

Week 19

IGCSE Mathematics

Homework bundle for this week

本周作业合集

exercises.lol

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5. Mensuration

Starter Quiz · 课前小测

IGCSE Mathematics Name: _____ Score: _____

- Convert 3 m^2 to cm^2 . ($1 \text{ m}^2 = 10\,000 \text{ cm}^2$)

- A trapezium has parallel sides 6 cm and 10 cm and height 4 cm. Find its area (cm^2).

- A circle has radius 7 cm. Its area is $k\pi \text{ cm}^2$. What is k?

- A cylinder has radius 5 cm and height 10 cm. Its volume is $k\pi \text{ cm}^3$. What is k?

- To find the area of a rectangle with a semicircle on one end, you:
☐ add the rectangle area and the semicircle area
☐ multiply them
☐ subtract the semicircle from the rectangle
☐ take the larger one only
- How many cm are there in 1 m?

- The volume of a hemisphere of radius r is:
☐ $(2/3)\pi r^3$
☐ $(4/3)\pi r^3$
☐ $(1/3)\pi r^3$
☐ $4\pi r^2$
- The volume of a sphere of radius r is:
☐ $(4/3)\pi r^3$
☐ $4\pi r^2$
☐ $\pi r^2 h$
☐ $(1/3)\pi r^2 h$
- A circle with diameter 10 cm has area $100\pi \text{ cm}^2$.
☐ True ☐ False

10. $1 \text{ m}^2 = 100 \text{ cm}^2$.

☐ True ☐ False

5. Mensuration

IGCSE Mathematics

1. **30000**

$3 \times 10\,000 = 30\,000 \text{ cm}^2$.

2. **32**

$\frac{1}{2}(6 + 10) \times 4 = \frac{1}{2} \times 16 \times 4 = 32 \text{ cm}^2$.

3. **49**

Area = $\pi r^2 = \pi(7^2) = 49\pi$, so $k = 49$.

4. **250**

Volume = $\pi r^2 h = \pi(25)(10) = 250\pi$, so $k = 250$.

5. **add the rectangle area and the semicircle area**

The shape is the rectangle plus the semicircle, so add the two areas.

6. **100**

$1 \text{ m} = 100 \text{ cm}$.

7. **$(2/3)\pi r^3$**

Half of $(4/3)\pi r^3$ is $(2/3)\pi r^3$.

8. **$(4/3)\pi r^3$**

Sphere volume = $(4/3)\pi r^3$; $4\pi r^2$ is its surface area.

9. **False**

Radius = 5 cm, so area = $\pi(5^2) = 25\pi$, not 100π . You must halve the diameter first.

10. **False**

$1 \text{ m}^2 = 100^2 = 10\,000 \text{ cm}^2$, not 100 cm^2 . The conversion factor must be squared for area.

E5.3 Circles, arcs and sectors

Name: _____ Class: _____ Date: _____ **Total: 12 marks**

KEY WORDS circle 圆 • radius 半径 • circumference 圆周 • sector area 扇形面积
• arc length 弧长

Objective

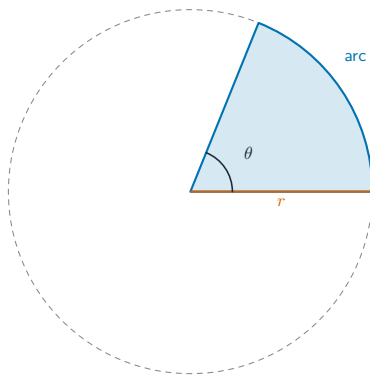
Build the skills to answer Extended exam questions on **circles, arcs and sectors** 圆、弧和扇形 — **circumference and area** 周长和面积, and the **arc length** 弧长 and **sector area** 扇形面积.

You must be able to:

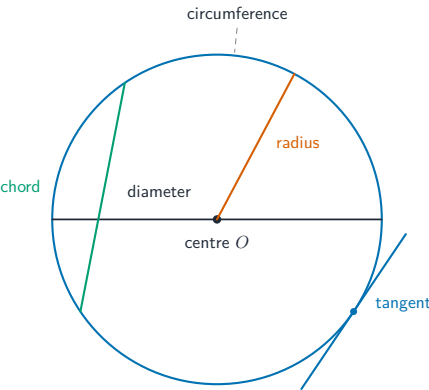
- find the circumference and area of a circle
- find an arc length and a sector area
- find the perimeter of a sector

1 Worked examples

■ Arcs and sectors



Take the fraction $\frac{\theta}{360}$ of the whole circle



The named parts of a circle used in arc and sector work

$$\text{circumference} = 2\pi r \quad \text{area} = \pi r^2$$

$$\text{arc} = \frac{\theta}{360} \times 2\pi r \quad \text{sector area} = \frac{\theta}{360} \times \pi r^2$$

- **Worked** ($\theta = 90^\circ$, $r = 8$): fraction $\frac{1}{4} \rightarrow \text{arc} = \frac{1}{4} \times 16\pi = 4\pi$ cm; sector area $= \frac{1}{4} \times 64\pi = 16\pi$ cm².

Answer shapes: **M1** a correct method, **A1** the correct answer.

2 Practice

2.1 Find the circumference of a circle of radius 5 cm, in terms of π . [1]

2.2 Find the area of a circle of radius 6 cm, in terms of π . [2]

2.3 Find the arc length of a sector with angle 90° and radius 12 cm, in terms of π . [2]

3 Exam-style questions

3.1 A sector has angle 120° and radius 9 cm.

(a) Find its area, in terms of π . [2]

(b) Find its perimeter, in terms of π . [3]

3.2 A circle has area $36\pi \text{ cm}^2$. Find its radius. [2]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $2\pi \times 5 = 10\pi \text{ cm}$.

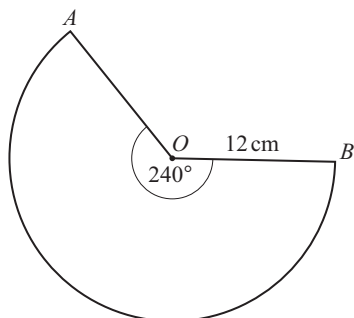
2.2 $\pi \times 6^2 = 36\pi \text{ cm}^2$.

2.3 $\frac{90}{360} \times 2\pi \times 12 = \frac{1}{4} \times 24\pi = 6\pi \text{ cm}$.

3.1 (a) $\frac{120}{360} \times \pi \times 9^2 = \frac{1}{3} \times 81\pi = 27\pi \text{ cm}^2$. (b) $\text{arc} = \frac{1}{3} \times 2\pi \times 9 = 6\pi$; perimeter = arc + $2r = 6\pi + 18 \text{ cm}$.

3.2 $\pi r^2 = 36\pi \rightarrow r^2 = 36 \rightarrow r = 6 \text{ cm}$.

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NOT TO
SCALE

The diagram shows a major sector, AOB , of a circle.
The sector angle is 240° and the radius is 12 cm.

- (a) Show that the length of the major arc AB is 16π cm.

[1]

- (b) OA is joined to OB to form a cone.

Work out the volume of the cone.

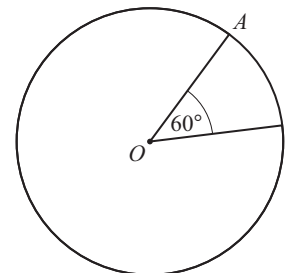
Give your answer in the form $\frac{(a\sqrt{b})\pi}{3}$ where a is an integer and b is a prime number.

..... cm^3 [6]



Question 24 is printed on the next page

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NOT TO
SCALE

The diagram shows a circle, centre O , radius 12 cm.

- (a) Work out the area of the **minor** sector AOB .
Give your answer in terms of π in its simplest form.

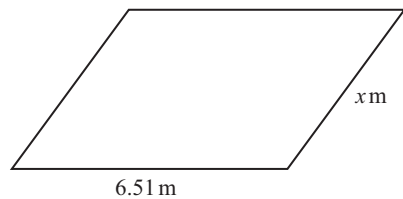
..... cm^2 [2]

- (b) Calculate the length of the **major** arc AB .
Give your answer in terms of π in its simplest form.

..... cm [3]



7 The diagram shows a parallelogram.



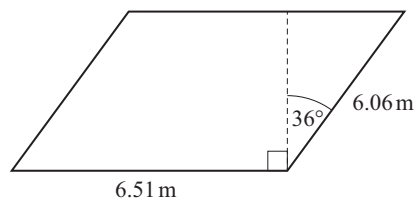
NOT TO
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The parallelogram has the same perimeter as a circle with radius 4 m.

(a) Show that $x = 6.06$ m, correct to 2 decimal places.

[4]

(b)

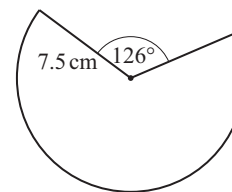


NOT TO
SCALE

The floor of a room is in the shape of this parallelogram.
It costs \$18 per square metre to tile the floor.

Calculate the total cost of tiling the floor.

19



NOT TO
SCALE

The diagram shows a major sector of a circle with radius 7.5 cm.

Calculate the perimeter of the major sector.

..... cm [4]

E5.4 Surface area and volume

Name: _____ Class: _____ Date: _____ Total: 33 marks

KEY WORDS cylinder 圆柱 • cuboid 长方体 • surface area 表面积 • sphere 球
• cross-section 横截面

Objective

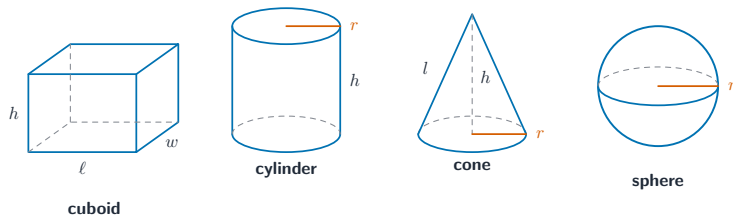
Build the skills to answer Extended exam questions on **surface area and volume** 表面积和体积 — the **cylinder, cone and sphere** 圆柱、圆锥和球, and parts of solids such as a **hemisphere** 半球.

You must be able to:

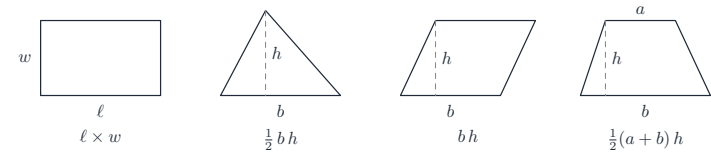
- find the volume of a cylinder, cone, sphere and pyramid
- find the surface area of a cylinder and cone
- work with parts of solids (hemispheres)

1 Worked examples

■ Volumes and surface areas



Learn the key formulas — some are on the formula list



Face areas build up the surface area of a solid

cylinder: $V = \pi r^2 h$, $SA = 2\pi r h + 2\pi r^2$ cone: $V = \frac{1}{3}\pi r^2 h$

sphere: $V = \frac{4}{3}\pi r^3$, $SA = 4\pi r^2$ hemisphere: $V = \frac{2}{3}\pi r^3$

- **Worked (cylinder $r = 5$, $h = 10$):** $V = \pi(25)(10) = 250\pi$; $SA = 100\pi + 50\pi = 150\pi \text{ cm}^2$.

Answer shapes: M1 a correct method, A1 the correct answer.

■ The formulas, and the three traps

Solid	Volume	Curved / total surface area
cylinder	$\pi r^2 h$	curved $2\pi r h$; total $2\pi r h + 2\pi r^2$
cone	$\frac{1}{3}\pi r^2 h$	curved $\pi r l$; total $\pi r l + \pi r^2$
sphere	$\frac{4}{3}\pi r^3$	$4\pi r^2$
prism	area of cross-section $\times$ length	perimeter of cross-section $\times$ length, plus the two ends

Trap 1 — the cone uses the SLANT height. The curved surface area needs l , not the vertical height h . If you are given h and r , find l by Pythagoras: $l = \sqrt{r^2 + h^2}$. Using h in $\pi r l$ is the single most common error on this topic.

Trap 2 — read “curved” against “total”. A closed cylinder has two circular ends; an open one (a pipe, a tank with no lid) has one or none. Count the faces the object actually has before adding anything.

Trap 3 — a hemisphere’s total surface area is not half a sphere’s. Cutting a sphere in half gives $\frac{1}{2}(4\pi r^2) = 2\pi r^2$ of curved surface **plus** the flat circular face πr^2 , so the total is $3\pi r^2$.

Composite solids — split the shape, work out each part, then add or subtract. Say which you are doing, because that reasoning carries a mark.

Worked example. A solid is a cylinder of radius 5 cm and height 12 cm with a cone of the same radius and height 12 cm on top. Find its total volume and its total surface area.

1. Volume of cylinder = $\pi(5)^2(12) = 300\pi$.
2. Volume of cone = $\frac{1}{3}\pi(5)^2(12) = 100\pi$; total = $400\pi = 1260 \text{ cm}^3$ (3 s.f.).

3. Slant height $l = \sqrt{5^2 + 12^2} = 13$.
4. Surface area: base circle $\pi(5)^2 = 25\pi$, curved cylinder $2\pi(5)(12) = 120\pi$, curved cone $\pi(5)(13) = 65\pi$. The join is inside the solid, so it is **not** a surface.
5. Total $= 210\pi = 660 \text{ cm}^2$ (3 s.f.).

2 Practice

2.1 Find the volume of a cuboid 4 cm by 3 cm by 5 cm. [1]

2.2 Find the volume of a cylinder of radius 3 cm and height 10 cm, in terms of π . [2]

2.3 Find the volume of a sphere of radius 3 cm, in terms of π . [2]

2.4 Find the curved surface area of a cone with radius 6 cm and slant height 10 cm. [2]

2.5 A cone has radius 8 cm and vertical height 15 cm. Find its slant height. [2]

2.6 Show that the total surface area of a solid hemisphere of radius r is $3\pi r^2$. [2]

3 Exam-style questions

3.1 A cone has radius 6 cm and height 8 cm. Find its volume, in terms of π . [2]

3.2 Find the total surface area of a cylinder of radius 4 cm and height 9 cm, in terms of π . [3]

3.3 A solid hemisphere has radius 6 cm. Find its volume, in terms of π . [2]

3.4 A grain silo is a cylinder of radius 3 m and height 8 m, with a cone of the same radius and vertical height 4 m on top. It is open at the bottom.

(a) Calculate the volume of the silo. Give your answer to 3 significant figures. [4]

(b) Calculate the slant height of the cone. [2]

(c) Calculate the total area of metal needed to make the silo, given that it has no base.[4]

(d) The silo is filled with grain of density 0.75 g/cm^3 . Calculate the mass of grain, in tonnes. [3]

(e) A second silo is mathematically similar, with all lengths twice as large. State the ratio of their volumes and of their surface areas. [2]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $4 \times 3 \times 5 = 60 \text{ cm}^3$.

2.2 $\pi \times 3^2 \times 10 = 90\pi \text{ cm}^3$.

2.3 $\frac{4}{3}\pi \times 3^3 = \frac{4}{3}\pi \times 27 = 36\pi \text{ cm}^3$.

2.4 $\pi r l = \pi(6)(10) = 60\pi = 188 \text{ cm}^2$.

2.5 $l = \sqrt{8^2 + 15^2} = \sqrt{289} = 17 \text{ cm}$.

2.6 Curved surface $= \frac{1}{2}(4\pi r^2) = 2\pi r^2$; plus the flat circular face πr^2 , giving $3\pi r^2$.

3.1 $\frac{1}{3}\pi \times 6^2 \times 8 = \frac{1}{3}\pi \times 288 = 96\pi \text{ cm}^3$.

3.2 $SA = 2\pi(4)(9) + 2\pi(4)^2 = 72\pi + 32\pi = 104\pi \text{ cm}^2$.

3.3 $\frac{2}{3}\pi \times 6^3 = \frac{2}{3}\pi \times 216 = 144\pi \text{ cm}^3$.

3.4 (a) Cylinder: $\pi(3)^2(8) = 72\pi$; cone: $\frac{1}{3}\pi(3)^2(4) = 12\pi$; total $= 84\pi = 264 \text{ m}^3$.

(b) $l = \sqrt{3^2 + 4^2} = 5 \text{ m}$.

(c) Curved cylinder $= 2\pi(3)(8) = 48\pi$; curved cone $= \pi(3)(5) = 15\pi$; no base and the join is internal, so total $= 63\pi = 198 \text{ m}^2$.

(d) $264 \text{ m}^3 = 264 \times 10^6 \text{ cm}^3$; mass $= 264 \times 10^6 \times 0.75 = 1.98 \times 10^8 \text{ g} = 198 \text{ tonnes}$.

(e) Volumes $1 : 8$; surface areas $1 : 4$.

14 A cube contains a solid metal sphere.
The sphere touches all the faces of the cube.
The side length of the cube is 8 cm.

(a) Show that the volume of the sphere is $\frac{256}{3}\pi \text{ cm}^3$.

[1]

(b) Calculate the percentage of the cube that is **not** occupied by the sphere.

..... % [3]

(c) The density of the metal of the sphere is 7.86 g/cm^3 .

Calculate the mass of the sphere.
Give your answer in kilograms.
[Density = mass ÷ volume]

..... kg [2]



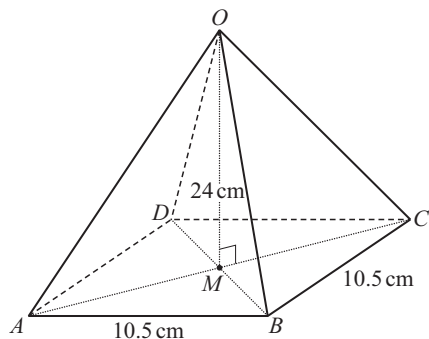
(d) The sphere is melted down and made into a solid cylinder with radius 3.1 cm.

Calculate the **total** surface area of the cylinder.

..... cm^2 [4]



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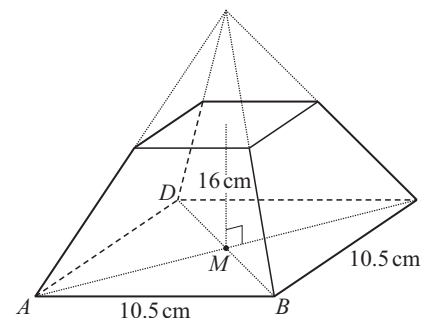
NOT TO
SCALE

The diagram shows a pyramid $OABCD$.
The pyramid has a square base, $ABCD$, with sides 10.5 cm.
The vertex O is vertically above the centre of the base, M .
The height of the pyramid is 24 cm.

- (a) Calculate the angle that OA makes with the base.

..... [4]

(b)



NOT TO
SCALE

The diagram shows a frustum of the pyramid $OABCD$.
The height of the frustum is 16 cm.

Calculate the volume of the frustum.

..... cm^3 [5]

Question 21 is on page 16.