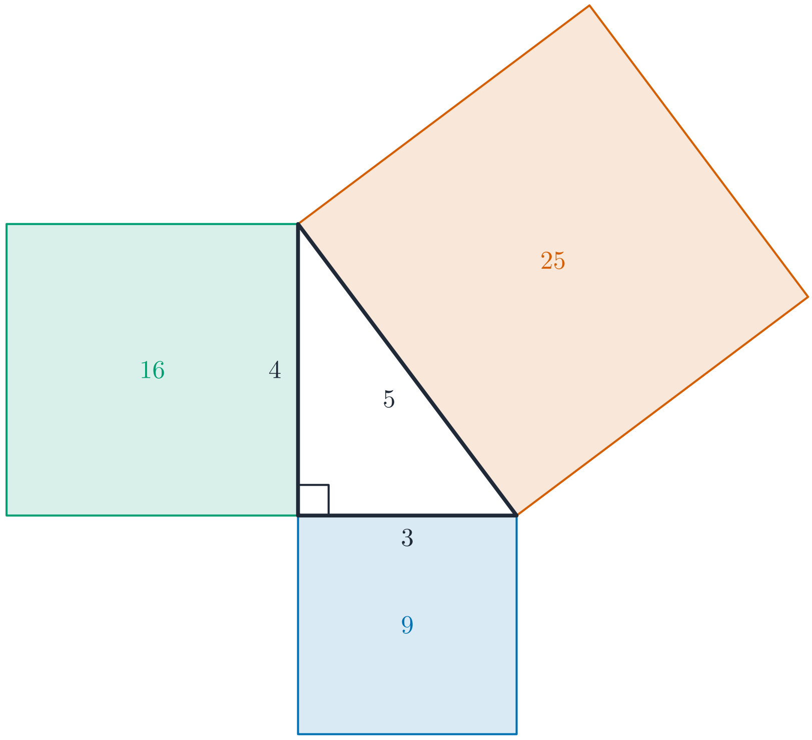
Build the skills to answer Extended exam questions on **Pythagoras' theorem** 勾股定理 — finding a missing side of a **right-angled triangle** 直角三角形.

You must be able to:

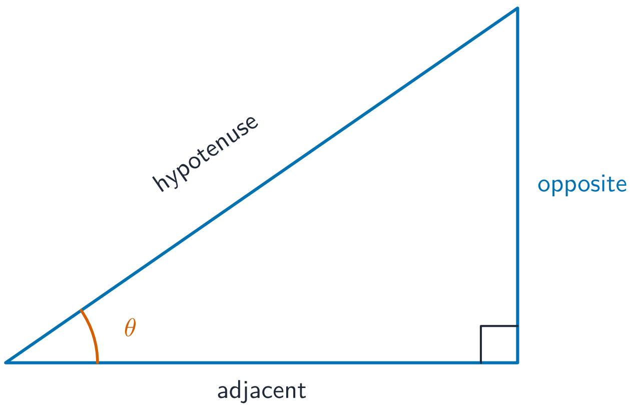
- use $a^2 + b^2 = c^2$ to find the hypotenuse
- rearrange to find a shorter side
- apply Pythagoras to a practical problem

1 Worked examples

■ Pythagoras' theorem



The hypotenuse is the longest side, opposite the right angle



The same right triangle is used in trigonometry

$$a^2 + b^2 = c^2$$

- **Find hypotenuse:** legs 3 and 4 $\rightarrow c = \sqrt{9+16} = 5$.
- **Find a short side:** hyp 13, leg 5 $\rightarrow$ other $= \sqrt{169-25} = \sqrt{144} = 12$.

Answer shapes: M1 a correct method, **A1** the correct answer.

2 Practice

2.1 A right-angled triangle has legs 6 cm and 8 cm. Find the hypotenuse. [2]

2.2 A right-angled triangle has hypotenuse 17 cm and one leg 8 cm. Find the other leg. [2]

2.3 A right-angled triangle has legs 5 cm and 12 cm. Find the hypotenuse. [2]

3 Exam-style questions

3.1 A ladder of length 5 m leans against a wall. Its foot is 1.4 m from the wall. Find how high up the wall it reaches, correct to 2 decimal places. [3]

3.2 A rectangle is 9 cm by 12 cm. Find the length of its diagonal. [2]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $c = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$ cm.

2.2 other leg $= \sqrt{17^2 - 8^2} = \sqrt{289 - 64} = \sqrt{225} = 15$ cm.

2.3 $c = \sqrt{5^2 + 12^2} = \sqrt{169} = 13$ cm.

3.1 height $= \sqrt{5^2 - 1.4^2} = \sqrt{25 - 1.96} = \sqrt{23.04} = 4.80$ m.

3.2 diagonal $= \sqrt{9^2 + 12^2} = \sqrt{81 + 144} = \sqrt{225} = 15$ cm.

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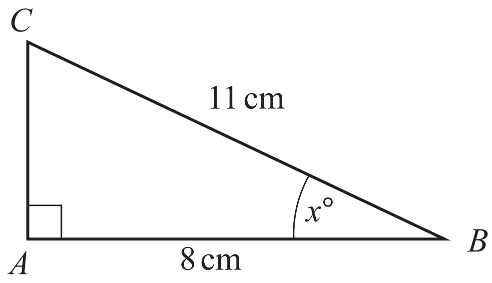


NOT TO SCALE

The diagram shows a right-angled triangle *ABC*.

(a) Work out the exact length of *AC*.

.... cm [3]

(b) $\cos x = k$

Write down the value of *k*.

$k =$ .... [1]

E6.2 Right-angled triangles

Name: ______________ Class: __________ Date: ____________ **Total: 12 marks**

KEY WORDS adjacent 邻边 • sine 正弦 • right-angled triangles 直角三角形 • cosine 余弦 • triangle 三角形

Objective

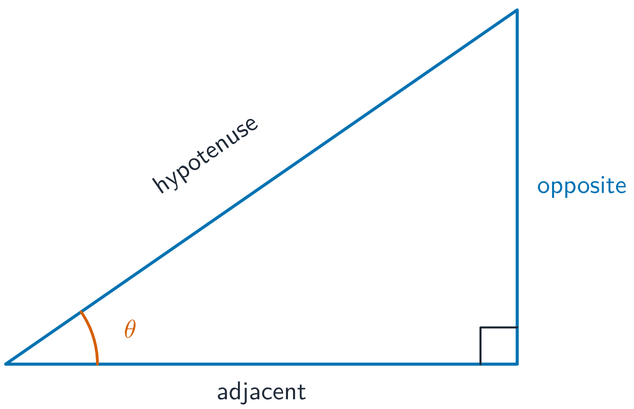
Build the skills to answer Extended exam questions on **right-angled triangles** 直角三角形 — the **sine, cosine and tangent** 正弦、余弦、正切 ratios (SOH-CAH-TOA), to find a side or an angle.

You must be able to:

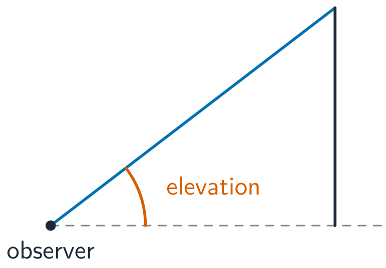
- label opposite, adjacent and hypotenuse
- use sin, cos, tan to find a missing side
- use the inverse ratios to find a missing angle

1 Worked examples

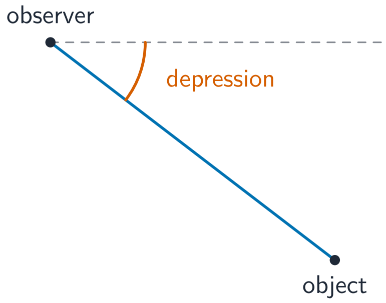
■ SOH-CAH-TOA



Give angle answers to 1 decimal place



angle of elevation



angle of depression

Angles of elevation and depression apply right-triangle trig

$$\sin\theta = \frac{\text{opp}}{\text{hyp}} \qquad \cos\theta = \frac{\text{adj}}{\text{hyp}} \qquad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

- **Find a side:** hyp 10, angle 30° $\to$ opp $= 10\sin 30° = 5$.
- **Find an angle:** opp 4, adj 3 $\to \tan\theta = \frac{4}{3} \to \theta = \tan^{-1}\frac{4}{3} = 53.1°$.

Answer shapes: M1 a correct method, **A1** the correct answer.

2 Practice

2.1 A right-angled triangle has hypotenuse 12 cm and angle 40°. Find the opposite side. [2]

2.2 A right-angled triangle has opposite 5 cm and hypotenuse 10 cm. Find the angle. [2]

2.3 A right-angled triangle has adjacent 8 cm and angle 35°. Find the opposite side. [2]

3 Exam-style questions

3.1 In a right-angled triangle, the side opposite an angle is 7 cm and the adjacent side is 10 cm. Find the angle. [2]

3.2 A right-angled triangle has hypotenuse 15 cm and an angle of $52°$. Find the side adjacent to the angle. [2]

3.3 A ramp rises 1.2 m over a horizontal distance of 5 m. Find the angle the ramp makes with the horizontal. [2]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 opp $= 12 \times \sin 40° = 12 \times 0.643 = 7.71$ cm.

2.2 $\sin\theta = \frac{5}{10} = 0.5 \to \theta = 30°$.

2.3 opp $= 8 \times \tan 35° = 8 \times 0.700 = 5.60$ cm.

3.1 $\tan\theta = \frac{7}{10} = 0.7 \to \theta = \tan^{-1}(0.7) = 35.0°$.

3.2 adj $= 15 \times \cos 52° = 15 \times 0.616 = 9.23$ cm.

3.3 $\tan\theta = \frac{1.2}{5} = 0.24 \to \theta = \tan^{-1}(0.24) = 13.5°$.

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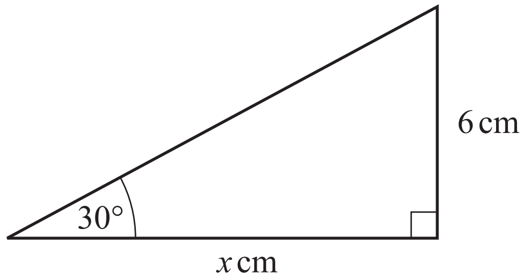


NOT TO SCALE

Find the exact value of x.

$x =$ .... [4]

E6.4 Trigonometric functions

Name: __________ Class: ________ Date: __________ **Total: 24 marks**

KEY WORDS exact values 精确值 • sine and cosine rules 正弦和余弦定理 • trigonometric equation 三角方程 • trigonometric functions 三角函数 • sine 正弦

Objective

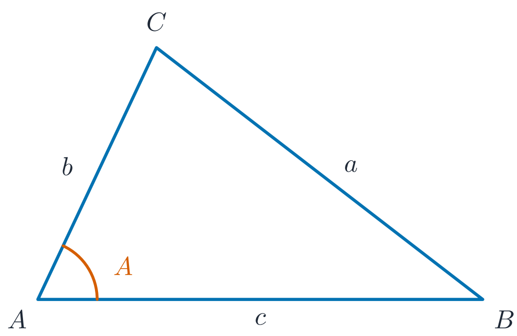
Build the skills to answer Extended exam questions on **trigonometric functions** 三角函数 — **exact values** 精确值, solving **trigonometric equations** 三角方程, and the **sine and cosine rules** 正弦和余弦定理.

You must be able to:

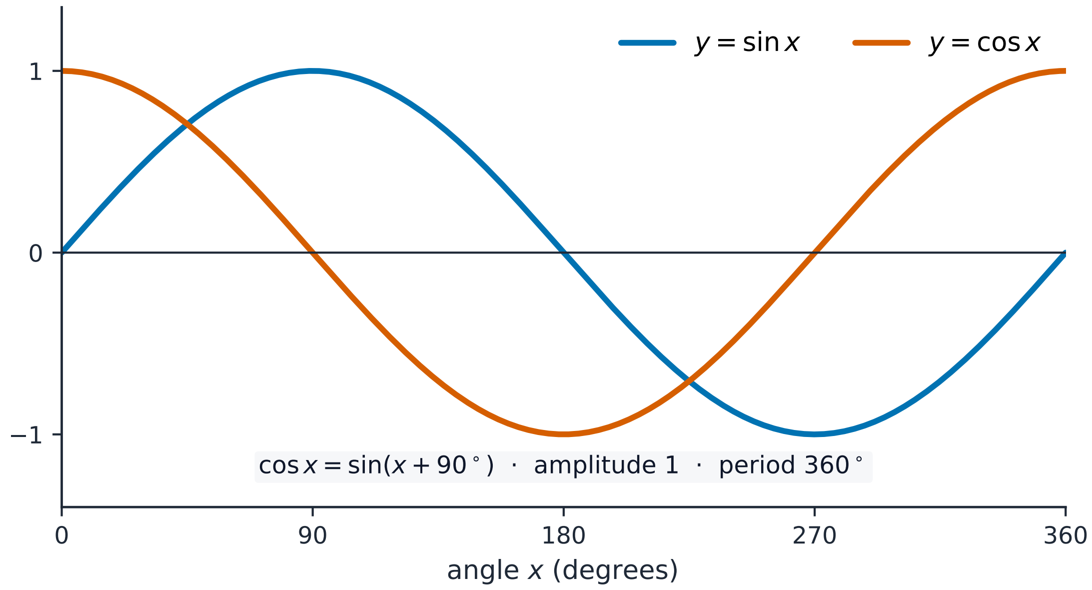
- recall exact values of sin, cos, tan for 0–90°
- use the cosine rule and the sine rule in any triangle
- find the area of a triangle with $\frac{1}{2}ab\sin C$

1 Worked examples

■ The sine and cosine rules



For any triangle, not just right-angled ones



The sine and cosine function graphs

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc\cos A \qquad \text{area} = \tfrac{1}{2}ab\sin C$$

- **Cosine rule (two sides + included angle):** $b = 7,\ c = 8,\ A = 40° \to a^2 = 49 + 64 - 112\cos 40° = 27.2 \to a = 5.2$ cm.
- **Exact:** $\sin 60° = \frac{\sqrt{3}}{2}$; solve $\sin x = \frac{\sqrt{3}}{2}$ (0–360°) $\to x = 60°$ or $120°$.

Answer shapes: M1 a correct method, **A1** the correct answer.

2 Practice

2.1 Write down the exact value of $\cos 30°$. [1]

2.2 Solve $\sin x = \frac{1}{2}$ for $0° \leqslant x \leqslant 360°$. [2]

2.3 Find the area of a triangle with sides $a = 6$ cm, $b = 9$ cm and included angle $C = 50°$. [2]

3 Exam-style questions

3.1 A triangle has $b = 5$ cm, $c = 7$ cm and the angle between them $A = 60°$. Find side a. [3]

3.2 In a triangle, $a = 8$ cm, angle $A = 40°$ and angle $B = 75°$. Use the sine rule to find side b. [3]

3.3 (a) Write down the exact values of $\sin 30°$, $\cos 45°$ and $\tan 60°$. [3]

(b) Solve $2\sin x = 1$ for $0° \leq x \leq 360°$. [3]

(c) Solve $\cos x = -0.5$ for $0° \leq x \leq 360°$. [3]

(d) Sketch $y = \sin x$ for $0° \leq x \leq 360°$, mark its maximum and minimum, and use the sketch to explain why $\sin x = 1.4$ has no solution. [4]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $\cos 30° = \dfrac{\sqrt{3}}{2}$.

2.2 $x = 30°$ or $x = 180 - 30 = 150°$.

2.3 area $= \frac{1}{2}(6)(9)\sin 50° = 27 \times 0.766 = 20.7$ cm^2.

3.1 $a^2 = 5^2 + 7^2 - 2(5)(7)\cos 60° = 25 + 49 - 70(0.5) = 39$; $a = \sqrt{39} = 6.24$ cm.

3.2 $\dfrac{b}{\sin 75°} = \dfrac{8}{\sin 40°} \rightarrow b = \dfrac{8\sin 75°}{\sin 40°} = \dfrac{8 \times 0.966}{0.643} = 12.0$ cm.

3.3 (a) $\sin 30° = \dfrac{1}{2}$, $\cos 45° = \dfrac{\sqrt{2}}{2}$ (or $\dfrac{1}{\sqrt{2}}$), $\tan 60° = \sqrt{3}$.
(b) $\sin x = \dfrac{1}{2}$; the first solution is $x = 30°$; sine is also positive in the second quadrant, so $x = 180 - 30 = 150°$.
(c) The related acute angle is $\cos^{-1} 0.5 = 60°$; cosine is negative in the second and third quadrants; $x = 180 - 60 = 120°$ and $x = 180 + 60 = 240°$.
(d) A correct sine curve starting at $(0, 0)$, rising to $(90, 1)$, back through $(180, 0)$, down to $(270, -1)$ and back to $(360, 0)$; maximum and minimum marked. The curve never rises above $y = 1$, so no value of x gives $\sin x = 1.4$ — the sine of an angle always lies between -1 and 1.

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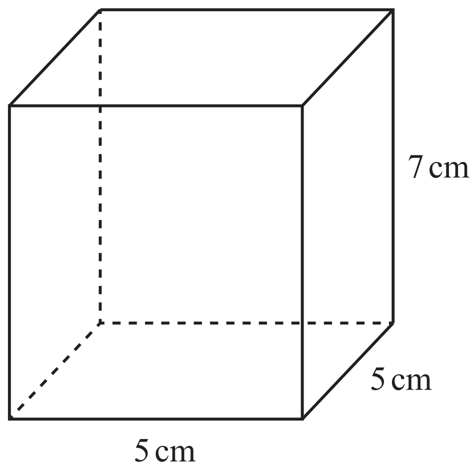

NOT TO SCALE

The diagram shows a box in the shape of a cuboid.
Mala has a straight rod of length 10 cm.

Show that this rod does **not** fit completely inside the box.

[3]

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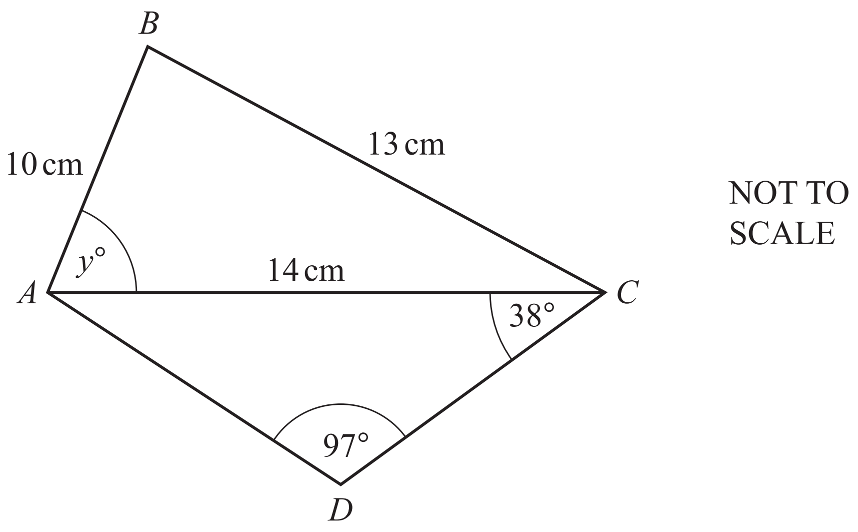

(a) Calculate the value of y.

$y =$ .... [3]

(b) Calculate BD.

$BD =$ .... cm [5]

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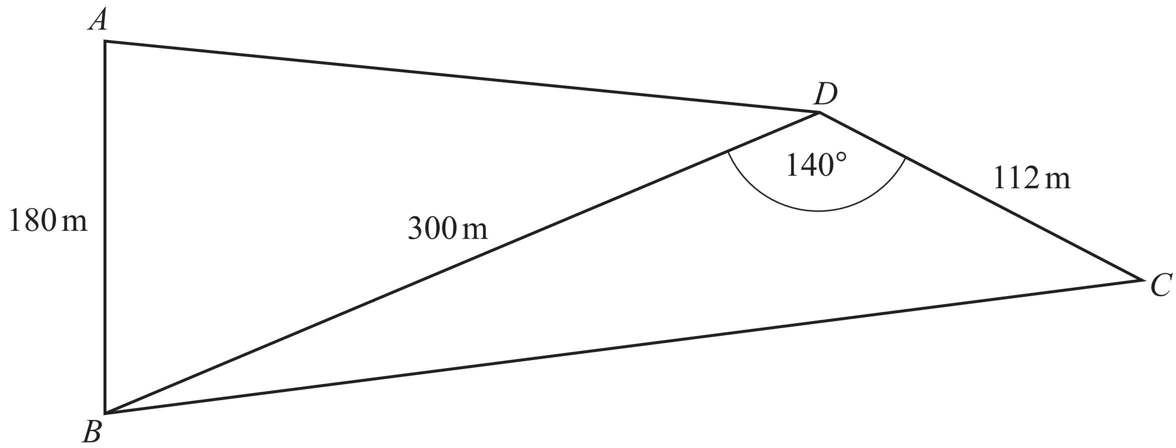

NOT TO SCALE

The diagram shows a field, *ABCD*, in the shape of a quadrilateral.
BD is a straight path across the field.

(a) Calculate *BC*.

$BC =$ .... m [3]

(b) Calculate angle *DBC*.

Angle $DBC =$ .... [3]

(c) The total area of the field, *ABCD*, is $35\,900\,\text{m}^2$.

Work out the length of the shortest distance from *D* to *AB*.

m [4]

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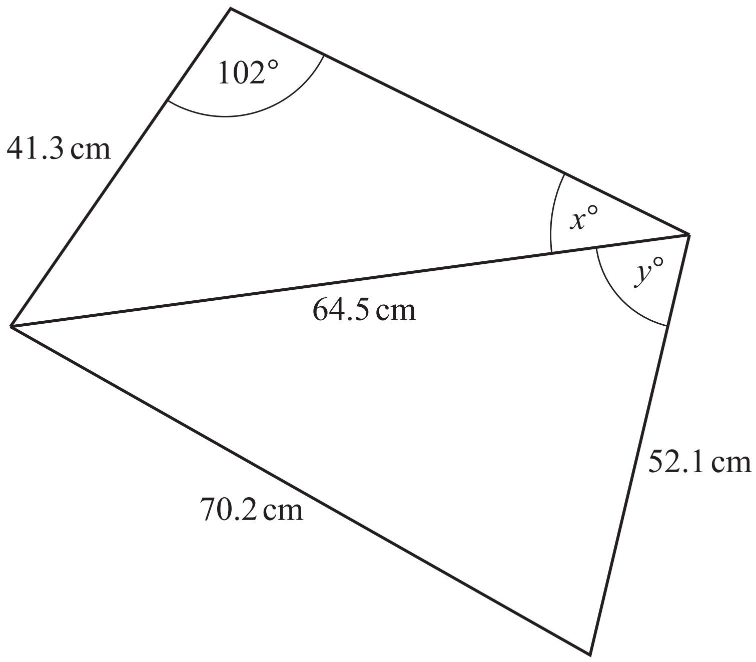


NOT TO
SCALE

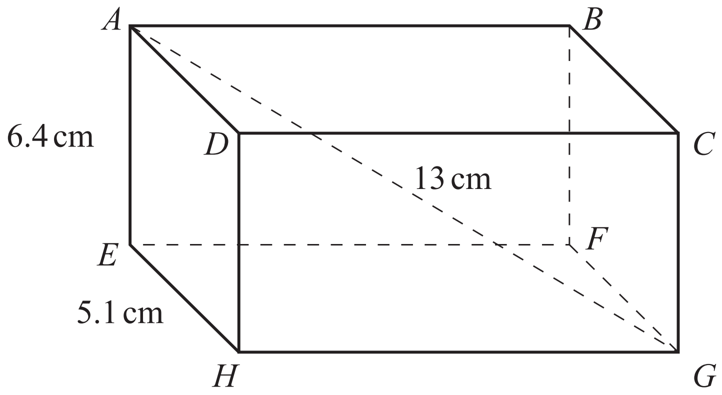
(a) Calculate the value of x.

$x =$ .... [3]

(b) Calculate the value of y.

$y =$ .... [3]

24



NOT TO SCALE

The diagram shows a cuboid *ABCDEFGH*.
$AE = 6.4$ cm, $EH = 5.1$ cm and $AG = 13$ cm.

(a) Calculate *EF*.

$EF =$ .... cm [3]

(b) Calculate the angle between the line *AG* and the base *EFGH* of the cuboid.

.... [3]

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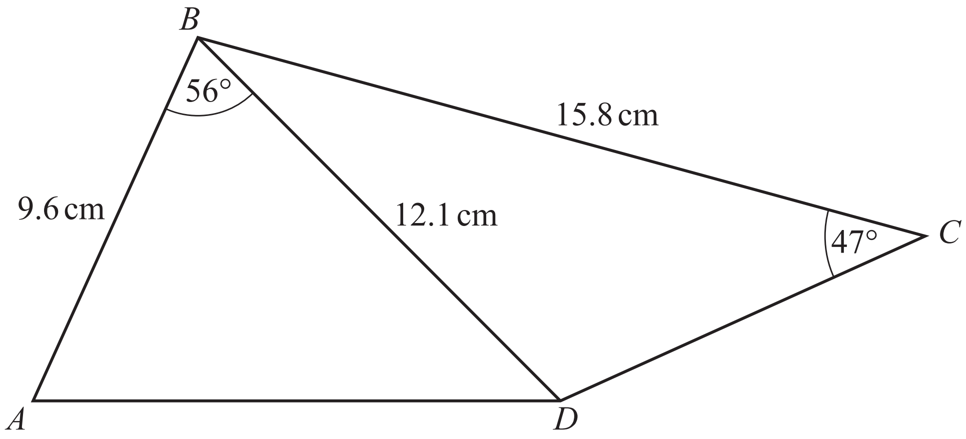

The diagram shows a quadrilateral *ABCD*.

(a) Calculate *AD*.

AD = ... cm [3]

(b) Calculate the obtuse angle *BDC*.

Angle *BDC* = ... [4]

(c) Calculate the area of the quadrilateral.

.... cm^2 [3]