

Week 7

IGCSE Mathematics

Homework bundle for this week

本周作业合集

exercises.lol

Contents 目录

Topic 2 quiz	3
Exercise sheet E2.1	6
Past-paper questions E2.1	10
Exercise sheet E2.2	11
Past-paper questions E2.2	16
Exercise sheet E2.4	17
Past-paper questions E2.4	20
Exercise sheet E2.5	21
Past-paper questions E2.5	26
Exercise sheet E2.6	31
Past-paper questions E2.6	36
Exercise sheet E2.10	40

Past-paper questions E2.10	46
Exercise sheet E2.11	50
Past-paper questions E2.11	54

2. Algebra and graphs

Starter Quiz · 课前小测

IGCSE Mathematics

Name: _____ Score: _____

1. Find the value of $3x^2 - 2y$ when $x = 4$ and $y = 5$.

2. Simplify $(x^2 - 2x)/(x^2 - 5x + 6)$.

☐ $x/(x - 3)$

☐ $x/(x - 2)$

☐ $(x - 2)/(x - 3)$

☐ $x - 3$

3. $(5x^3)^2 = k \cdot x^6$. What is the coefficient k ?

4. Solve $2x + y = 7$ and $3x - y = 8$. What is x ?

5. Solve $-3 \leq 3x - 2 < 7$. The solution is $-1/3 \leq x < ?$ What is the upper bound?

6. The n th term of 2, 5, 8, 11, ... is $3n - 1$. What is the 10th term?

7. On a distance–time graph, the gradient represents the:

☐ speed

☐ acceleration

☐ distance

☐ time

8. The roots of a graph are where it:

☐ crosses the x -axis ($y = 0$)

☐ crosses the y -axis

☐ is steepest

☐ turns around

9. Add $x/3 + (x-4)/2$ over a common denominator of 6.

☐ $(5x - 12)/6$

☐ $(2x - 4)/6$

☐ $(5x + 12)/6$

☐ $(x - 4)/6$

10. Solve $x^2 + 5x + 6 = 0$ by factorising.

☐ $x = -2$ or $x = -3$

☐ $x = 2$ or $x = 3$

☐ $x = -1$ or $x = -6$

☐ $x = 5$ or $x = 6$

1. 38

$$3 \times 4^2 - 2 \times 5 = 3 \times 16 - 10 = 48 - 10 = 38.$$

2. $x/(x-3)$

Factorise: $x(x-2) / [(x-2)(x-3)]$, cancel $(x-2) \rightarrow x/(x-3)$.

3. 25

$$(5x^3)^2 = 5^2 \times (x^3)^2 = 25x^6, \text{ so } k = 25.$$

4. 3

Adding the equations: $5x = 15$, so $x = 3$ (then $y = 1$).

5. 3

Add 2: $-1 \leq 3x < 9$; divide by 3: $-1/3 \leq x < 3$.

6. 29

$$3 \times 10 - 1 = 30 - 1 = 29.$$

7. speed

Distance $\div$ time = speed, which is the gradient.

8. crosses the x-axis ($y = 0$)

Roots are the x-values where $y = 0$ — where the curve meets the x-axis.

9. $(5x - 12)/6$

$$2x/6 + 3(x-4)/6 = (2x + 3x - 12)/6 = (5x - 12)/6.$$

10. $x = -2$ or $x = -3$

$$(x+2)(x+3) = 0, \text{ so } x = -2 \text{ or } x = -3.$$

E2.1 Introduction to algebra

Name: _____ Class: _____ Date: _____ **Total: 12 marks**

KEY WORDS like terms 同类项 • substitute 代入 • terms and like terms 项和同类项
• introduction to algebra 代数入门 • expression 表达式

Objective

Build the skills to answer Extended exam questions on **introduction to algebra** 代数入门 — **substitution** 代入, **terms and like terms** 项和同类项, and simplifying expressions.

You must be able to:

- substitute numbers into an expression
- collect like terms to simplify
- use correct order of operations with letters

1 Worked examples

■ Substituting and simplifying



Letters stand for numbers; like terms can be combined

- **Substitute:** $3x^2 - 2y$ with $x = 4$, $y = 5 \rightarrow 3(16) - 10 = 38$.
- **Like terms** 同类项: same letters $\rightarrow$ combine. $2a^2 + 5a^2 = 7a^2$; $3ab - 9ab = -6ab$.

$$2a^2 + 3ab - 1 + 5a^2 - 9ab + 4$$

↓ group like terms

$$7a^2 - 6ab + 3$$

terms that share a colour are like terms

like terms: same letters/powers · add coefficients only

Collecting like terms: group the terms that share a colour (the same letters)

- **Worked:** $2a^2 + 3ab - 1 + 5a^2 - 9ab + 4 = 7a^2 - 6ab + 3$.

Answer shapes: **M1** a correct method, **A1** the correct answer.

2 Practice

2.1 Find the value of $4x - 3y$ when $x = 5$ and $y = 2$. [2]

2.2 Simplify $7p + 2q - 3p + 5q$. [2]

2.3 Find the value of $2x^2 + 1$ when $x = -3$. [2]

3 Exam-style questions

3.1 Simplify $5a^2 - 2ab + 3 - a^2 + 6ab$. [2]

3.2 Find the value of $\frac{2a+b}{a-b}$ when $a = 7$ and $b = 1$. [2]

3.3 Find the value of $3x^2 - 5x$ when $x = -2$. [2]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $4(5) - 3(2) = 20 - 6 = 14.$

2.2 $7p - 3p + 2q + 5q = 4p + 7q.$

2.3 $2(-3)^2 + 1 = 2(9) + 1 = 19.$

3.1 $5a^2 - a^2 = 4a^2$, $-2ab + 6ab = 4ab$, so $4a^2 + 4ab + 3.$

3.2 $\frac{2(7)+1}{7-1} = \frac{15}{6} = \frac{5}{2}.$

3.3 $3(-2)^2 - 5(-2) = 3(4) + 10 = 22.$

Name:

IGCSE Mathematics: **w25 22**

- 8** Kemi buys p pens that each cost 40 cents.
She pays with \$20.

Write an expression, in terms of p , for the change, in **cents**, Kemi receives from the \$20.

..... cents [2]

E2.2 Algebraic manipulation

Name: _____ Class: _____ Date: _____

Total: 25 marks

KEY WORDS term 项 • expand 展开 • area 面积 • difference of two squares 平方差
• factorising 因式分解

Objective

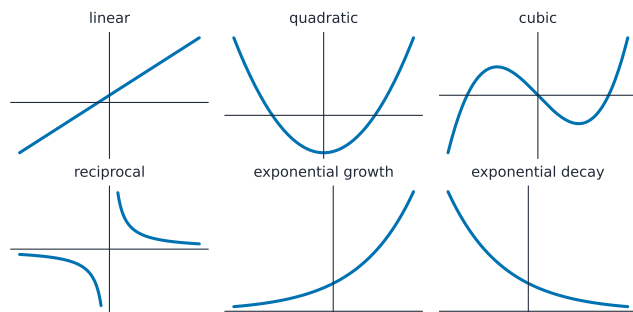
Build the skills to answer Extended exam questions on **algebraic manipulation** 代数运算 — **expanding brackets** 展开括号 and **factorising** 因式分解 (common factors, difference of two squares, quadratics).

You must be able to:

- expand single and double brackets
- factorise by taking out a common factor
- factorise a difference of two squares and a quadratic

1 Worked examples

■ Expand and factorise

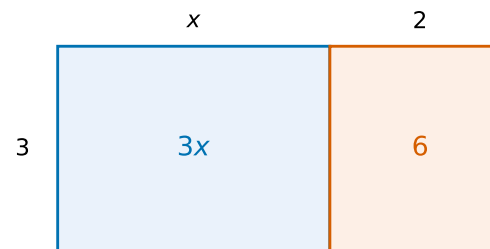


recognise $y = x^2$, $y = 1/x$, $y = 2^x$ from shape

Factorising is the reverse of expanding

- **Expand:** $(2x + 1)(x - 4) = 2x^2 - 7x - 4$.

$$3(x + 2) = 3x + 6$$



$a(b + c) = ab + ac$ · area model of expansion

The area model of expanding a bracket: $3(x + 2) = 3x + 6$

- **Common factor:** $9x^2 + 15xy = 3x(3x + 5y)$.
- **Difference of two squares** 平方差: $9x^2 - 16 = (3x + 4)(3x - 4)$.
- **Quadratic:** $2x^2 + 7x + 3 = (x + 3)(2x + 1)$ (split the middle term).

Answer shapes: M1 a correct method, A1 the correct answer.

■ Choosing the factorising method

Work through these in order; the first one that applies is the right one.

1. **Common factor first, every time.** $6x^2 + 9x = 3x(2x + 3)$. Missing it makes everything afterwards harder, and the mark for the common factor is lost even if the rest is right.
2. **Difference of two squares**, when there are two terms and both are squares with a minus between: $a^2 - b^2 = (a + b)(a - b)$. So $x^2 - 49 = (x + 7)(x - 7)$ and $9x^2 - 25 = (3x + 5)(3x - 5)$.
3. **A quadratic with $a = 1$:** find two numbers that multiply to the constant and add to the coefficient of x . For $x^2 + 3x - 10$ they are +5 and -2, so it factorises as $(x + 5)(x - 2)$.
4. **A quadratic with $a \neq 1$:** multiply a by c , find two numbers multiplying to that and adding to b , split the middle term and factorise in pairs.
5. **Four terms:** factorise in pairs, then take out the common bracket: $xy + 2x + 3y + 6 = x(y + 2) + 3(y + 2) = (x + 3)(y + 2)$.

Worked example (the $a \neq 1$ method). Factorise $2x^2 + 5x - 3$.

1. $a \times c = 2 \times (-3) = -6$.
2. Two numbers multiplying to -6 and adding to +5: +6 and -1.
3. Split the middle term: $2x^2 + 6x - x - 3$.

4. Factorise in pairs: $2x(x + 3) - 1(x + 3)$.
5. Take out the common bracket: $(2x - 1)(x + 3)$.

Always **check by expanding**. It takes one line and catches a sign error, which is the commonest loss on this topic.

2 Practice

2.1 Expand $3x(2x - 5)$. [1]

2.2 Factorise $6a + 9ab$. [1]

2.3 Expand and simplify $(x + 2)(x - 5)$. [2]

2.4 Factorise fully $12x^2 - 27$. [2]

2.5 Factorise $ab + 4a + 3b + 12$. [2]

3 Exam-style questions

3.1 Factorise $x^2 - 49$. [1]

3.2 Factorise $x^2 + 3x - 10$. [2]

3.3 Factorise $2x^2 + 5x - 3$. [2]

3.4 (a) Factorise fully $18x^2 - 8$. [3]

(b) Factorise $3x^2 - 11x + 6$. [3]

(c) Simplify $\frac{x^2 - 9}{x^2 + x - 12}$, showing your factorising. [3]

(d) Hence solve $\frac{x^2 - 9}{x^2 + x - 12} = 2$. [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $3x(2x - 5) = 6x^2 - 15x$.

2.2 $6a + 9ab = 3a(2 + 3b)$.

2.3 $(x + 2)(x - 5) = x^2 - 5x + 2x - 10 = x^2 - 3x - 10$.

2.4 Common factor 3: $3(4x^2 - 9)$; difference of two squares: $3(2x + 3)(2x - 3)$.

2.5 $a(b + 4) + 3(b + 4); (a + 3)(b + 4)$.

3.1 $x^2 - 49 = (x + 7)(x - 7)$.

3.2 two numbers multiplying to -10 and adding to 3 are 5 and -2 : $(x + 5)(x - 2)$.

3.3 $a \times c = -6$, middle $+5$: numbers 6 and $-1 \rightarrow 2x^2 + 6x - x - 3 = 2x(x + 3) - (x + 3) = (2x - 1)(x + 3)$.

3.4 (a) Common factor 2 : $2(9x^2 - 4)$; this is a difference of two squares; $2(3x + 2)(3x - 2)$.

(b) $a \times c = 18$; two numbers multiplying to 18 and adding to -11 are -9 and -2 ; $3x^2 - 9x - 2x + 6 = 3x(x - 3) - 2(x - 3); (3x - 2)(x - 3)$.

(c) Numerator $= (x + 3)(x - 3)$; denominator $= (x + 4)(x - 3)$; cancelling the common $(x - 3)$ gives $\frac{x + 3}{x + 4}$.

(d) $\frac{x + 3}{x + 4} = 2$, so $x + 3 = 2(x + 4)$; $x + 3 = 2x + 8$, so $-5 = x$; $x = -5$.

17 $I = M(k^2 + c^2)$

(a) Find the value of I when $M = 7$, $k = 3$ and $c = 2$.

$I = \dots\dots\dots$ [2]

(b) Rearrange the formula to write k in terms of I , M and c .

$k = \dots\dots\dots$ [3]

10 $b = dm + 2mk$

(a) $d = 3.14$, $m = 7.92$ and $k = 10.16$.

By rounding each value correct to 1 significant figure, work out an estimate for b .

$\dots\dots\dots$ [3]

(b) Rearrange the formula to make m the subject.

$m = \dots\dots\dots$ [2]

E2.4 Indices II

Name: _____ Class: _____ Date: _____

Total: 11 marks

KEY WORDS indices 指数 • equation 方程 • indices in algebra 代数中的指数
 • laws of indices 指数定律 • base 底数

Objective

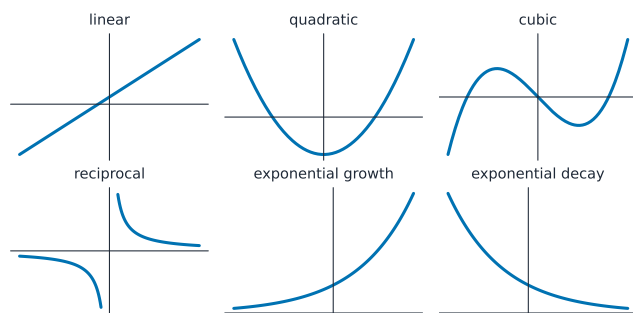
Build the skills to answer Extended exam questions on **indices in algebra** 代数中的指数 — applying the **laws of indices** 指数定律 to algebraic terms, and solving simple index equations.

You must be able to:

- simplify products, quotients and powers of algebraic terms
- handle negative powers in algebra
- solve $a^x = b$ by writing both sides with the same base

1 Worked examples

■ Indices with letters



recognise $y = x^2$, $y = 1/x$, $y = 2^x$ from shape

The index laws work with letters too

$$a^m \times a^n = a^{m+n} \quad a^m \div a^n = a^{m-n} \quad (a^m)^n = a^{mn}$$

- **Worked:** $(5x^3)^2 = 25x^6$; $12a^5 \div 3a^{-2} = 4a^7$; $6x^7y^4 \times 5x^{-5}y = 30x^2y^5$.
- **Index equation:** $2^x = 32 = 2^5 \rightarrow x = 5$.

Answer shapes: **M1** a correct method, **A1** the correct answer.

2 Practice

2.1 Simplify $(3a^2)^3$. [2]

2.2 Simplify $10x^6 \div 2x^2$. [2]

2.3 Simplify $a^4 \times a^{-7}$. [1]

3 Exam-style questions

3.1 Simplify $(2x^2y)^3$. [2]

3.2 Simplify $\frac{15a^7b^2}{5a^3b^2}$. [2]

3.3 Solve $3^x = 81$. [2]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $(3a^2)^3 = 3^3 a^6 = 27a^6$.

2.2 $10 \div 2 = 5$, $x^6 \div x^2 = x^4$, so $5x^4$.

2.3 $a^4 \times a^{-7} = a^{-3}$ (or $\frac{1}{a^3}$).

3.1 $(2x^2y)^3 = 2^3 x^6 y^3 = 8x^6 y^3$.

3.2 $\frac{15}{5} = 3$, $a^7 \div a^3 = a^4$, $b^2 \div b^2 = 1$, so $3a^4$.

3.3 $81 = 3^4$, so $3^x = 3^4 \rightarrow x = 4$.

Name:

IGCSE Mathematics: **w25 43**

6 Simplify.

(a) $\frac{y^5}{y^2}$

..... [1]

(b) $3x^3 \times 5x^5$

..... [2]

E2.5 Equations

Name: _____ Class: _____ Date: _____ **Total: 29 marks**

KEY WORDS equations 方程 • simultaneous 联立 • simultaneous equations 联立方程
• linear 一次 • quadratic 二次

Objective

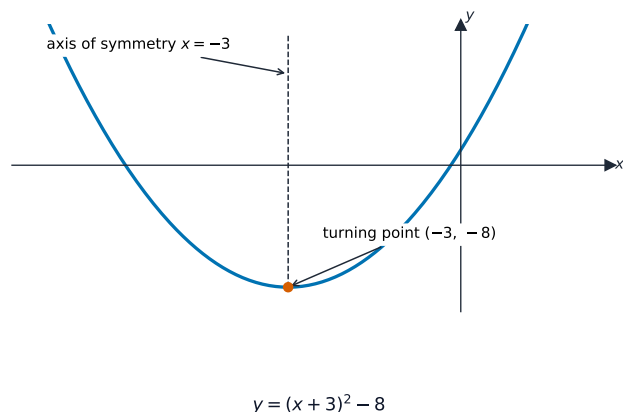
Build the skills to answer Extended exam questions on **equations** 方程 — **linear** 一次, **simultaneous** 联立, and **quadratic** 二次 equations (factorising and the formula).

You must be able to:

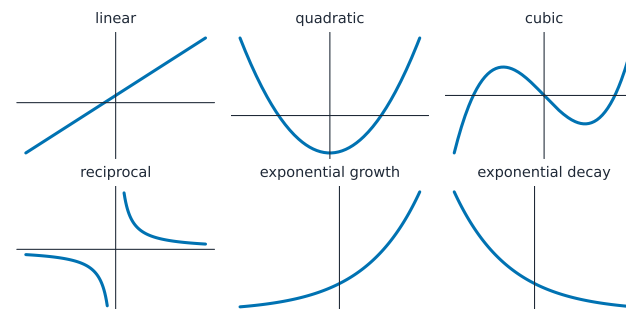
- solve a linear equation with brackets
- solve simultaneous linear equations
- solve a quadratic by factorising or the formula

1 Worked examples

■ Solving equations



Do the same to both sides until the unknown is alone



recognise $y = x^2$, $y = 1/x$, $y = 2^x$ from shape

An equation is solved graphically where curves cross

- **Linear:** $5 - 2x = 3(x + 7) \rightarrow -16 = 5x \rightarrow x = -\frac{16}{5}$.
- **Simultaneous:** $2x + y = 7$, $3x - y = 8 \rightarrow$ add: $5x = 15$, $x = 3$, $y = 1$.
- **Quadratic formula:** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Answer shapes: M1 a correct method, A1 the correct answer.

■ The three routes through a quadratic, and how to pick one

Try them in this order.

1. **Factorising.** Find two numbers that multiply to give the constant and add to give the coefficient of x . For $x^2 - 2x - 8 = 0$ they are -4 and 2 , so $(x - 4)(x + 2) = 0$ and $x = 4$ or $x = -2$. Fastest when the numbers are whole.
2. **The formula.** Use it whenever the question says “correct to 2 decimal places”, because that is a sign the roots are not whole numbers:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

3. **Completing the square.** Use it when the question asks for the turning point or for an exact (surd) answer.

Worked example (formula). Solve $2x^2 + 3x - 4 = 0$ to 2 decimal places. Here $a = 2$, $b = 3$, $c = -4$.

1. $b^2 - 4ac = 9 - 4(2)(-4) = 9 + 32 = 41$.
2. $x = \frac{-3 \pm \sqrt{41}}{4}$.
3. $\sqrt{41} = 6.403$, so $x = \frac{3.403}{4} = 0.85$ or $x = \frac{-9.403}{4} = -2.35$.

Show the substituted formula before evaluating: that line earns the method mark even if the arithmetic slips.

Simultaneous equations. With one linear and one quadratic, **substitute** the linear into the quadratic; with two linear equations, **eliminate** by adding or subtracting once the coefficients match. Always find both unknowns and pair them correctly.

2 Practice

2.1 Solve $3x - 7 = 11$. [2]

2.2 Solve $x^2 + 5x + 6 = 0$. [2]

2.3 Solve $4(x - 1) = 2x + 6$. [2]

2.4 Solve $x^2 - 7x + 12 = 0$ by factorising. [2]

2.5 Write down the values of a , b and c for $3x^2 - 5x - 2 = 0$. [1]

3 Exam-style questions

3.1 Solve the simultaneous equations $3x + 2y = 12$ and $x - 2y = 4$. [3]

3.2 Solve $x^2 - 2x - 8 = 0$. [2]

3.3 Solve $2x^2 + 3x - 4 = 0$, giving your answers correct to 2 decimal places. [3]

3.4 (a) Solve the simultaneous equations $y = x + 3$ and $x^2 + y^2 = 29$. [5]

(b) Solve $3x^2 - 7x + 2 = 0$ by factorising. [3]

(c) Solve $x^2 + 6x + 4 = 0$, giving your answers correct to 2 decimal places. [4]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $3x = 18 \rightarrow x = 6$.

2.2 $(x + 2)(x + 3) = 0 \rightarrow x = -2$ or $x = -3$.

2.3 $4x - 4 = 2x + 6 \rightarrow 2x = 10 \rightarrow x = 5$.

2.4 $(x - 3)(x - 4) = 0$; $x = 3$ or $x = 4$.

2.5 $a = 3$, $b = -5$, $c = -2$.

3.1 add the two equations: $4x = 16 \rightarrow x = 4$; then $4 - 2y = 4 \rightarrow y = 0$.

3.2 $(x - 4)(x + 2) = 0 \rightarrow x = 4$ or $x = -2$.

3.3 $a = 2$, $b = 3$, $c = -4$: $x = \frac{-3 \pm \sqrt{9 + 32}}{4} = \frac{-3 \pm \sqrt{41}}{4} \rightarrow x = 0.85$ or $x = -2.35$.

3.4 (a) Substitute $y = x + 3$: $x^2 + (x + 3)^2 = 29$; $2x^2 + 6x + 9 = 29$, so $2x^2 + 6x - 20 = 0$ and $x^2 + 3x - 10 = 0$; $(x + 5)(x - 2) = 0$; $x = -5$ or $x = 2$; the matching values are $y = -2$ and $y = 5$.

(b) $(3x - 1)(x - 2) = 0$; $x = \frac{1}{3}$ or $x = 2$.

(c) $a = 1$, $b = 6$, $c = 4$, so $x = \frac{-6 \pm \sqrt{36 - 16}}{2}$; $\sqrt{20} = 4.472$; $x = -0.76$ or $x = -5.24$.

15 Factorise.

(a) $x^2 - 7x + 12$

..... [2]

(b) $5x + 10y + 6ny + 3nx$

..... [2]

16 The table shows three sequences.

	1st term	2nd term	3rd term	4th term	5th term	n th term
Sequence A	8	13	18	23	28	
Sequence B	$\frac{3}{2}$	$\frac{4}{3}$	$\frac{5}{4}$	$\frac{6}{5}$	$\frac{7}{6}$	
Sequence C	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	

Complete the table to show the n th term of each sequence.

17 Expand and simplify.

$(x - 2)(2x + 3)(x + 4)$

..... [3]

14 These expressions are all equal in value.

$\frac{5x - 2}{3}$ $10 - x$ $y + 11$

Find the value of y .

$y =$ [5]

15 The population of a town is 54 000.
The population is **decreasing** exponentially at a rate of 2% per year.

(a) Calculate the decrease in the population at the end of 4 years.

..... [3]

(b) Find the number of complete years it takes for the population of 54 000 to first fall below 44 000.

..... years [2]

16 Expand and simplify.

(a) $7(x+2)+4(3x-5)$

..... [2]

(b) $(3x-y)(5x+2y)$

..... [2]

17 Make t the subject of the formula.

$$x = \frac{7t}{5-t}$$

$t =$ [3]

6 Solve.

(a) $8x+7=39$

$x =$ [2]

(b) $2(5y-1)=24$

$y =$ [3]

6 Expand.

$$5x^2(3x-2)$$

..... [2]

E2.6 Inequalities

Name: _____ Class: _____ Date: _____

Total: 29 marks

KEY WORDS inequality 不等式 • number line 数轴 • region 区域 • term 项 • equation 方程

Objective

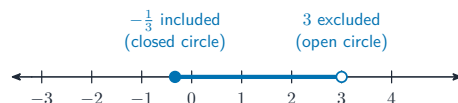
Build the skills to answer Extended exam questions on **inequalities** 不等式 — solving them, the rule for **multiplying or dividing by a negative** 乘除负数, and showing a solution on a **number line** 数轴.

You must be able to:

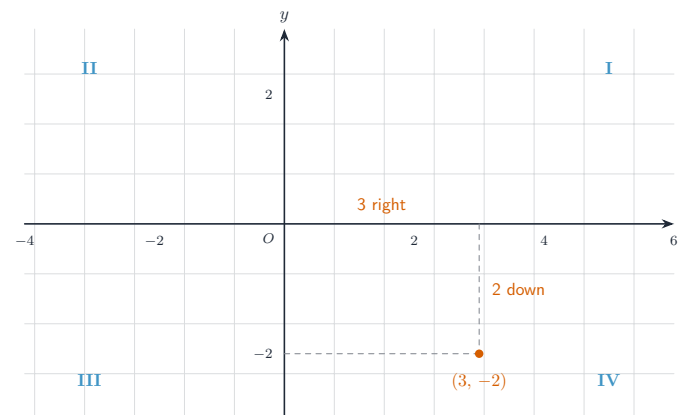
- solve a linear inequality (and reverse the sign when needed)
- solve a double inequality
- list the integers that satisfy an inequality

1 Worked examples

■ Solving inequalities



Solve like an equation, but reverse the sign if you $\div$ or $\times$ by a negative



Two-variable inequalities shade regions of the plane

- **Worked:** $-3 \leq 3x - 2 < 7 \rightarrow$ add 2: $-1 \leq 3x < 9 \rightarrow \div 3$: $-\frac{1}{3} \leq x < 3$.
- **Reverse:** $-2x < 6 \rightarrow x > -3$ (sign flips when dividing by -2).
- **Number line:** closed circle for $\leq / \geq$, open circle for $< / >$.

Answer shapes: M1 a correct method, A1 the correct answer.

■ Inequalities: the same steps as an equation, with one extra rule

Solve an inequality exactly as you would the matching equation — **except** that multiplying or dividing both sides by a **negative** number **reverses** the inequality sign.

$$-3x > 12 \Rightarrow x < -4$$

That single rule is the difference between full marks and none on half the questions in this topic. If you would rather not risk it, move the x term to the side where it is positive instead: $-3x > 12$ becomes $-12 > 3x$, so $-4 > x$, which is the same answer with no sign flip.

A **double inequality** is solved by doing the same operation to all three parts: $5 \leq 2x + 1 \leq 11$ gives $4 \leq 2x \leq 10$, then $2 \leq x \leq 5$.

Reading the answer as integers. For $-2 < x \leq 3$: the $<$ EXCLUDES -2 and the $\leq$ INCLUDES 3 , so the integers are $-1, 0, 1, 2, 3$. Getting the endpoints right is usually a mark by itself.

On a number line, an open circle means the value is excluded ($<$ or $>$) and a filled circle means it is included ($\leq$ or $\geq$).

Worked example. Find the integers n for which $-9 < 3n - 3 \leq 6$.

1. Add 3 throughout: $-6 < 3n \leq 9$.
2. Divide by 3 throughout: $-2 < n \leq 3$.
3. The endpoints: -2 is excluded, 3 is included.
4. So $n = -1, 0, 1, 2, 3$.

2 Practice

2.1 Solve $2x + 1 > 9$. [2]

2.2 Solve $-3x \leq 12$. [2]

2.3 Solve $5x - 4 < 11$. [2]

2.4 Solve $-4x + 3 > 19$. [2]

2.5 Write down the inequality shown by an open circle at -1 and a filled circle at 4 , joined by a line. [2]

3 Exam-style questions

3.1 Solve the inequality $5 \leq 2x + 1 \leq 11$. [3]

3.2 Write down the integers that satisfy $-2 < x \leq 3$. [2]

3.3 Solve $7 - 2x \geq 1$. [2]

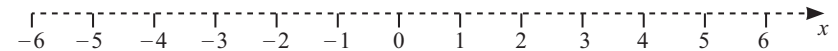
3.4 (a) Solve the inequality $-11 < 4x + 5 \leq 13$. [3]

(b) Write down all the integers that satisfy your answer to part (a). [2]

(c) Solve $9 - 4x \geq 21$, and explain the step in which the inequality sign changes. [3]

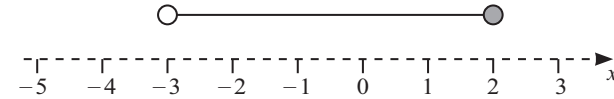
(d) A rectangle has width x cm and length $(x + 5)$ cm. Its perimeter is at least 30 cm and less than 50 cm. Form an inequality in x and solve it. [4]

7 Represent the inequality $-4 < x \leq 3$ on the number line.



[2]

7 (a)



Write down the inequality represented in the diagram.

..... [2]

(b) Write down the integer values of x that satisfy the inequality $-4 < 2x \leq 8$.

..... [2]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $2x > 8 \rightarrow x > 4$.

2.2 dividing by -3 reverses the sign: $x \geq -4$.

2.3 $5x < 15 \rightarrow x < 3$.

2.4 $-4x > 16$; dividing by -4 reverses the sign, so $x < -4$.

2.5 $-1 < x \leq 4$.

3.1 subtract 1: $4 \leq 2x \leq 10$; divide by 2: $2 \leq x \leq 5$.

3.2 the integers are $-1, 0, 1, 2, 3$.

3.3 $-2x \geq -6 \rightarrow$ dividing by -2 reverses the sign $\rightarrow x \leq 3$.

3.4 (a) Subtract 5 throughout: $-16 < 4x \leq 8$; divide by 4 throughout: $-4 < x \leq 2$.

(b) $-3, -2, -1, 0, 1, 2$.

(c) $-4x \geq 12$; dividing both sides by -4 reverses the inequality; $x \leq -3$.

(d) Perimeter $= 2(x) + 2(x+5) = 4x + 10$; $30 \leq 4x + 10 < 50$; $20 \leq 4x < 40$; $5 \leq x < 10$.

24 Martha walks a distance of 10 km at a speed of x km/h.
She then runs a distance of 5 km at a speed of $(x + 4)$ km/h.
The total time taken for the whole journey is 3.5 hours.

(a) Write down an expression in terms of x for the time Martha is walking.

..... h [1]

(b) Show that $7x^2 - 2x - 80 = 0$.

(c) Solve $7x^2 - 2x - 80 = 0$, giving your answers correct to 2 decimal places.
You must show all your working.

[4]

$x = \dots\dots\dots$ or $x = \dots\dots\dots$ [3]

(d) Calculate the difference between the time Martha is walking and the time she is running.
Give your answer in hours and minutes correct to the nearest minute.



18 One day, Anya runs 12 km at a speed of x km/h.
The next day she walks 10 km at a speed of $(x - 4)$ km/h.

(a) Write down an expression, in terms of x , for the time she spends running.

..... h [1]

(b) Write down an expression, in terms of x , for the time she spends walking.

..... h [1]

(c) The time Anya spends walking is 1 hour more than the time she spends running.
Write an equation in terms of x and show that it simplifies to $x^2 - 2x - 48 = 0$.

(d) Use factorisation to solve the equation $x^2 - 2x - 48 = 0$.

[4]

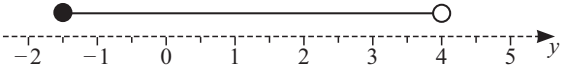
$x = \dots\dots\dots$ or $x = \dots\dots\dots$ [3]

(e) Find the time Anya spends running.

..... h [1]



7



Write down the inequality in y shown by the number line.

..... [2]

E2.10 Graphs of functions

Name: _____ Class: _____ Date: _____ **Total: 28 marks**

KEY WORDS table of values 数值表 • coordinates 坐标 • axis 坐标轴 • function 函数 • roots 根

Objective

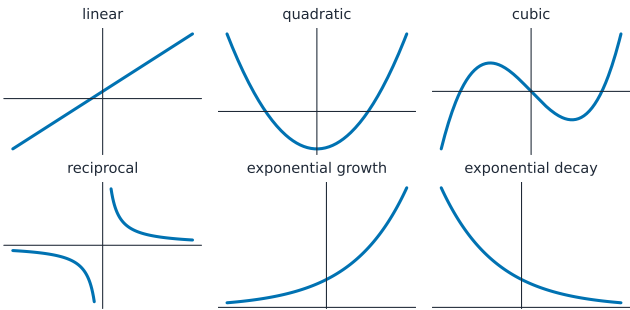
Build the skills to answer Extended exam questions on **graphs of functions** 函数图象 — making a **table of values** 数值表, finding **roots** 根 (where the graph crosses the x -axis), and reading solutions from a graph.

You must be able to:

- complete a table of values for a function
- identify the roots and the y -intercept of a graph
- use a graph to solve an equation

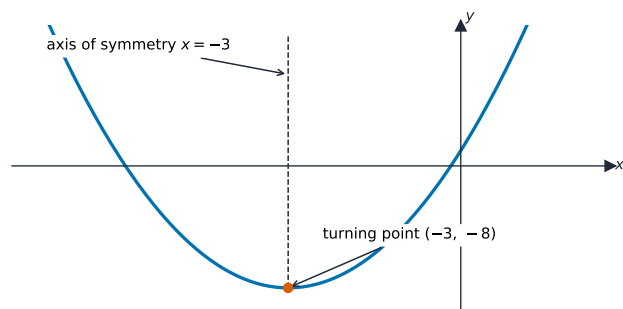
1 Worked examples

■ Tables, roots and intersections



recognise $y = x^2$, $y = 1/x$, $y = 2^x$ from shape

Plot points from a table, then join with a smooth curve



$$y = (x + 3)^2 - 8$$

The quadratic is one key function graph

- **Table:** for $y = x^2 - 3$, when $x = 2$, $y = 4 - 3 = 1$.
- **Roots** 根: where $y = 0$ (the graph crosses the x -axis).
- **Solve from a graph:** the **intersection** 交点 of a line and a curve solves the two together.

Answer shapes: **M1** a correct method, **A1** the correct answer.

■ Reading a graph, and solving with a line

The **roots** are where the curve crosses the x -axis, so they are the solutions of $y = 0$. The **y -intercept** is the value at $x = 0$. The **turning point** of $y = ax^2 + bx + c$ lies on the line of symmetry $x = -\frac{b}{2a}$; substitute that x back to get its y .

Solving with a drawn line. To solve an equation using a drawn curve, rearrange it so that one side is exactly the curve's equation; the other side is then the line to draw, and the solutions are the x -coordinates where the curve and line cross.

Worked example. The curve $y = x^2 - 2x - 3$ is drawn. Use it to solve $x^2 - 2x - 8 = 0$.

1. Rearrange so the left side matches the drawn curve: $x^2 - 2x - 8 = 0$ becomes $x^2 - 2x - 3 = 5$.
2. So draw the horizontal line $y = 5$.
3. Read the x -coordinates where the curve meets it: $x = -2$ and $x = 4$.
4. Check by factorising: $(x - 4)(x + 2) = 0$, which agrees.

Sketching. Give the shape (a positive x^2 term opens upwards, a negative one opens downwards), the roots, the y -intercept and the turning point. For a cubic, the shape rises from bottom-left to top-right when the x^3 term is positive, and the roots are still where it crosses the x -axis.

2 Practice

2.1 For $y = x^2 - 4$, find the value of y when $x = 3$. [1]

2.2 Write down the y -intercept of $y = 2x^2 - 5$. [1]

2.3 Find the roots of $y = x^2 - 9$. [2]

2.4 Find the coordinates of the turning point of $y = x^2 - 6x + 5$. [3]

2.5 State the line you would draw on the graph of $y = x^2 + x$ to solve $x^2 + x - 4 = 0$. [1]

3 Exam-style questions

3.1 $y = x^2 - 2x - 3$.

(a) Complete the value of y when $x = -2$. [1]

(b) Find the roots of $x^2 - 2x - 3 = 0$. [2]

3.2 The graph of $y = x^2 - 2x - 3$ is drawn. By solving a quadratic, find the x -values where the graph meets the line $y = 5$. [3]

3.3 The curve $y = x^2 - 4x + 1$ is to be drawn for $-1 \leq x \leq 5$.

(a) Complete the table of values. [3]

x	-1	0	1	2	3	4	5
y							

(b) Write down the coordinates of the turning point of the curve. [3]

(c) Use the curve to solve $x^2 - 4x + 1 = 0$, giving your answers correct to 1 decimal place. [2]

(d) By drawing a suitable straight line, explain how you would solve $x^2 - 4x - 2 = 0$, and give the equation of that line. [2]

(e) Solve $x^2 - 4x - 2 = 0$ algebraically, giving your answers correct to 2 decimal places. [4]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $y = 3^2 - 4 = 5$.

2.2 the y -intercept is at $x = 0$: $y = -5$.

2.3 $x^2 - 9 = 0 \rightarrow (x + 3)(x - 3) = 0 \rightarrow x = 3$ or $x = -3$.

2.4 $x = -\frac{-6}{2} = 3$; $y = 9 - 18 + 5 = -4$; the turning point is $(3, -4)$.

2.5 $y = 4$.

3.1 (a) $y = (-2)^2 - 2(-2) - 3 = 4 + 4 - 3 = 5$. (b) $(x - 3)(x + 1) = 0 \rightarrow x = 3$ or $x = -1$.

3.2 set $x^2 - 2x - 3 = 5 \rightarrow x^2 - 2x - 8 = 0 \rightarrow (x - 4)(x + 2) = 0 \rightarrow x = 4$ or $x = -2$.

3.3 (a) 6, 1, -2, -3, -2, 1, 6.

(b) $x = -\frac{-4}{2} = 2$; $y = 4 - 8 + 1 = -3$; the turning point is $(2, -3)$.

(c) The curve crosses the x -axis at about $x = 0.3$ and $x = 3.7$.

(d) Rearranging, $x^2 - 4x - 2 = 0$ becomes $x^2 - 4x + 1 = 3$, so draw the line $y = 3$ and read where it meets the curve.

(e) $a = 1$, $b = -4$, $c = -2$, so $x = \frac{4 \pm \sqrt{16 + 8}}{2}$; $\sqrt{24} = 4.899$; $x = 4.45$ or $x = -0.45$.

Name: _____

IGCSE Mathematics: w25 42

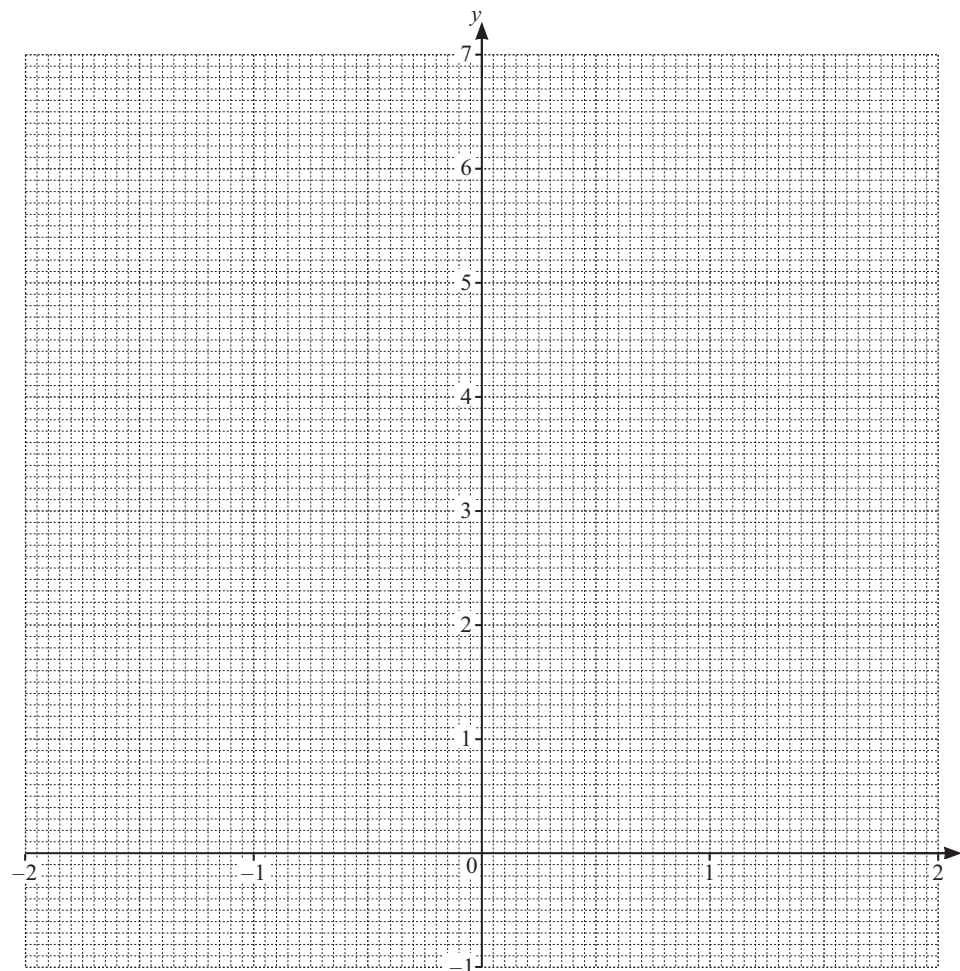
- 11 The table shows some values for $y = x^3 - 2x + 3$.
Where appropriate, values of y are given correct to 2 decimal places.

x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
y	-1		4	3.88	3	2.13	2		7

(a) Complete the table.

[2]

(b) Draw the graph of $y = x^3 - 2x + 3$ for $-2 \leq x \leq 2$.



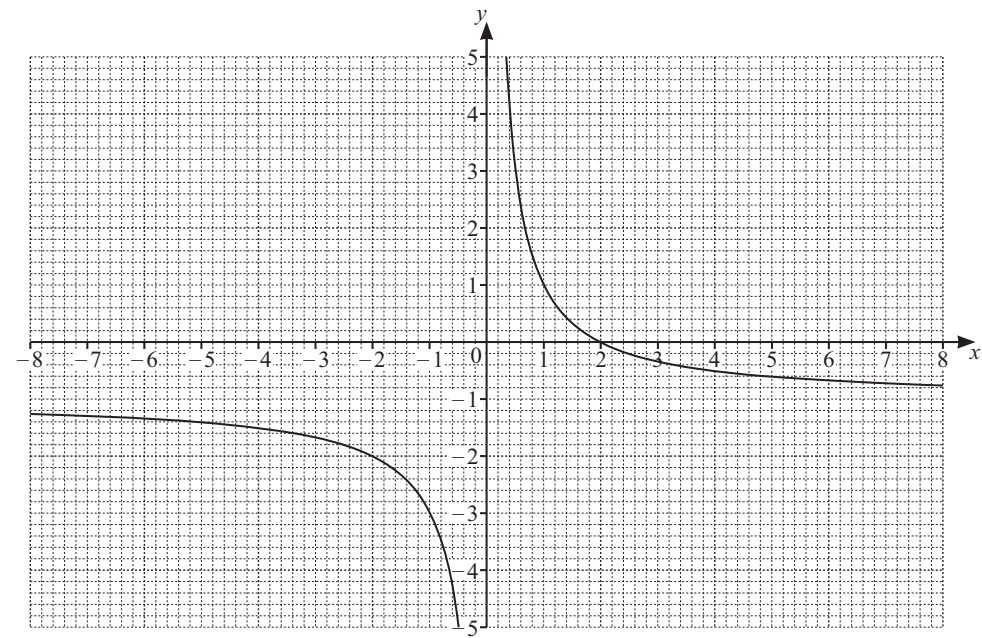
[4]

(c) By drawing a suitable straight line on the grid, solve the equation $x^3 - 2.5x + 1 = 0$.

$x = \dots\dots\dots$ or $x = \dots\dots\dots$ or $x = \dots\dots\dots$ [4]



15



The diagram shows the graph of $y = \frac{2}{x} - 1$.

(a) Write down the coordinates of the point where the graph crosses the x -axis.

($\dots\dots\dots$, $\dots\dots\dots$) [1]

(b) Write down the equation of each asymptote.

$\dots\dots\dots$
 $\dots\dots\dots$

[2]

(c) By drawing a suitable straight line on the grid, solve $\frac{2}{x} - x - 1 = 0$.

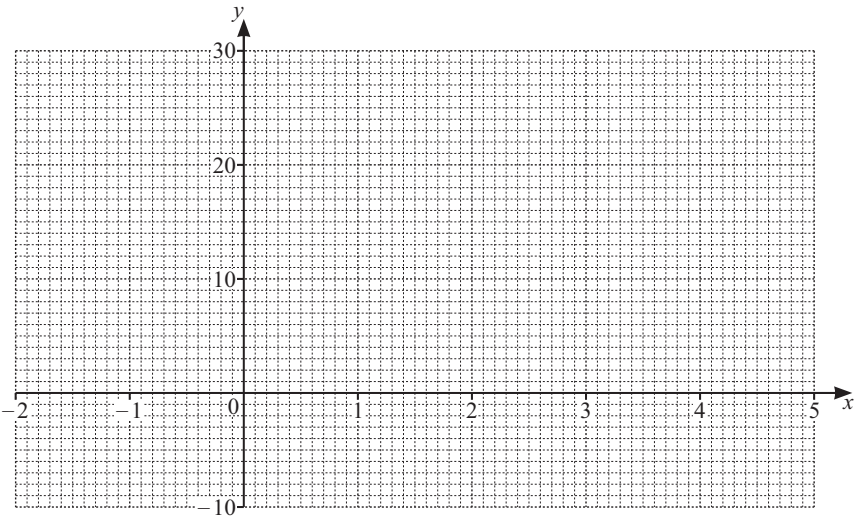
$x = \dots\dots\dots$ or $x = \dots\dots\dots$ [3]



14 The table shows some values for $y = 5x^2 - x^3 - 4$.

x	-2	-1	0	1	2	3	4	5
y	24		-4		8	14		-4

- (a) Complete the table. [3]
- (b) On the grid, draw the graph of $y = 5x^2 - x^3 - 4$ for $-2 \leq x \leq 5$.



- (c) By drawing a suitable straight line on the grid, solve the equation $x^3 - 5x^2 - x + 14 = 0$.

x = or x = or x = [4]

E2.11 Sketching curves

Name: _____ Class: _____ Date: _____ **Total: 7 marks**

KEY WORDS turning point 转折点 • parabola 抛物线 • shapes 形状 • line of symmetry 对称轴
• symmetry 对称

Objective

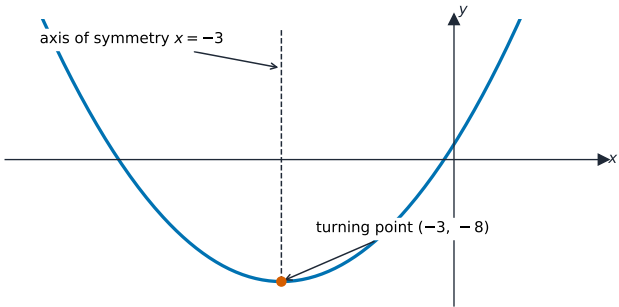
Build the skills to answer Extended exam questions on **sketching curves** 草绘曲线 — the **shapes** 形状 of common graphs, the **turning point** 转折点 of a parabola, and key features.

You must be able to:

- recognise the shape of linear, quadratic, cubic and reciprocal graphs
- find the turning point of a parabola by completing the square
- state the line of symmetry of a parabola

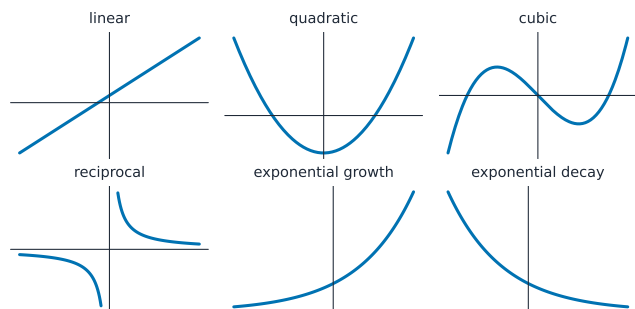
1 Worked examples

■ Shapes and turning points



$y = (x + 3)^2 - 8$

Completing the square gives the turning point



recognise $y = x^2$, $y = 1/x$, $y = 2^x$ from shape

The family of standard curve shapes to recognise

- **Shapes:** linear (line), **quadratic** (a **parabola** 抛物线, U if $a > 0$), cubic (S-shape), reciprocal (two curves).
- **Turning point:** $y = (x + 3)^2 - 8$ has its turning point at $(-3, -8)$.
- **Line of symmetry** 对称轴: the vertical line through the turning point ($x = -3$ here).

Answer shapes: **M1** a correct method, **A1** the correct answer.

2 Practice

2.1 Write down the shape of the graph of $y = x^2 + 1$. [1]

2.2 Write down the turning point of $y = (x - 4)^2 + 2$. [1]

2.3 Write down the y -intercept of $y = 3x - 5$. [1]

3 Exam-style questions

3.1 Write $x^2 + 6x + 5$ in the form $(x + p)^2 + q$. [2]

3.2 Hence write down the coordinates of the turning point of $y = x^2 + 6x + 5$. [1]

3.3 Write down the equation of the line of symmetry of $y = x^2 + 6x + 5$. [1]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 a parabola (a U-shaped curve).

2.2 (4, 2).

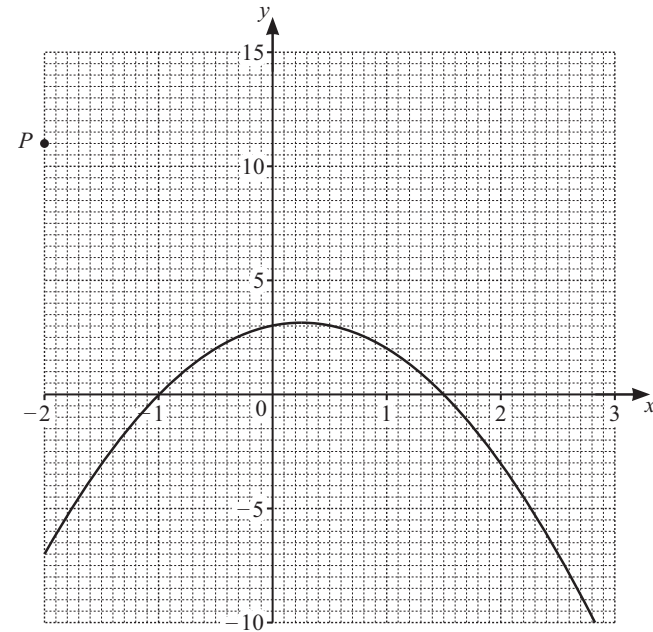
2.3 the y -intercept is -5 .

3.1 half of 6 is 3, $3^2 = 9$: $x^2 + 6x + 5 = (x + 3)^2 - 9 + 5 = (x + 3)^2 - 4$.

3.2 the turning point is $(-3, -4)$.

3.3 the line of symmetry is $x = -3$.

11 The diagram shows the graph of $y = f(x)$ and the point $P(-2, 11)$.



The tangent from P touches the graph of $y = f(x)$ at the point (a, b) .
The values of a and b are integers.

(a) By drawing this tangent, find the value of a and the value of b .

$a = \dots\dots\dots$, $b = \dots\dots\dots$ [2]

(b) Find the equation of the tangent.
Give your answer in the form $y = mx + c$.

$y = \dots\dots\dots$ [3]