Week 15

IGCSE Mathematics

Homework bundle for this week

本周作业合集

exercises.lol

Contents 目录

IGCSE Mathematics Name: ______________ Score: ______

1. An angle of 40° is called:

   - ☐ acute
   - ☐ right
   - ☐ obtuse
   - ☐ reflex

2. Three angles on a straight line are x, 50° and 70°. Find x (degrees).

   ____

3. What three-figure bearing points due east?

   ____

4. How many lines of symmetry does a square have?

   ____

5. Two similar shapes have lengths in the ratio 2:3. The area ratio is 4:b. What is b?

   ____

6. The angle in a semicircle is how many degrees?

   ____

7. To construct a triangle from three given sides you mainly use a ruler and:

   - ☐ a pair of compasses
   - ☐ a protractor only
   - ☐ a calculator
   - ☐ a set of scales

8. A flat shape that folds up into a solid is called a ________.

   ____

9. Bearings are measured clockwise from north.

   ☐ True ☐ False

10. Line symmetry means a mirror line splits the shape into:

    - ☐ two matching halves
    - ☐ four equal parts

☐ a smaller copy

☐ a rotation

IGCSE Mathematics

1. **acute**

   Less than 90° is acute.

2. **60**

   x + 50 + 70 = 180, so x = 60°.

3. **90**

   Clockwise from north, east is 090°.

4. **4**

   Two diagonals and two through the side midpoints: 4 lines.

5. **9**

   Area scales by k^2: $2^2{:}3^2 = 4{:}9$, so b = 9.

6. **90**

   An angle standing on a diameter is always 90°.

7. **a pair of compasses**

   Compasses swing arcs of the correct length for the other two sides.

8. **net**

   A net is the unfolded, flat version of a solid.

9. **True**

   Always clockwise from north, written as three figures.

10. **two matching halves**

    A line of symmetry reflects one half exactly onto the other.

E4.2 Geometrical constructions

Name: ______________ Class: __________ Date: ____________ **Total: 23 marks**

KEY WORDS bearing 方位角 • scale drawing 比例图 • construct 作图 • compasses 圆规 • geometrical constructions 几何作图

Objective

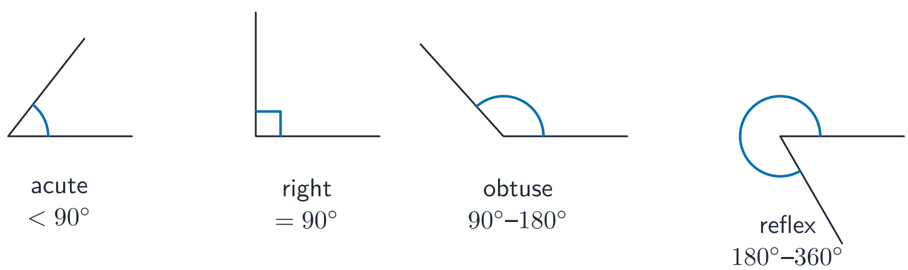
Build the skills to answer Extended exam questions on **geometrical constructions 几何作图** — constructing triangles with **compasses 圆规**, **scale drawings 比例图**, and **bearings 方位角**.

You must be able to:

- describe how to construct a triangle from given sides
- use a scale to convert between drawing and real lengths
- work out a bearing and a back bearing

1 Worked examples

Constructions, scale and bearings



Leave construction arcs showing; measure bearings clockwise from north

- **Construct a triangle (3 sides):** draw the base, then mark each other side as a compass **arc**; the arcs cross at the third vertex.
- **Scale drawing 比例图:** e.g. 1 cm represents 5 m → a 6 cm line is 30 m.
- **Bearing 方位角:** three figures, clockwise from north. **Back bearing** differs by 180°.

Answer shapes: M1 a correct method, **A1** the correct answer.

2 Practice

2.1 A map uses a scale of 1 cm to 4 km. A road is 7 cm long on the map. Find its real length. [2]

2.2 The bearing of Q from P is 060°. Find the bearing of P from Q. [2]

2.3 Write down the three-figure bearing of due west. [1]

3 Exam-style questions

3.1 A scale drawing uses 1 cm to represent 20 m. A field is 250 m long. Find the length on the drawing. [2]

3.2 The bearing of B from A is 145°. Find the bearing of A from B. [2]

3.3 Describe how to construct a triangle with sides 6 cm, 5 cm and 4 cm. [2]

3.4 A triangular plot ABC has $AB = 8$ cm, $AC = 6$ cm and angle $BAC = 50°$.

(a) Using ruler and compasses only, construct triangle ABC. Leave all construction arcs visible. [3]

(b) Construct the perpendicular bisector of BC, and the bisector of angle ABC. Label the point P where they cross. [4]

(c) State the two properties that make P special — one from each construction. [2]

(d) Shade the region inside the triangle that is **both** nearer to B than to C **and** less than 5 cm from A. [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $7 \times 4 = 28$ km.

2.2 $060 + 180 = 240°$.

2.3 270°.

3.1 $250 \div 20 = 12.5$ cm.

3.2 $145 + 180 = 325°$.

3.3 draw the 6 cm base with a ruler; from one end set compasses to 5 cm and from the other end to 4 cm, draw two arcs that cross — the crossing point is the third vertex; join it to both ends.

3.4 (a) AB drawn accurately at 8 cm; angle 50° at A measured with a protractor and $AC = 6$ cm marked; BC joined and all arcs left visible.
(b) Perpendicular bisector: equal arcs drawn from B and from C, above and below BC, and the two crossings joined. Angle bisector at B: an arc from B cutting both arms, then equal arcs from those two points, joined to B. P labelled at the intersection.
(c) P is **equidistant from** B **and** C — every point on the perpendicular bisector of BC is; and P is **equidistant from** BA **and** BC — every point on the bisector of angle B is.
(d) Nearer to B than to C is the side of the perpendicular bisector of BC containing B; less than 5 cm from A is inside a circle of radius 5 cm centred on A; the required region is the overlap of those two, inside the triangle, correctly shaded.

5

(a) In triangle ABC, $AB = 8\,\text{cm}$, $AC = 7\,\text{cm}$ and $BC = 5\,\text{cm}$.

Using a ruler and compasses only, construct triangle ABC.
The side AB has been drawn for you. [2]

(b) Measure angle ACB.

.... [1]

(c) Triangle ABC is a scale drawing of a field.

(i) The scale is 1 : 10 000.

Find the actual distance from A to B.
Give your answer in kilometres.

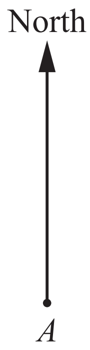
.... km [1]

(ii) B is due east of A.
Find the bearing of A from B.

.... [1]

4 **(a)** The scale diagram shows the position of town A on a map.
Town B is 12 km from town A on a bearing of 080°.

Using a scale of 1 cm represents 2 km, mark the position of town B on the diagram.



Scale: 1 cm to 2 km

[2]

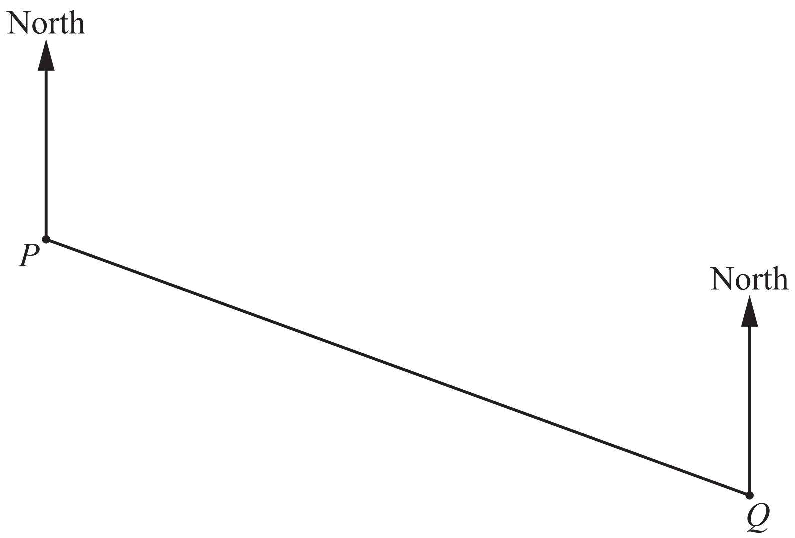
(b) The bearing of C from D is 130°.

Work out the bearing of D from C.

.... [2]

2 The scale drawing shows the positions of two villages, P and Q.

The scale is 1 cm represents 0.5 km.



(a) Find the actual distance between village P and village Q.

.... km [2]

(b) Measure the bearing of village Q from village P.

.... [1]

IGCSE MATHEMATICS • EXERCISE SHEET

E4.4 Similarity

Name: ____________ Class: ________ Date: __________ **Total: 21 marks**

KEY WORDS similar 相似 • similar shapes 相似形 • scale factor 比例因子 • area and volume 面积和体积 • triangle 三角形

Objective

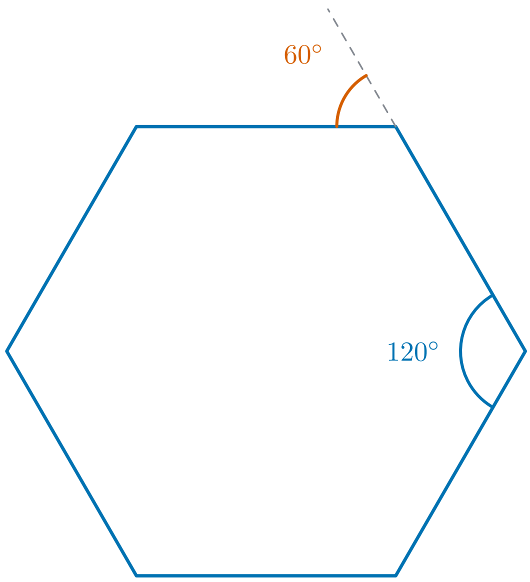
Build the skills to answer Extended exam questions on **similarity** 相似 — **similar shapes** 相似形, the **scale factor** 比例因子, and the **area and volume** 面积和体积 ratios k^2 and k^3.

You must be able to:

- find a missing length in similar shapes
- use the area ratio k^2 for similar shapes
- use the volume ratio k^3 for similar solids

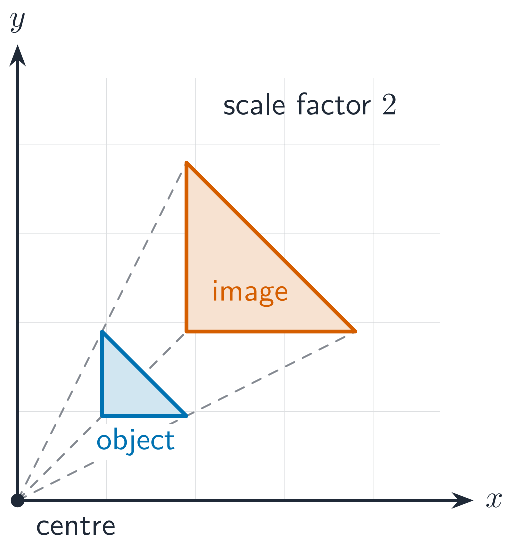
1 Worked examples

■ Similar shapes and solids



interior + exterior = $180°$

Similar shapes have equal angles and sides in the same ratio



Similar shapes are linked by an enlargement

- **Similar** 相似: same angles, all sides × the **scale factor** 比例因子 k.
- $\dfrac{\text{area } A}{\text{area } B} = k^2$; $\dfrac{\text{volume } A}{\text{volume } B} = k^3$.
- **Worked:** lengths $2:3$, smaller volume $40\text{ cm}^3 \to$ ratio $8:27 \to$ larger $= 40 \times \frac{27}{8} = 135\text{ cm}^3$.

Answer shapes: M1 a correct method, **A1** the correct answer.

2 Practice

2.1 Two similar triangles have a scale factor of 3. A side of the small triangle is 4 cm. Find the matching side of the large triangle. [1]

2.2 Two similar shapes have lengths in the ratio 2 : 5. Write down the ratio of their areas. [1]

2.3 Two similar solids have lengths in the ratio 1 : 2. Write down the ratio of their volumes. [1]

3 Exam-style questions

3.1 Two similar shapes have lengths in the ratio 3 : 4. The area of the smaller is 18 cm^2. Find the area of the larger. [3]

3.2 Two similar bottles have heights 10 cm and 15 cm. The smaller holds 400 ml. Find the volume of the larger. [3]

3.3 Triangle ABC has $AB = 6$ cm, $BC = 9$ cm and area 20 cm^2. Triangle PQR is similar to ABC, with PQ corresponding to AB and $PQ = 15$ cm.

(a) Write down the scale factor from ABC to PQR, and find QR. [3]

(b) Find the area of triangle PQR. [3]

(c) A solid cone has a volume of 54 cm^3. A mathematically similar cone has a volume of 128 cm^3. Find the ratio of their heights. [3]

(d) Explain why two shapes with equal angles must be similar, but two shapes with equal areas need not be. [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $4 \times 3 = 12$ cm.

2.2 $2^2 : 5^2 = 4 : 25$.

2.3 $1^3 : 2^3 = 1 : 8$.

3.1 area ratio $= 3^2 : 4^2 = 9 : 16 \to$ larger $= 18 \times \frac{16}{9} = 32$ cm^2.

3.2 length ratio $10 : 15 = 2 : 3 \to$ volume ratio $2^3 : 3^3 = 8 : 27 \to$ larger $= 400 \times \frac{27}{8} = 1350$ ml.

3.3 (a) scale factor $= \dfrac{15}{6} = 2.5$; $QR = 9 \times 2.5 = 22.5$ cm.
(b) Areas scale by the **square** of the length scale factor; $2.5^2 = 6.25$; area $= 20 \times 6.25 = 125$ cm^2.
(c) Volumes scale by the cube of the length scale factor, so $\left(\dfrac{h_2}{h_1}\right)^3 = \dfrac{128}{54} = \dfrac{64}{27}$; $\dfrac{h_2}{h_1} = \sqrt[3]{\dfrac{64}{27}} = \dfrac{4}{3}$, so the ratio is $3 : 4$.
(d) Equal angles fix the **shape**, so one figure is an enlargement of the other and corresponding sides are in a fixed ratio. Equal areas fix only the amount of space enclosed — a 2×8 rectangle and a 4×4 square both have area 16 but different shapes, so they are not similar.

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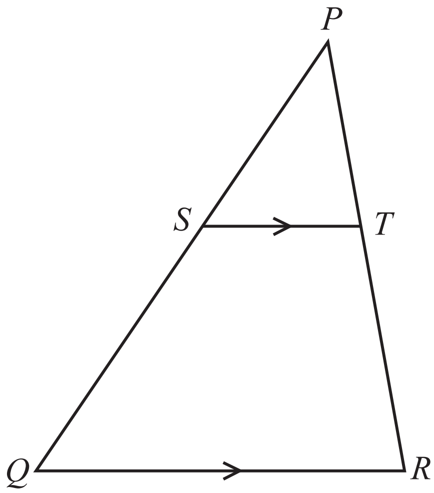


NOT TO SCALE

In the diagram, S lies on PQ and T lies on PR.
ST is parallel to QR.

(a) Explain why triangle PQR is mathematically similar to triangle PST.
Give a geometrical reason for each statement you make.

....

....

....

.... [3]

(b) $ST = 3$ cm, $QR = 9$ cm and $PS = 5$ cm.

Work out PQ.

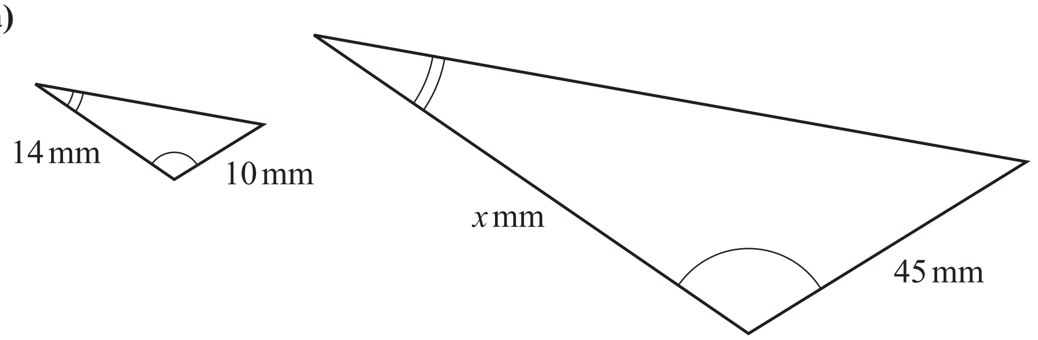
$PQ =$ .... cm [2]

(c) The area of triangle PST is $2k$ cm^2.

Find, in terms of k, the area of quadrilateral $QRTS$.

.... cm^2 [2]

22 **(a)**



NOT TO
SCALE

The diagram shows two mathematically similar triangles.

Find the value of x.

$x =$ .... [2]

(b) The surface areas of two mathematically similar containers are $124\,\text{cm}^2$ and $279\,\text{cm}^2$.
The capacity of the smaller container is 56 ml.

Find the capacity of the larger container.

.... ml [3]

E4.6 Angles

Name: ______________ Class: __________ Date: ____________ **Total: 26 marks**

KEY WORDS polygon 多边形 • angles 角 • co-interior 同旁内角 • alternate 内错角 • corresponding 同位角

Objective

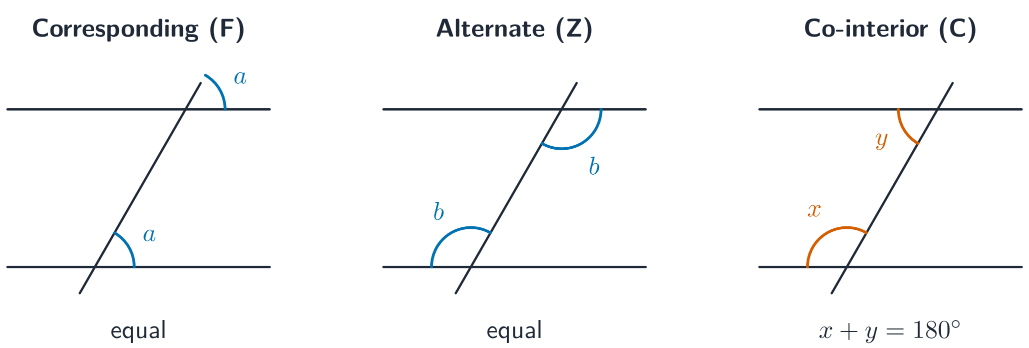
Build the skills to answer Extended exam questions on **angles 角 — angles in parallel lines** 平行线中的角, **triangle and quadrilateral** 三角形和四边形 angles, and **polygon** 多边形 angles. Always **give a reason**.

You must be able to:

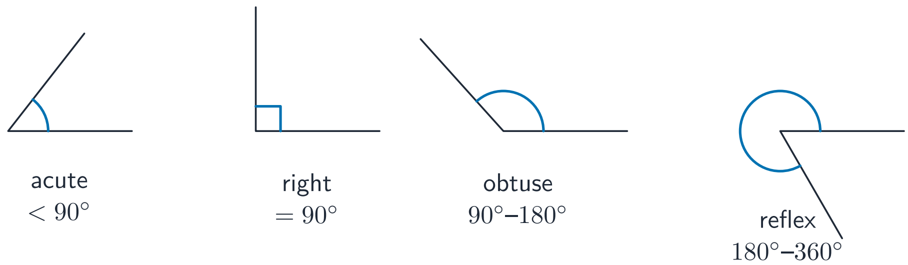
- use corresponding, alternate and co-interior angles
- use the angle sums of triangles and quadrilaterals
- find interior and exterior angles of a regular polygon

1 Worked examples

■ Angle facts



Give the correct reason, not just the number



The basic types of angle

- Angles on a line $= 180°$; at a point $= 360°$; **vertically opposite** 对顶角 equal.
- **Corresponding** 同位角 (F) and **alternate** 内错角 (Z) angles equal; **co-interior** 同旁内角 (C) add to $180°$.
- Polygon: sum of interior angles $= (n-2) \times 180°$; exterior angles sum to $360°$.
- **Worked:** regular hexagon — exterior $= \frac{360}{6} = 60°$, interior $= 120°$.

Answer shapes: M1 a correct method, **A1** the correct answer.

2 Practice

2.1 Three angles on a straight line are x, $55°$ and $65°$. Find x. [2]

2.2 A triangle has angles x, $2x$ and $60°$. Find x. [2]

2.3 Find the exterior angle of a regular pentagon. [2]

3 Exam-style questions

3.1 A line crosses two parallel lines. One angle is $72°$. Find the co-interior angle, giving a reason. [2]

3.2 Find the size of each interior angle of a regular octagon. [3]

3.3 The interior angle of a regular polygon is $156°$. Find the number of sides. [3]

3.4 In the diagram, PQ is parallel to RS. A straight line crosses PQ at A and RS at B. The angle between the line and PQ on the upper right at A is $(3x + 10)°$, and the angle between the line and RS on the lower left at B is $(5x - 30)°$.

(a) Explain why these two angles are equal, naming the angle fact you use, and find x. [4]

(b) Find the size of each of the two angles. [2]

(c) A regular polygon has an exterior angle of $24°$. Find the number of sides and the sum of its interior angles. [4]

(d) Explain why a regular polygon cannot have an interior angle of $160.5°$. [2]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $x + 55 + 65 = 180 \to x = 60°$.

2.2 $x + 2x + 60 = 180 \to 3x = 120 \to x = 40°$.

2.3 exterior angle $= \frac{360°}{5} = 72°$.

3.1 co-interior angle $= 180 - 72 = 108°$ (reason: co-interior angles add to 180°).

3.2 exterior angle $= \frac{360}{8} = 45° \to$ interior $= 180 - 45 = 135°$.

3.3 exterior angle $= 180 - 156 = 24°$; number of sides $= \frac{360}{24} = 15$.

3.4 (a) They are **alternate angles** between the parallel lines PQ and RS, cut by the same transversal, so they are equal; $3x + 10 = 5x - 30$; $40 = 2x$, so $x = 20$.
(b) $3(20) + 10 = 70°$; $5(20) - 30 = 70°$ — the same, as expected.
(c) The exterior angles of any polygon add to 360°; $n = \dfrac{360}{24} = 15$ sides; the interior angle sum $= (n-2) \times 180 = 13 \times 180 = 2340°$.
(d) An interior angle of 160.5° gives an exterior angle of $180 - 160.5 = 19.5°$; $\dfrac{360}{19.5} = 18.46\ldots$, which is not a whole number, so no such regular polygon exists.

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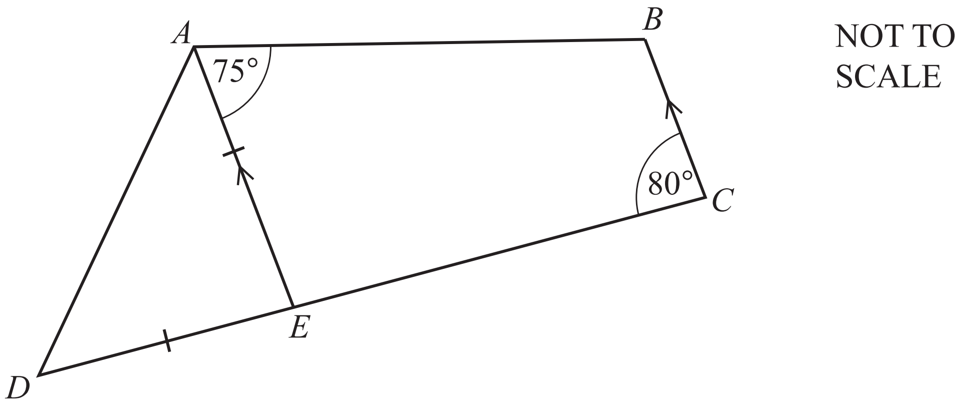

ABCD is a quadrilateral.
E lies on *CD* and *AE* is parallel to *BC*.
EA = *ED*.

Find

(a) angle *ABC*

Angle *ABC* = ... [1]

(b) angle *AED*

Angle *AED* = ... [1]

(c) angle *DAB*.

Angle *DAB* = ... [2]

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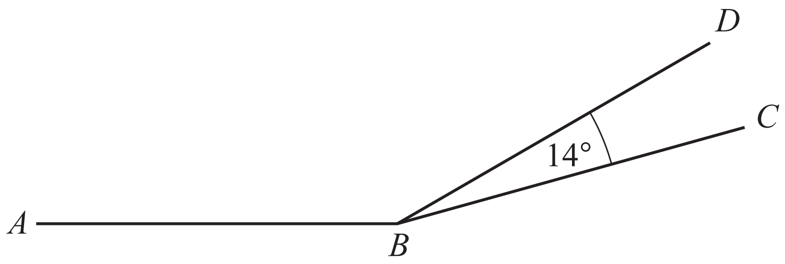


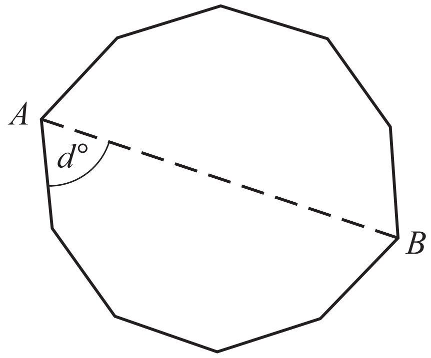
NOT TO SCALE

AB and BD are two sides of a regular 15-sided polygon.
AB and BC are two sides of a regular n-sided polygon.
Angle $DBC = 14°$.

Work out the value of n.

$n =$ .... [4]

5 **(a)** The diagram shows a regular decagon.
AB is a line of symmetry of the decagon.



NOT TO
SCALE

Work out the value of *d*.

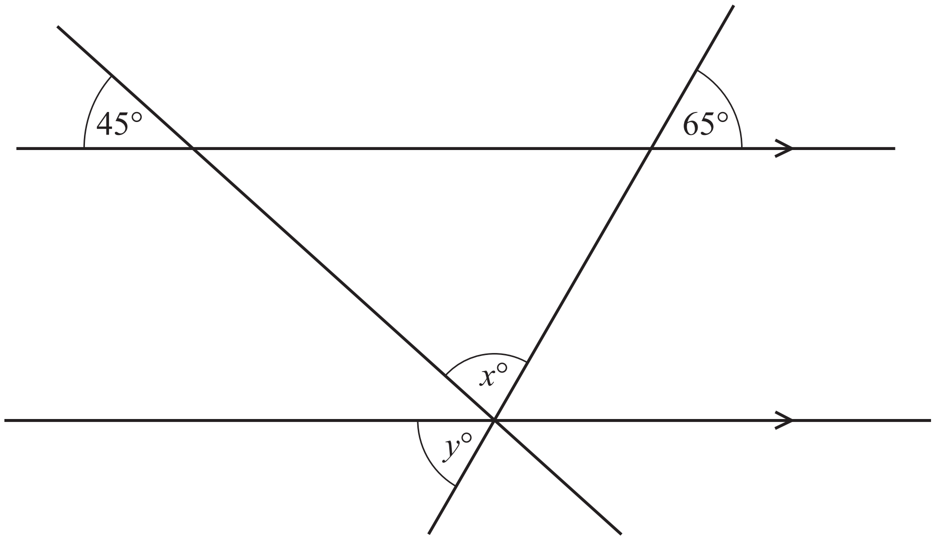
$d =$ .... [3]

(b) The exterior angle of a regular polygon with *n* sides is 45°.

Work out the value of *n*.

$n =$ .... [1]

3



NOT TO SCALE

The diagram shows two straight lines intersecting two parallel lines.

Find the value of x and the value of y.

$x =$ ....

$y =$ .... [3]

IGCSE MATHEMATICS • EXERCISE SHEET

E4.7 Circle theorems I

Name: ____________ Class: ________ Date: __________ **Total: 20 marks**

KEY WORDS cyclic quadrilateral 圆内接四边形 • circle theorems 圆定理 • alternate segment theorem 弦切角定理 • circle 圆 • centre 圆心

Objective

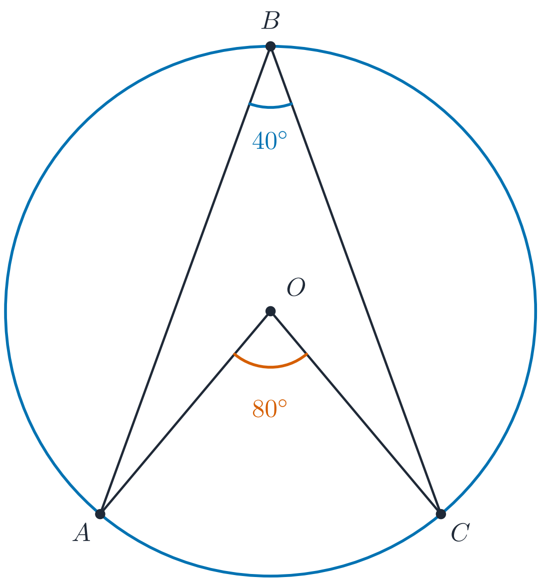
Build the skills to answer Extended exam questions on **circle theorems** 圆定理 — the angle at the **centre**, angles in the **same segment**, the **cyclic quadrilateral** 圆内接四边形, and the semicircle/tangent facts. Always **give a reason**.

You must be able to:

- use "angle at the centre = twice the angle at the circumference"
- use "angles in the same segment are equal" and the cyclic-quadrilateral rule
- use the semicircle and tangent–radius right angles

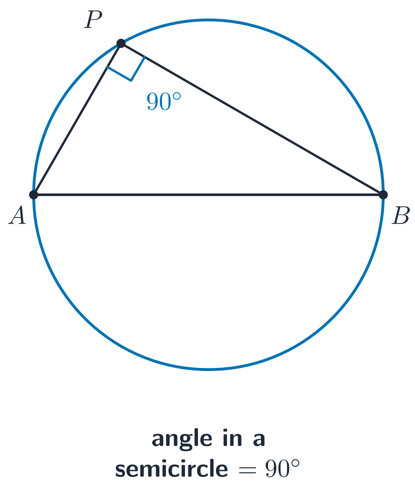
1 Worked examples

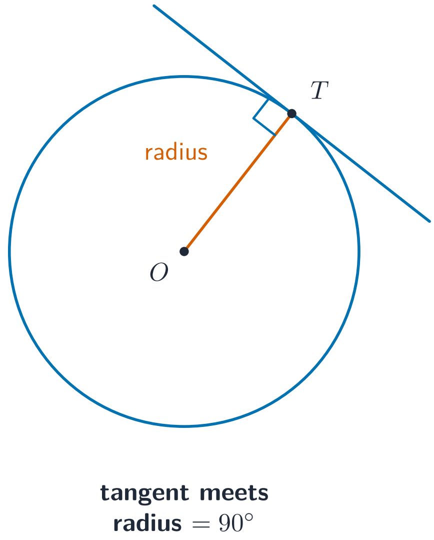
■ Circle theorems



angle at centre $= 2\times$ angle at circumference

Standing on the same arc





Further circle theorems: angle in a semicircle and the tangent

- Angle at **centre** $= 2\times$ angle at **circumference** (same arc).
- Angles in the **same segment** are equal; opposite angles of a **cyclic quadrilateral** add to $180°$.
- Angle in a **semicircle** $= 90°$; tangent meets radius at $90°$.
- **Worked:** circumference angle $40°$ $\to$ centre angle $= 2 \times 40 = 80°$.

Answer shapes: M1 a correct method, **A1** the correct answer.

2 Practice

2.1 The angle at the circumference standing on an arc is $35°$. Find the angle at the centre on the same arc. [1]

2.2 A triangle is drawn with one side a diameter. State the angle at the point on the circle. [1]

2.3 In a cyclic quadrilateral, one angle is $85°$. Find the opposite angle. [2]

3 Exam-style questions

3.1 A, B, C lie on a circle, centre O. The angle AOC at the centre is $130°$. Find the angle ABC at the circumference, giving a reason. [2]

3.2 $PQRS$ is a cyclic quadrilateral. Angle $PQR = 95°$ and angle $QRS = 70°$. Find angles PSR and SPQ. [3]

3.3 A, B, C and D lie on a circle with centre O. BD is a diameter. Angle $BAC = 32°$ and angle $ADB = 54°$.

(a) Write down, with a reason, the size of angle BCD. [2]

(b) Find angle BDC, giving your reason. [2]

(c) Find angle ABD, showing your working. [2]

(d) Find angle AOC, giving a reason. [2]

(e) TC is a tangent to the circle at C. Find the angle between TC and the chord CD, naming the theorem you use. [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $2 \times 35 = 70°$.

2.2 $90°$ (angle in a semicircle).

2.3 opposite angles of a cyclic quadrilateral add to $180°$: $180 - 85 = 95°$.

3.1 angle $ABC = \frac{1}{2} \times 130 = 65°$ (reason: angle at the centre is twice the angle at the circumference).

3.2 opposite angles add to $180°$: $PSR = 180 - 95 = 85°$; $SPQ = 180 - 70 = 110°$.

3.3 (a) $90°$ — the angle in a semicircle is a right angle, since BD is a diameter.
(b) Angle BDC and angle BAC are angles in the **same segment**, standing on the chord BC, so they are equal: $32°$.
(c) In triangle ABD, angle $BAD = 90°$ (angle in a semicircle) and angle $ADB = 54°$; so angle $ABD = 180 - 90 - 54 = 36°$.
(d) Angle ABC is the angle at the circumference standing on arc AC, and angle AOC is the angle at the centre on the same arc, so $AOC = 2 \times ABC$. $ABC = ABD + DBC$; in triangle BCD, $DBC = 180 - 90 - 32 = 58°$, so $ABC = 36 + 58 = 94°$ and $AOC = 188°$ — the reflex angle; the angle marked at the centre on the other side is $360 - 188 = 172°$.
(e) By the **alternate segment theorem**, the angle between the tangent TC and the chord CD equals the angle in the alternate segment, angle CBD; $CBD = 58°$ from part (d), so the required angle is $58°$.

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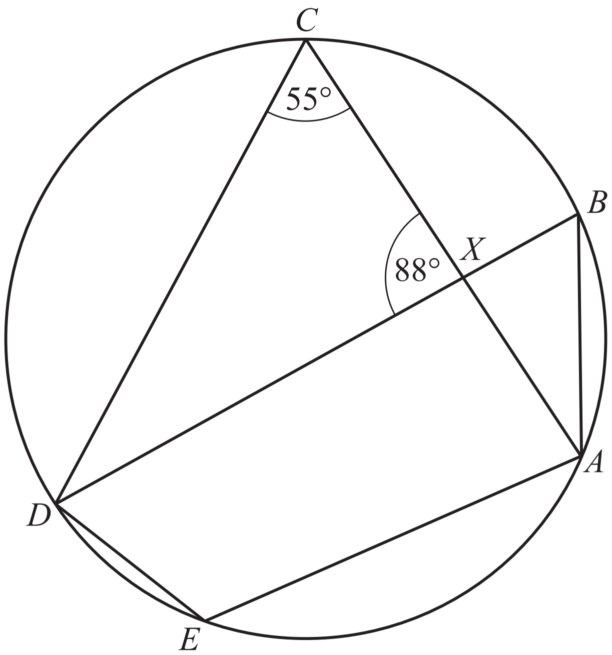


NOT TO SCALE

A, *B*, *C*, *D* and *E* lie on the circle.
AC and *BD* intersect at *X*.
Angle *ACD* = 55° and angle *CXD* = 88°.

(a) Complete the statements, giving a geometrical reason in each part.

Angle *CDB* = ... because ....

....

Angle *ABD* = ... because ....

....

Angle *AED* = ... because ....

....

[6]

(b) Triangle CXD is mathematically similar to triangle BXA.
$DX = 8.0$ cm, $BX = 2.7$ cm and $AX = 4.0$ cm.

(i) Work out the length of CX.

$CX =$ .... cm [2]

(ii) Complete the statement.

Area of triangle CXD : area of triangle BXA = ... : .... [1]

15 (a) Write 66 000 in standard form.

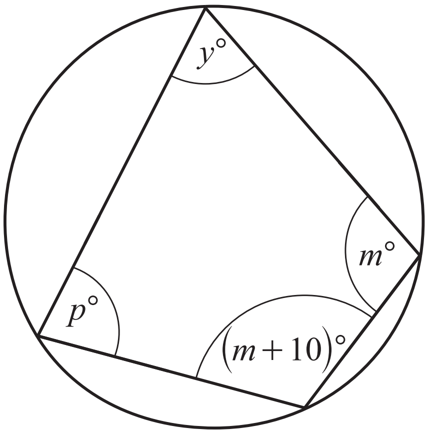
.... [1]

(b) Work out $(3.7 \times 10^8) + (3.7 \times 10^7)$.

Give your answer in standard form.

.... [2]

21



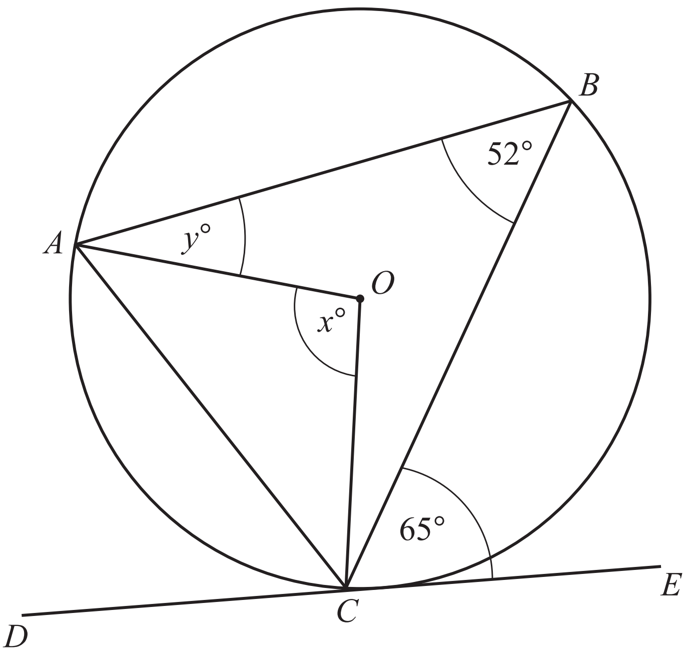
NOT TO SCALE

The diagram shows a cyclic quadrilateral.
The ratio $p : m = 2 : 3$.

Find the value of y.

$y =$ .... [4]

12



NOT TO
SCALE

A, *B* and *C* lie on a circle centre *O*.
DE is a tangent to the circle at *C*.
Angle *ABC* = 52° and angle *BCE* = 65°.

(a) Find the value of *x*.
Give a geometrical reason for your answer.

x = ... because ....

.... [2]

(b) Find the value of *y*.

y = ... [2]

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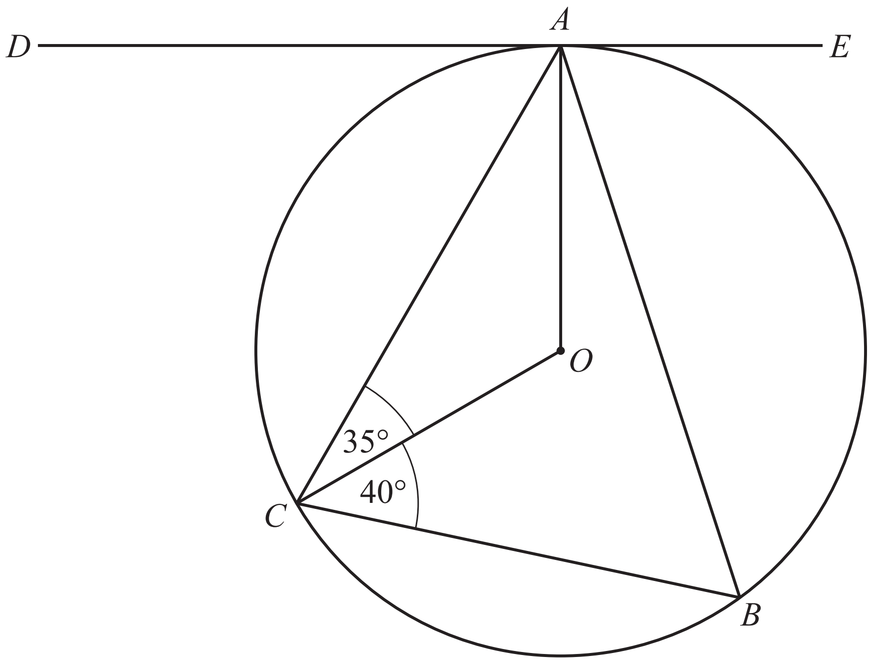

NOT TO SCALE

A, B and C are three points on a circle, centre O.
DE is a tangent to the circle at A.
Angle $ACO = 35°$ and angle $BCO = 40°$.

Find

(a) angle AOC

Angle AOC = ... [1]

(b) angle ABC

Angle ABC = ... [1]

(c) angle DAC

Angle DAC = ... [1]

(d) angle OAB.

Angle OAB = ... [1]