

Week 16

IGCSE Mathematics

Homework bundle for this week

本周作业合集

exercises.lol

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1. An angle of 40° is called:
☐ acute
☐ right
☐ obtuse
☐ reflex
2. Three angles on a straight line are x , 50° and 70° . Find x (degrees).

3. What three-figure bearing points due east?

4. How many lines of symmetry does a square have?

5. Two similar shapes have lengths in the ratio 2:3. The area ratio is 4:b. What is b?

6. The angle in a semicircle is how many degrees?

7. To construct a triangle from three given sides you mainly use a ruler and:
☐ a pair of compasses
☐ a protractor only
☐ a calculator
☐ a set of scales
8. A flat shape that folds up into a solid is called a _____.

9. Bearings are measured clockwise from north.
☐ True ☐ False
10. Line symmetry means a mirror line splits the shape into:
☐ two matching halves
☐ four equal parts

☐ a smaller copy

☐ a rotation

1. **acute**

Less than 90° is acute.

2. **60**

$x + 50 + 70 = 180$, so $x = 60^\circ$.

3. **90**

Clockwise from north, east is 090° .

4. **4**

Two diagonals and two through the side midpoints: 4 lines.

5. **9**

Area scales by k^2 : $2^2:3^2 = 4:9$, so $b = 9$.

6. **90**

An angle standing on a diameter is always 90° .

7. **a pair of compasses**

Compasses swing arcs of the correct length for the other two sides.

8. **net**

A net is the unfolded, flat version of a solid.

9. **True**

Always clockwise from north, written as three figures.

10. **two matching halves**

A line of symmetry reflects one half exactly onto the other.

E4.2 Geometrical constructions

Name: _____ Class: _____ Date: _____

Total: 11 marks

KEY WORDS bearing 方位角 • scale drawing 比例图 • construct 作图 • compasses 圆规 • geometrical constructions 几何作图

Objective

Build the skills to answer Extended exam questions on **geometrical constructions** 几何作图 — constructing triangles with **compasses** 圆规, **scale drawings** 比例图, and **bearings** 方位角.

You must be able to:

- describe how to construct a triangle from given sides
- use a scale to convert between drawing and real lengths
- work out a bearing and a back bearing

1 Worked examples

■ Constructions, scale and bearings



acute
 $< 90^\circ$



right
 $= 90^\circ$



obtuse
 $90^\circ - 180^\circ$



reflex
 $180^\circ - 360^\circ$

Leave construction arcs showing; measure bearings clockwise from north

- **Construct a triangle (3 sides):** draw the base, then mark each other side as a compass **arc**; the arcs cross at the third vertex.
- **Scale drawing** 比例图: e.g. 1 cm represents 5 m $\rightarrow$ a 6 cm line is 30 m.
- **Bearing** 方位角: three figures, clockwise from north. **Back bearing** differs by 180° .

Answer shapes: M1 a correct method, A1 the correct answer.

2 Practice

2.1 A map uses a scale of 1 cm to 4 km. A road is 7 cm long on the map. Find its real length. [2]

2.2 The bearing of Q from P is 060° . Find the bearing of P from Q . [2]

2.3 Write down the three-figure bearing of due west. [1]

3 Exam-style questions

3.1 A scale drawing uses 1 cm to represent 20 m. A field is 250 m long. Find the length on the drawing. [2]

3.2 The bearing of B from A is 145° . Find the bearing of A from B . [2]

3.3 Describe how to construct a triangle with sides 6 cm, 5 cm and 4 cm. [2]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $7 \times 4 = 28$ km.

2.2 $060 + 180 = 240^\circ$.

2.3 270° .

3.1 $250 \div 20 = 12.5$ cm.

3.2 $145 + 180 = 325^\circ$.

3.3 draw the 6 cm base with a ruler; from one end set compasses to 5 cm and from the other end to 4 cm, draw two arcs that cross — the crossing point is the third vertex; join it to both ends.



- (a) In triangle ABC , $AB = 8$ cm, $AC = 7$ cm and $BC = 5$ cm.

Using a ruler and compasses only, construct triangle ABC .
The side AB has been drawn for you. [2]
- (b) Measure angle ACB .

..... [1]
- (c) Triangle ABC is a scale drawing of a field.

(i) The scale is 1 : 10 000.

Find the actual distance from A to B .
Give your answer in kilometres.

..... km [1]

(ii) B is due east of A .
Find the bearing of A from B .

..... [1]

- 4 (a) The scale diagram shows the position of town A on a map.
Town B is 12 km from town A on a bearing of 080° .

Using a scale of 1 cm represents 2 km, mark the position of town B on the diagram.



Scale: 1 cm to 2 km

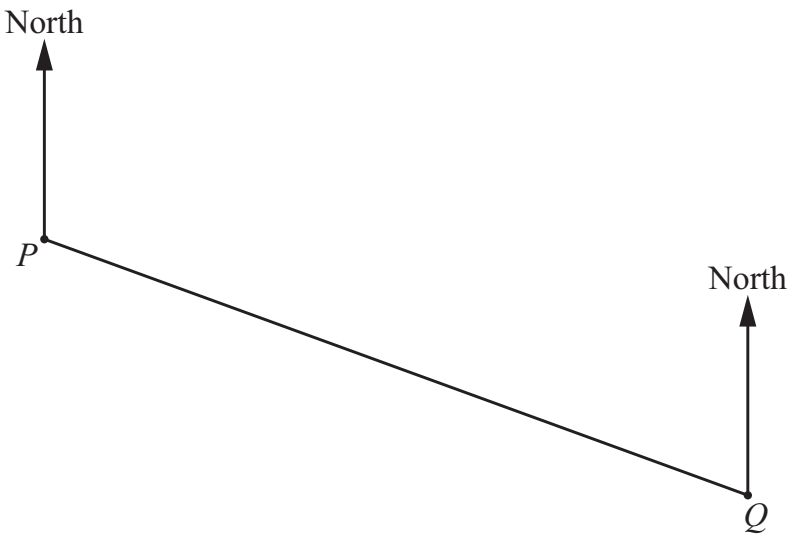
[2]

- (b) The bearing of C from D is 130° .
Work out the bearing of D from C .

..... [2] -

2 The scale drawing shows the positions of two villages, P and Q .

The scale is 1 cm represents 0.5 km.



(a) Find the actual distance between village P and village Q .

..... km [2]

(b) Measure the bearing of village Q from village P .

..... [1]

E4.4 Similarity

Name: _____ Class: _____ Date: _____

Total: 9 marks

KEY WORDS similar 相似 • similar shapes 相似形 • scale factor 比例因子 • area and volume 面积和体积
• triangle 三角形

Objective

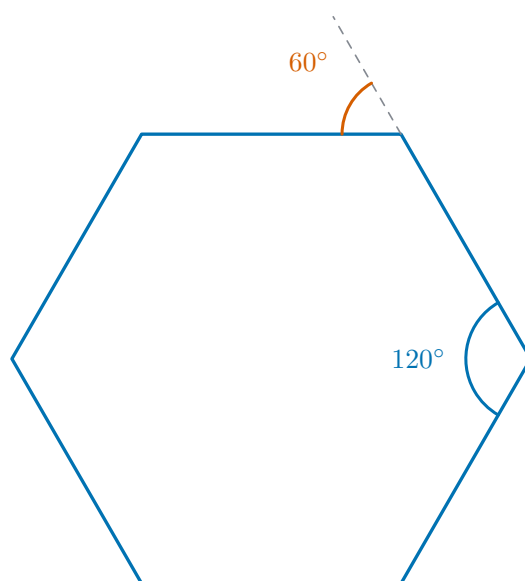
Build the skills to answer Extended exam questions on **similarity** 相似 — **similar shapes** 相似形, the **scale factor** 比例因子, and the **area and volume** 面积和体积 ratios k^2 and k^3 .

You must be able to:

- find a missing length in similar shapes
- use the area ratio k^2 for similar shapes
- use the volume ratio k^3 for similar solids

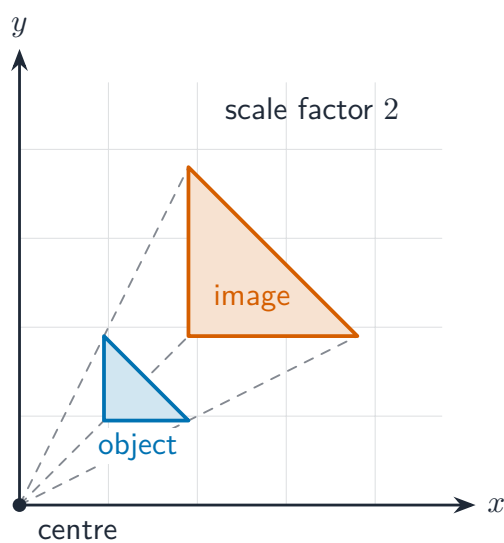
1 Worked examples

- Similar shapes and solids



$$\text{interior} + \text{exterior} = 180^\circ$$

Similar shapes have equal angles and sides in the same ratio



Similar shapes are linked by an enlargement

- **Similar** 相似: same angles, all sides $\times$ the **scale factor** 比例因子 k .
- $\frac{\text{area } A}{\text{area } B} = k^2$; $\frac{\text{volume } A}{\text{volume } B} = k^3$.
- **Worked:** lengths 2 : 3, smaller volume 40 cm³ $\rightarrow$ ratio 8 : 27 $\rightarrow$ larger = $40 \times \frac{27}{8} = 135$ cm³.

Answer shapes: M1 a correct method, A1 the correct answer.

2 Practice

2.1 Two similar triangles have a scale factor of 3. A side of the small triangle is 4 cm. Find the matching side of the large triangle. [1]

2.2 Two similar shapes have lengths in the ratio 2 : 5. Write down the ratio of their areas. [1]

2.3 Two similar solids have lengths in the ratio 1 : 2. Write down the ratio of their volumes. [1]

3 Exam-style questions

3.1 Two similar shapes have lengths in the ratio 3 : 4. The area of the smaller is 18 cm². Find the area of the larger. [3]

3.2 Two similar bottles have heights 10 cm and 15 cm. The smaller holds 400 ml. Find the volume of the larger. [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $4 \times 3 = 12$ cm.

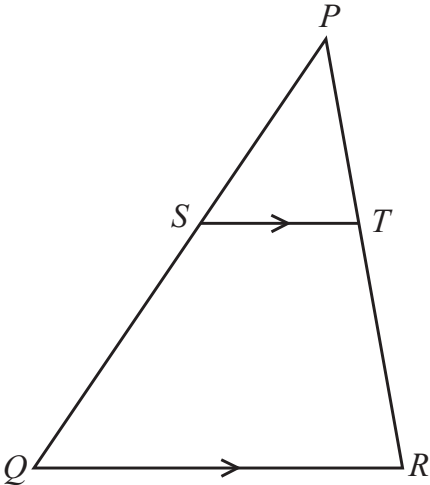
2.2 $2^2 : 5^2 = 4 : 25$.

2.3 $1^3 : 2^3 = 1 : 8$.

3.1 area ratio $= 3^2 : 4^2 = 9 : 16 \rightarrow \text{larger} = 18 \times \frac{16}{9} = 32$ cm².

3.2 length ratio $10 : 15 = 2 : 3 \rightarrow \text{volume ratio } 2^3 : 3^3 = 8 : 27 \rightarrow \text{larger} = 400 \times \frac{27}{8} = 1350$ ml.

11



NOT TO
SCALE

In the diagram, S lies on PQ and T lies on PR .
 ST is parallel to QR .

- (a) Explain why triangle PQR is mathematically similar to triangle PST .
Give a geometrical reason for each statement you make.

.....
.....
.....
..... [3]

- (b) $ST = 3\text{ cm}$, $QR = 9\text{ cm}$ and $PS = 5\text{ cm}$.

Work out PQ .

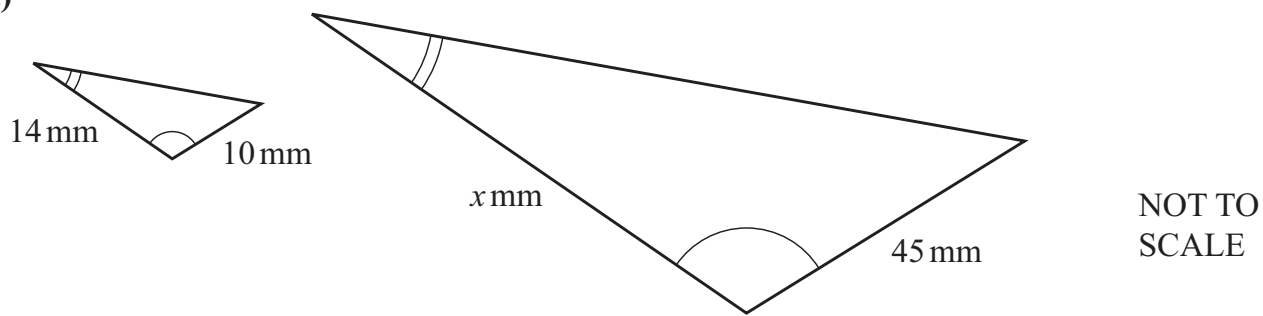
$PQ = \text{.....cm}$ [2]

- (c) The area of triangle PST is $2k\text{ cm}^2$.

Find, in terms of k , the area of quadrilateral $QRTS$.

..... cm^2 [2]

22 (a)



The diagram shows two mathematically similar triangles.

Find the value of x .

$x =$ [2]

- (b) The surface areas of two mathematically similar containers are 124 cm^2 and 279 cm^2 .
The capacity of the smaller container is 56 ml.

Find the capacity of the larger container.

..... ml [3]

E4.6 Angles

Name: _____ Class: _____ Date: _____

Total: 14 marks

KEY WORDS polygon 多边形 • angles 角 • co-interior 同旁内角 • alternate 内错角 • corresponding 同位角

Objective

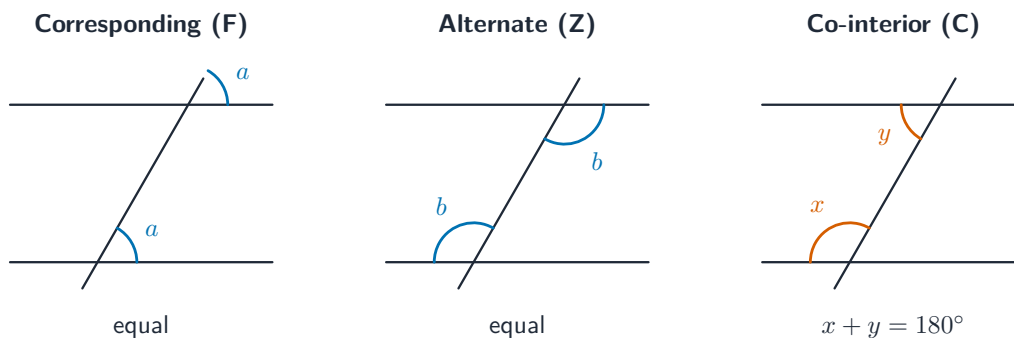
Build the skills to answer Extended exam questions on **angles** 角 — **angles in parallel lines** 平行线中的角, **triangle and quadrilateral** 三角形和四边形 angles, and **polygon** 多边形 angles. Always **give a reason**.

You must be able to:

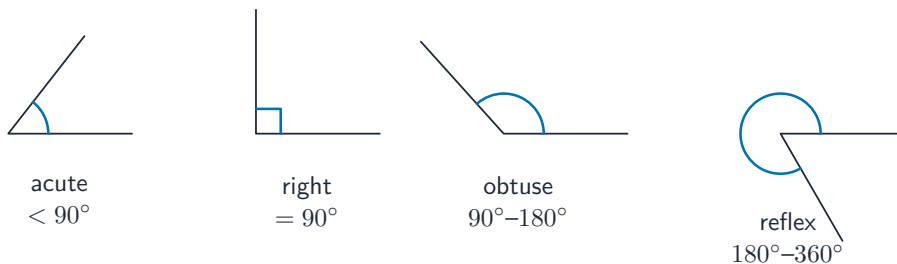
- use corresponding, alternate and co-interior angles
- use the angle sums of triangles and quadrilaterals
- find interior and exterior angles of a regular polygon

1 Worked examples

■ Angle facts



Give the correct reason, not just the number



The basic types of angle

- Angles on a line $= 180^\circ$; at a point $= 360^\circ$; **vertically opposite** 对顶角 equal.
- **Corresponding** 同位角 (F) and **alternate** 内错角 (Z) angles equal; **co-interior** 同旁内角 (C) add to 180° .
- Polygon: sum of interior angles $= (n - 2) \times 180^\circ$; exterior angles sum to 360° .
- **Worked:** regular hexagon — exterior $= \frac{360}{6} = 60^\circ$, interior $= 120^\circ$.

Answer shapes: M1 a correct method, A1 the correct answer.

2 Practice

2.1 Three angles on a straight line are x , 55° and 65° . Find x . [2]

2.2 A triangle has angles x , $2x$ and 60° . Find x . [2]

2.3 Find the exterior angle of a regular pentagon. [2]

3 Exam-style questions

3.1 A line crosses two parallel lines. One angle is 72° . Find the co-interior angle, giving a reason. [2]

3.2 Find the size of each interior angle of a regular octagon. [3]

3.3 The interior angle of a regular polygon is 156° . Find the number of sides. [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $x + 55 + 65 = 180 \rightarrow x = 60^\circ$.

2.2 $x + 2x + 60 = 180 \rightarrow 3x = 120 \rightarrow x = 40^\circ$.

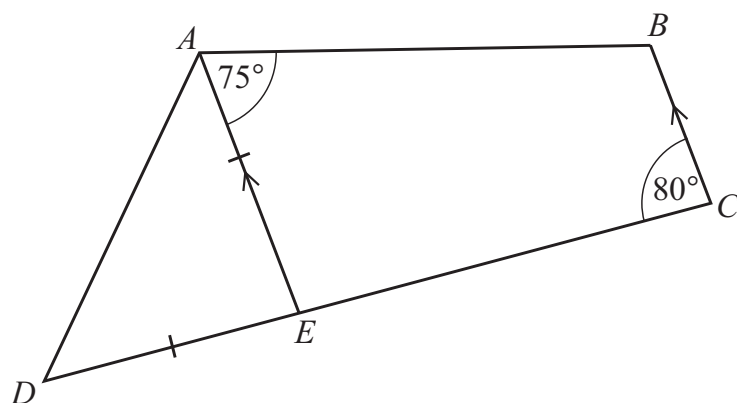
2.3 exterior angle $= \frac{360^\circ}{5} = 72^\circ$.

3.1 co-interior angle $= 180 - 72 = 108^\circ$ (reason: co-interior angles add to 180°).

3.2 exterior angle $= \frac{360}{8} = 45^\circ \rightarrow$ interior $= 180 - 45 = 135^\circ$.

3.3 exterior angle $= 180 - 156 = 24^\circ$; number of sides $= \frac{360}{24} = 15$.

6

NOT TO
SCALE

$ABCD$ is a quadrilateral.
 E lies on CD and AE is parallel to BC .
 $EA = ED$.

Find

(a) angle ABC

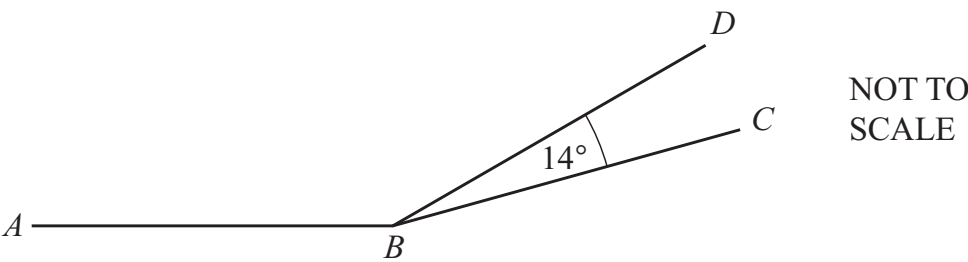
Angle $ABC = \dots\dots\dots$ [1]

(b) angle AED

Angle $AED = \dots\dots\dots$ [1]

(c) angle DAB .

Angle $DAB = \dots\dots\dots$ [2]

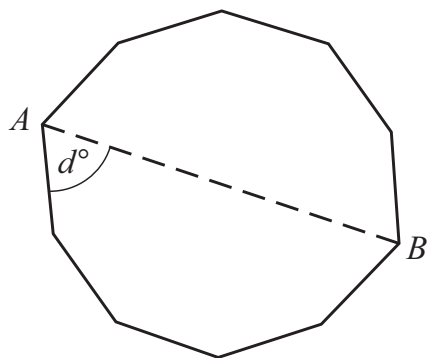


AB and BD are two sides of a regular 15-sided polygon.
 AB and BC are two sides of a regular n -sided polygon.
Angle $DBC = 14^\circ$.

Work out the value of n .

$n = \dots\dots\dots [4]$

- 5 (a) The diagram shows a regular decagon.
 AB is a line of symmetry of the decagon.



NOT TO
SCALE

Work out the value of d .

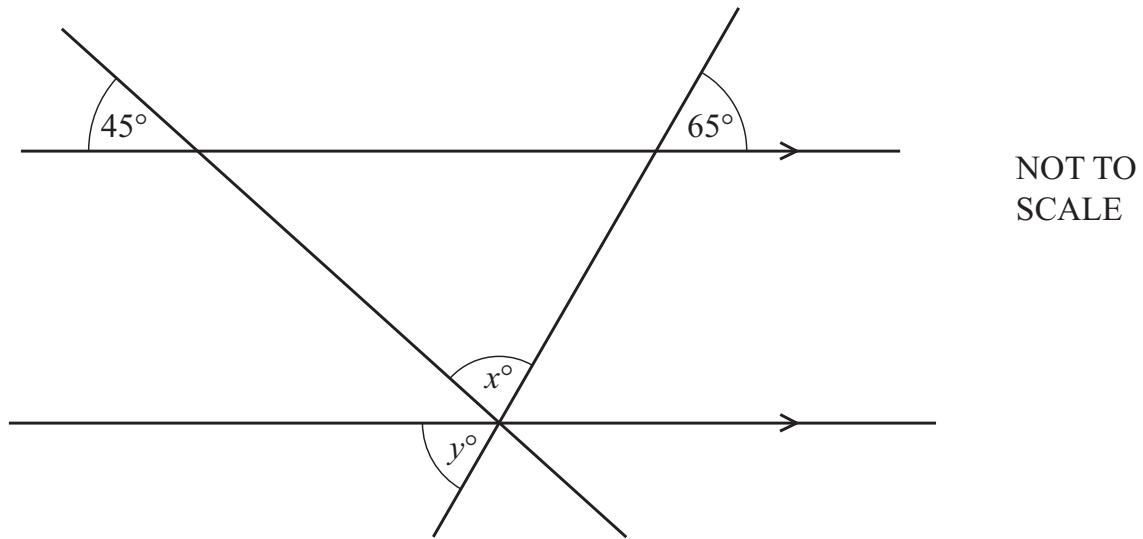
$$d = \dots\dots\dots [3]$$

- (b) The exterior angle of a regular polygon with n sides is 45° .

Work out the value of n .

$$n = \dots\dots\dots [1]$$

3



The diagram shows two straight lines intersecting two parallel lines.

Find the value of x and the value of y .

$x =$

$y =$

[3]

E4.7 Circle theorems I

Name: _____ Class: _____ Date: _____

Total: 9 marks

KEY WORDS cyclic quadrilateral 圆内接四边形 • circle theorems 圆定理 • circle 圆 • centre 圆心
• tangent 切线

Objective

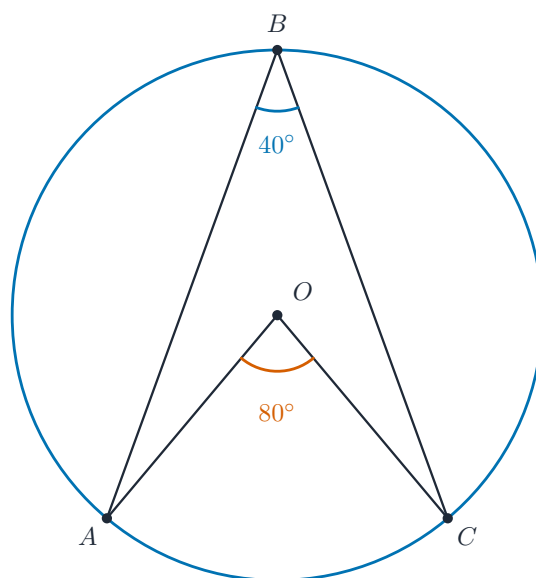
Build the skills to answer Extended exam questions on **circle theorems** 圆定理 — the angle at the **centre**, angles in the **same segment**, the **cyclic quadrilateral** 圆内接四边形, and the semicircle/tangent facts. Always **give a reason**.

You must be able to:

- use “angle at the centre = twice the angle at the circumference”
- use “angles in the same segment are equal” and the cyclic-quadrilateral rule
- use the semicircle and tangent–radius right angles

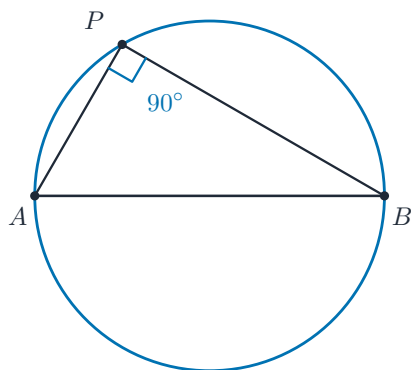
1 Worked examples

■ Circle theorems

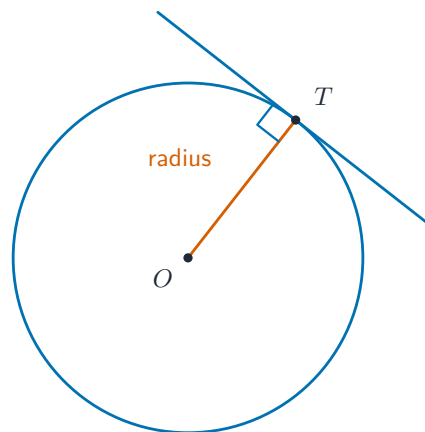


angle at centre = $2 \times$ angle at circumference

Standing on the same arc



angle in a
semicircle = 90°



tangent meets
radius = 90°

Further circle theorems: angle in a semicircle and the tangent

- Angle at **centre** = $2 \times$ angle at **circumference** (same arc).
- Angles in the **same segment** are equal; opposite angles of a **cyclic quadrilateral** add to 180° .
- Angle in a **semicircle** = 90° ; tangent meets radius at 90° .
- **Worked:** circumference angle $40^\circ \rightarrow$ centre angle = $2 \times 40 = 80^\circ$.

Answer shapes: M1 a correct method, A1 the correct answer.

2 Practice

2.1 The angle at the circumference standing on an arc is 35° . Find the angle at the centre on the same arc. [1]

2.2 A triangle is drawn with one side a diameter. State the angle at the point on the circle. [1]

2.3 In a cyclic quadrilateral, one angle is 85° . Find the opposite angle. [2]

3 Exam-style questions

3.1 A, B, C lie on a circle, centre O . The angle AOC at the centre is 130° . Find the angle ABC at the circumference, giving a reason. [2]

3.2 $PQRS$ is a cyclic quadrilateral. Angle $PQR = 95^\circ$ and angle $QRS = 70^\circ$. Find angles PSR and SPQ . [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $2 \times 35 = 70^\circ$.

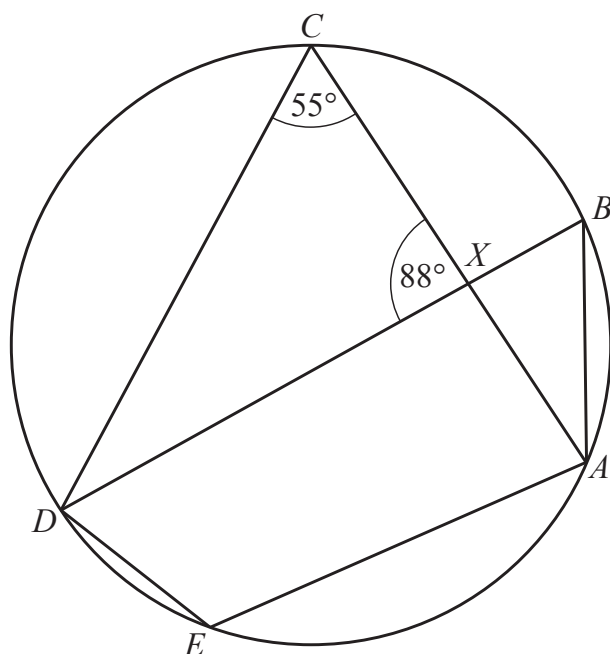
2.2 90° (angle in a semicircle).

2.3 opposite angles of a cyclic quadrilateral add to 180° : $180 - 85 = 95^\circ$.

3.1 angle $ABC = \frac{1}{2} \times 130 = 65^\circ$ (reason: angle at the centre is twice the angle at the circumference).

3.2 opposite angles add to 180° : $PSR = 180 - 95 = 85^\circ$; $SPQ = 180 - 70 = 110^\circ$.

14

NOT TO
SCALE

A, B, C, D and E lie on the circle.

AC and BD intersect at X .

Angle $ACD = 55^\circ$ and angle $CXD = 88^\circ$.

(a) Complete the statements, giving a geometrical reason in each part.

Angle $CDB = \dots\dots\dots$ because $\dots\dots\dots$

$\dots\dots\dots$

Angle $ABD = \dots\dots\dots$ because $\dots\dots\dots$

$\dots\dots\dots$

Angle $AED = \dots\dots\dots$ because $\dots\dots\dots$

$\dots\dots\dots$

[6]

- (b)** Triangle CXD is mathematically similar to triangle BXA .
 $DX = 8.0$ cm, $BX = 2.7$ cm and $AX = 4.0$ cm.

(i) Work out the length of CX .

$$CX = \dots\dots\dots \text{ cm [2]}$$

(ii) Complete the statement.

$$\text{Area of triangle } CXD : \text{area of triangle } BXA = \dots\dots\dots : \dots\dots\dots \text{ [1]}$$

- 15 (a)** Write 66 000 in standard form.

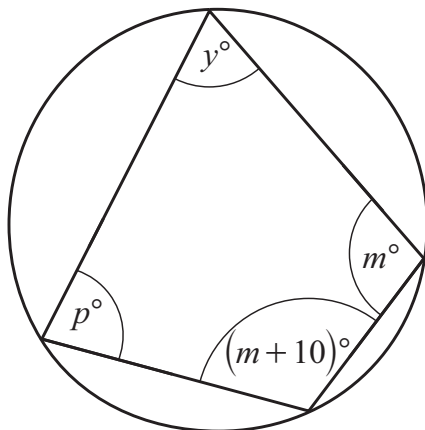
$$\dots\dots\dots \text{ [1]}$$

- (b)** Work out $(3.7 \times 10^8) + (3.7 \times 10^7)$.

Give your answer in standard form.

$$\dots\dots\dots \text{ [2]}$$

21

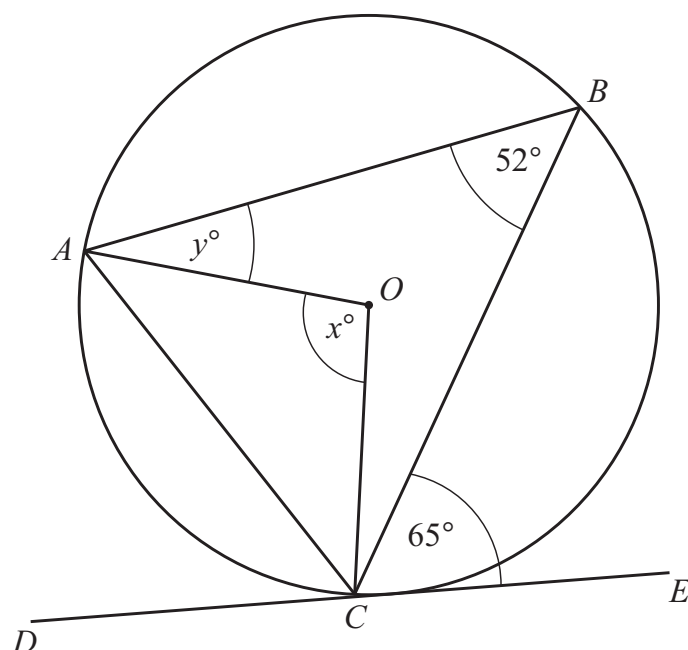
NOT TO
SCALE

The diagram shows a cyclic quadrilateral.

The ratio $p : m = 2 : 3$.

Find the value of y .

$y = \dots\dots\dots$ [4]

NOT TO
SCALE

A , B and C lie on a circle centre O .

DE is a tangent to the circle at C .

Angle $ABC = 52^\circ$ and angle $BCE = 65^\circ$.

- (a) Find the value of x .
Give a geometrical reason for your answer.

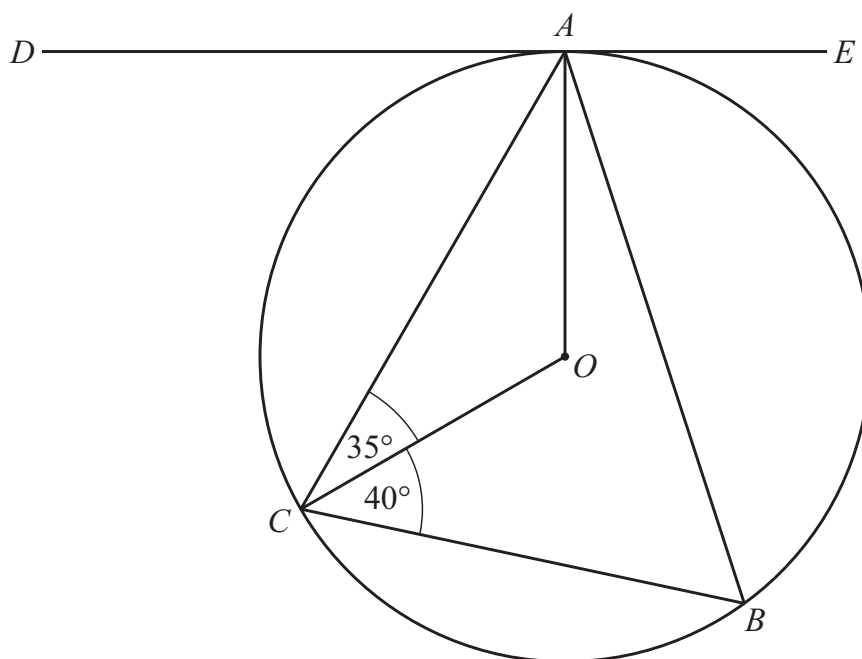
$x = \dots\dots\dots$ because $\dots\dots\dots$

$\dots\dots\dots$ [2]

- (b) Find the value of y .

$y = \dots\dots\dots$ [2]

10

NOT TO
SCALE

A , B and C are three points on a circle, centre O .

DE is a tangent to the circle at A .

Angle $ACO = 35^\circ$ and angle $BCO = 40^\circ$.

Find

(a) angle AOC

Angle $AOC = \dots\dots\dots$ [1]

(b) angle ABC

Angle $ABC = \dots\dots\dots$ [1]

(c) angle DAC

Angle $DAC = \dots\dots\dots$ [1]

(d) angle OAB .

Angle $OAB = \dots\dots\dots$ [1]