

Week 13

IGCSE Mathematics

Homework bundle for this week

本周作业合集

exercises.lol

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3. Coordinate geometry

Starter Quiz · 课前小测

IGCSE Mathematics

Name: _____ Score: _____

- The point $(-4, 3)$ is in which quadrant?
☐ I
☐ II
☐ III
☐ IV
- Find the gradient of the line through $(1, 2)$ and $(4, 11)$.

- Find the length of the segment from $(1, 2)$ to $(4, 6)$.

- Rearrange $5x + 4y = 8$ into $y = mx + c$. What is c (the y -intercept)?

- A line is parallel to $y = 4x - 1$ and passes through $(1, -3)$. In $y = 4x + c$, what is c ?

- The point where the x -axis and y -axis cross is called the _____.

- A line that goes down as you move to the right has a gradient that is:
☐ negative
☐ positive
☐ zero
☐ undefined
- The midpoint of $(1, 2)$ and $(4, 6)$ is $(a, 4)$. What is a ?

- A line has gradient $3/2$. The gradient of a line perpendicular to it is:
☐ $-2/3$
☐ $2/3$
☐ $-3/2$
☐ $3/2$

10. In $y = mx + c$, the letter m stands for the:

- ☐ gradient
- ☐ y-intercept
- ☐ x-value
- ☐ midpoint

3. Coordinate geometry

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1. II

Negative x (left) and positive y (up) → quadrant II (top-left).

2. 3

$(11 - 2)/(4 - 1) = 9/3 = 3.$

3. 5

$\sqrt{((4-1)^2 + (6-2)^2)} = \sqrt{(9 + 16)} = \sqrt{25} = 5.$

4. 2

$4y = -5x + 8$, so $y = -(5/4)x + 2$; the intercept is 2.

5. -7

Parallel → same gradient 4. $-3 = 4(1) + c$, so $c = -7$.

6. origin

The origin is the point (0,0) where the two axes meet.

7. negative

Falling to the right means the change in y is negative for a positive change in x , so the gradient is negative.

8. 2.5

Average the x -values: $(1 + 4)/2 = 2.5.$

9. -2/3

Flip and change sign: perpendicular gradient $= -1 \div (3/2) = -2/3.$

10. gradient

m is the gradient (steepness); c is the y -intercept.

E3.1 Coordinates

Name: _____ Class: _____ Date: _____

Total: 29 marks

KEY WORDS coordinates 坐标 • midpoint 中点 • length 长度 • origin 原点 • gradient 斜率

Objective

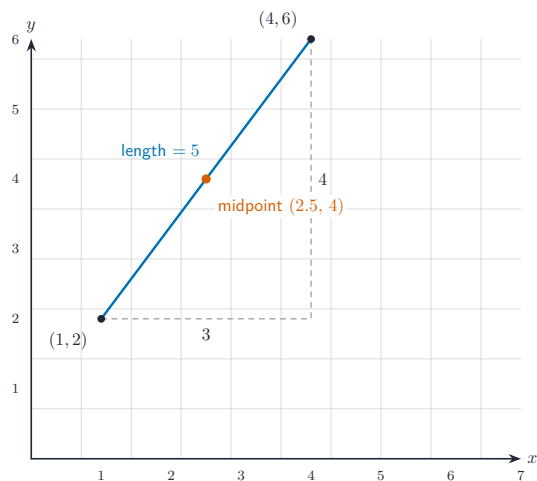
Build the skills to answer Extended exam questions on **coordinates** 坐标 — plotting points, and finding the **length** 长度 and **midpoint** 中点 of a line segment.

You must be able to:

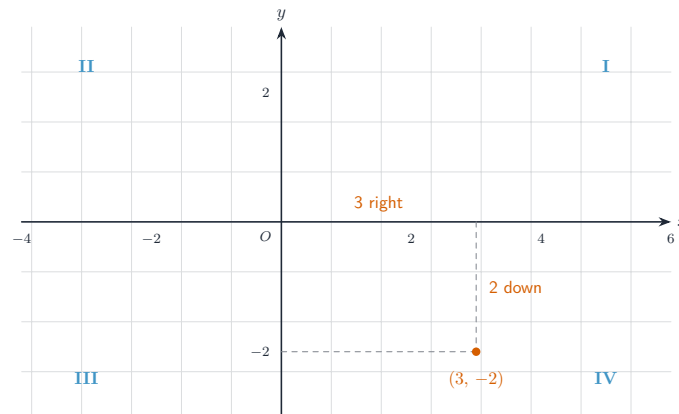
- plot and read coordinates (x, y)
- find the length of a segment using Pythagoras
- find the midpoint of a segment

1 Worked examples

■ Length and midpoint



(x, y) : first across, then up



Points are plotted on the coordinate plane

$$\text{length} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

- **Worked** (1, 2) to (4, 6): length = $\sqrt{3^2 + 4^2} = 5$; midpoint = (2.5, 4).

Answer shapes: M1 a correct method, A1 the correct answer.

■ The three coordinate formulas, and when each is wanted

For $A(x_1, y_1)$ and $B(x_2, y_2)$:

$$\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \quad \text{length} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

The midpoint **adds** and halves; the length and gradient **subtract**. Mixing the two is the commonest error, and it is easy to spot: a midpoint should lie between the two points.

Parallel and perpendicular. Parallel lines have the **same** gradient. Perpendicular lines have gradients whose product is -1 , so the perpendicular gradient is the **negative reciprocal**: if $m = \frac{2}{3}$ then the perpendicular gradient is $-\frac{3}{2}$.

The equation of a line is easiest from $y - y_1 = m(x - x_1)$ using any point on it; rearrange to $y = mx + c$ at the end if the question asks for that form.

Worked example. $A(1, 1)$ and $B(7, 9)$. Find the equation of the perpendicular bisector of AB .

1. Midpoint: $\left(\frac{1+7}{2}, \frac{1+9}{2} \right) = (4, 5)$.

2. Gradient of AB : $\frac{9-1}{7-1} = \frac{8}{6} = \frac{4}{3}$.

3. Perpendicular gradient: the negative reciprocal, $-\frac{3}{4}$.

4. Through the midpoint: $y - 5 = -\frac{3}{4}(x - 4)$.

5. Rearranged: $y = -\frac{3}{4}x + 8$.

The bisector must pass through the **midpoint** — using A or B instead gives a parallel line and loses every mark after step 3.

2 Practice

2.1 Find the midpoint of the segment joining $(2, 3)$ and $(6, 9)$. [2]

2.2 Find the length of the segment from $(0, 0)$ to $(3, 4)$. [2]

2.3 Write down the coordinates of the point 3 left and 2 up from the origin. [1]

2.4 Find the gradient of the line joining $(2, -1)$ and $(6, 7)$. [2]

2.5 State the gradient of a line perpendicular to $y = \frac{2}{5}x + 1$. [1]

2.6 Find the length of the line joining $(0, 0)$ and $(9, 12)$. [2]

3 Exam-style questions

3.1 $A = (1, 1)$ and $B = (7, 9)$.

(a) Find the midpoint of AB . [2]

(b) Find the length of AB . [2]

3.2 $M(3, 5)$ is the midpoint of PQ . Given $P = (1, 2)$, find the coordinates of Q . [2]

3.3 $P = (-2, 3)$ and $Q = (6, 9)$.

(a) Find the midpoint of PQ .

[2]

(b) Find the length of PQ .

[3]

(c) Find the gradient of PQ .

[2]

(d) Find the equation of the line through P and Q , giving your answer in the form $y = mx + c$.

[3]

(e) Find the equation of the perpendicular bisector of PQ .

[3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $\left(\frac{2+6}{2}, \frac{3+9}{2}\right) = (4, 6)$.

2.2 $\sqrt{3^2 + 4^2} = \sqrt{25} = 5$.

2.3 $(-3, 2)$.

2.4 $\frac{7 - (-1)}{6 - 2} = \frac{8}{4} = 2$.

2.5 $-\frac{5}{2}$.

2.6 $\sqrt{9^2 + 12^2} = \sqrt{81 + 144} = \sqrt{225} = 15$.

3.1 (a) $\left(\frac{1+7}{2}, \frac{1+9}{2}\right) = (4, 5)$. (b) $\sqrt{(7-1)^2 + (9-1)^2} = \sqrt{36 + 64} = \sqrt{100} = 10$.

3.2 if M is the midpoint, $Q = (2 \times 3 - 1, 2 \times 5 - 2) = (5, 8)$.

3.3 (a) $\left(\frac{-2+6}{2}, \frac{3+9}{2}\right) = (2, 6)$.

(b) $\sqrt{(6 - (-2))^2 + (9 - 3)^2} = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$.

(c) $\frac{9-3}{6-(-2)} = \frac{6}{8} = \frac{3}{4}$.

(d) $y - 3 = \frac{3}{4}(x + 2)$; $y = \frac{3}{4}x + \frac{3}{2} + 3$; $y = \frac{3}{4}x + 4.5$.

(e) Perpendicular gradient = $-\frac{4}{3}$; through the midpoint $(2, 6)$: $y - 6 = -\frac{4}{3}(x - 2)$;
 $y = -\frac{4}{3}x + \frac{26}{3}$.

- 17 B is the point $(-3, 1)$ and D is the point $(-5, 9)$.
 BD is a diagonal of the kite $ABCD$.
- (a) The ratio of the lengths of the diagonals $BD : AC = 2 : 3$.
- Work out the length of AC .
Give your answer as a surd in its simplest form.
- [5]

- (b) Find the coordinates of the midpoint of BD .
- (..... ,) [2]
- (c) The diagonal AC of the kite passes through the midpoint of BD .
- Find an equation of AC .
Give your answer in the form $y = mx + c$.
- $y =$ [4]



IGCSE MATHEMATICS • EXERCISE SHEET

E3.2 Drawing linear graphs

Name: _____ Class: _____ Date: _____ Total: 24 marks

KEY WORDS perpendicular垂直 • parallel平行 • gradient斜率
• parallel and perpendicular平行和垂直 • drawing linear graphs画直线图象

Objective

Build the skills to answer Extended exam questions on **drawing linear graphs** 画直线图象 — the equation $y = mx + c$, the **gradient** 斜率, finding the equation of a line, and **parallel and perpendicular** 平行和垂直 lines.

You must be able to:

- find the gradient and y -intercept of a line
- find the equation of a line from its gradient and a point
- use the gradient rules for parallel and perpendicular lines

1 Worked examples

■ Straight lines

$y = 2x + 1$

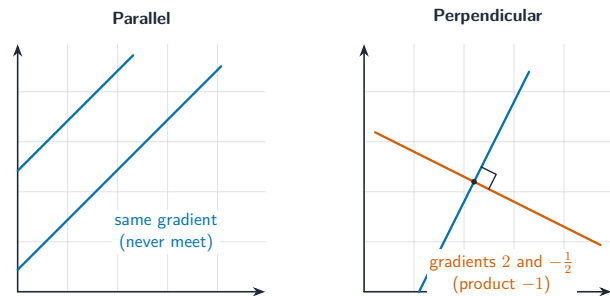
rise = 2

run = 1

gradient = $\frac{\text{rise}}{\text{run}} = 2$

y -intercept $c = 1$

$m = \text{gradient}, c = y\text{-intercept}$



Gradients of parallel and perpendicular lines

- **Gradient** 斜率: $m = \frac{\text{change in } y}{\text{change in } x}$. Through $(1, 2)$ and $(4, 11)$: $m = \frac{9}{3} = 3$.
- **Equation:** gradient 3 through $(1, 2) \rightarrow 2 = 3(1) + c \rightarrow c = -1 \rightarrow y = 3x - 1$.
- **Parallel** 平行: same gradient. **Perpendicular** 垂直: $m_1 m_2 = -1$.

Answer shapes: M1 a correct method, A1 the correct answer.

2 Practice

2.1 Write down the gradient and y -intercept of $y = 4x - 3$. [2]

2.2 Find the gradient of the line through $(2, 1)$ and $(5, 10)$. [2]

2.3 Write down the gradient of a line parallel to $y = 2x + 7$. [1]

3 Exam-style questions

3.1 Find the equation of the line with gradient 2 that passes through $(3, 1)$. [3]

3.2 Rearrange $3x + 2y = 12$ into the form $y = mx + c$ and write down its gradient. [2]

3.3 Find the gradient of a line perpendicular to $y = \frac{1}{2}x + 4$. [1]

3.4 The line L passes through $A(-2, 7)$ and $B(4, -5)$.

(a) Find the gradient of L and the equation of L in the form $y = mx + c$. [4]

(b) Find the coordinates of the points where L crosses the two axes. [3]

(c) Find the equation of the line through B that is perpendicular to L . [3]

(d) Show that the point $(1, 1)$ is the midpoint of AB , and state what this tells you about the perpendicular bisector of AB . [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 gradient 4; y -intercept -3 .

2.2 $m = \frac{10-1}{5-2} = \frac{9}{3} = 3$.

2.3 2.

3.1 $y = 2x + c$; substitute $(3, 1)$: $1 = 2(3) + c \rightarrow c = -5 \rightarrow y = 2x - 5$.

3.2 $2y = -3x + 12 \rightarrow y = -\frac{3}{2}x + 6$; gradient $-\frac{3}{2}$.

3.3 the gradient is $\frac{1}{2}$, so the perpendicular gradient is -2 (flip and change sign).

3.4 (a) gradient $= \frac{-5-7}{4-(-2)} = \frac{-12}{6} = -2$; using A : $7 = -2(-2) + c$, so $c = 3$; $y = -2x + 3$.

(b) y -axis: $x = 0$ gives $(0, 3)$. x -axis: $0 = -2x + 3$, so $x = 1.5$, giving $(1.5, 0)$.

(c) perpendicular gradient $= -\frac{1}{-2} = \frac{1}{2}$; through $B(4, -5)$: $-5 = \frac{1}{2}(4) + c$, so $c = -7$; $y = \frac{1}{2}x - 7$.

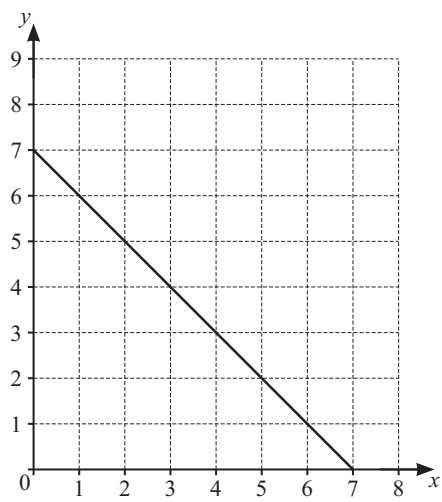
(d) midpoint $= \left(\frac{-2+4}{2}, \frac{7+(-5)}{2} \right) = (1, 1)$; the perpendicular bisector passes through $(1, 1)$ and has gradient $\frac{1}{2}$, so its equation is $y - 1 = \frac{1}{2}(x - 1)$, i.e. $y = \frac{1}{2}x + \frac{1}{2}$.

22 The equation of line L is $y = -\frac{1}{2}x + 7$.

Find an equation of the line perpendicular to line L that passes through the point $(3, 5)$.
Give your answer in the form $y = mx + c$.

$y = \dots\dots\dots$ [3]

6



The line $x + y = 7$ is drawn on the grid.

(a) On the grid, draw the line $y = 2x + 1$. [2]

(b) Use your graph to solve these simultaneous equations.

$x + y = 7$ $y = 2x + 1$

$x = \dots\dots\dots$

$y = \dots\dots\dots$

[1]

