**Week 7**

# IGCSE Mathematics

Homework bundle for this week

本周作业合集

exercises.lol

## Contents 目录

IGCSE Mathematics Name: ____________ Score: ______

1. Find the value of $3x^2 - 2y$ when $x = 4$ and $y = 5$.

______________

2. Simplify $(x^2 - 2x)/(x^2 - 5x + 6)$.

☐ $x/(x - 3)$

☐ $x/(x - 2)$

☐ $(x - 2)/(x - 3)$

☐ $x - 3$

3. $(5x^3)^2 = k \cdot x^6$. What is the coefficient k?

______________

4. Solve $2x + y = 7$ and $3x - y = 8$. What is x?

______________

5. Solve $-3 \leq 3x - 2 < 7$. The solution is $-1/3 \leq x <$ ? What is the upper bound?

______________

6. The nth term of 2, 5, 8, 11, … is $3n - 1$. What is the 10th term?

______________

7. On a distance–time graph, the gradient represents the:

☐ speed

☐ acceleration

☐ distance

☐ time

8. The roots of a graph are where it:

☐ crosses the x-axis $(y = 0)$

☐ crosses the y-axis

☐ is steepest

☐ turns around

9. Add $x/3 + (x-4)/2$ over a common denominator of 6.

☐ $(5x - 12)/6$

- ☐ $(2x - 4)/6$
- ☐ $(5x + 12)/6$
- ☐ $(x - 4)/6$

10. Solve $x^2 + 5x + 6 = 0$ by factorising.
    - ☐ $x = -2$ or $x = -3$
    - ☐ $x = 2$ or $x = 3$
    - ☐ $x = -1$ or $x = -6$
    - ☐ $x = 5$ or $x = 6$

1. **38**

   $3 \times 4^2 - 2 \times 5 = 3 \times 16 - 10 = 48 - 10 = 38$.

2. **x/(x − 3)**

   Factorise: x(x−2) / [(x−2)(x−3)], cancel (x−2) → x/(x−3).

3. **25**

   $(5x^3)^2 = 5^2 \times (x^3)^2 = 25x^6$, so k = 25.

4. **3**

   Adding the equations: 5x = 15, so x = 3 (then y = 1).

5. **3**

   Add 2: $-1 \leq 3x < 9$; divide by 3: $-1/3 \leq x < 3$.

6. **29**

   $3 \times 10 - 1 = 30 - 1 = 29$.

7. **speed**

   Distance ÷ time = speed, which is the gradient.

8. **crosses the x-axis (y = 0)**

   Roots are the x-values where y = 0 — where the curve meets the x-axis.

9. **(5x − 12)/6**

   $2x/6 + 3(x-4)/6 = (2x + 3x - 12)/6 = (5x - 12)/6$.

10. **x = −2 or x = −3**

    $(x + 2)(x + 3) = 0$, so $x = -2$ or $x = -3$.

IGCSE MATHEMATICS • EXERCISE SHEET

# E2.1 Introduction to algebra

Name: ______________ Class: __________ Date: ____________ **Total: 12 marks**

**KEY WORDS** like terms 同类项 • substitute 代入 • terms and like terms 项和同类项 • introduction to algebra 代数入门 • expression 表达式

## Objective

Build the skills to answer Extended exam questions on **introduction to algebra 代数入门** — **substitution 代入**, **terms and like terms 项和同类项**, and simplifying expressions.

**You must be able to:**

- substitute numbers into an expression
- collect like terms to simplify
- use correct order of operations with letters

## 1 Worked examples

### Substituting and simplifying

*Letters stand for numbers; like terms can be combined*

- **Substitute:** $3x^2 - 2y$ with $x = 4$, $y = 5 \to 3(16) - 10 = 38$.
- **Like terms 同类项**: same letters $\to$ combine. $2a^2 + 5a^2 = 7a^2$; $3ab - 9ab = -6ab$.

$$2a^2 \quad +3ab \quad -1 \quad +5a^2 \quad -9ab \quad +4$$

$\downarrow$ *group like terms*

$$7a^2 \quad -6ab \quad +3$$

terms that share a colour are like terms

like terms: same letters/powers · add coefficients only

*Collecting like terms: group the terms that share a colour (the same letters)*

- **Worked:** $2a^2 + 3ab - 1 + 5a^2 - 9ab + 4 = 7a^2 - 6ab + 3$.

**Answer shapes: M1** a correct method, **A1** the correct answer.

## 2 Practice

**2.1** Find the value of $4x - 3y$ when $x = 5$ and $y = 2$. [2]

**2.2** Simplify $7p + 2q - 3p + 5q$. [2]

**2.3** Find the value of $2x^2 + 1$ when $x = -3$. [2]

## 3 Exam-style questions

**3.1** Simplify $5a^2 - 2ab + 3 - a^2 + 6ab$. [2]

**3.2** Find the value of $\dfrac{2a+b}{a-b}$ when $a = 7$ and $b = 1$. [2]

**3.3** Find the value of $3x^2 - 5x$ when $x = -2$. [2]

## Solutions

Marking: **M1** a correct method, **A1** the correct answer.

**2.1** $4(5) - 3(2) = 20 - 6 = 14$.

**2.2** $7p - 3p + 2q + 5q = 4p + 7q$.

**2.3** $2(-3)^2 + 1 = 2(9) + 1 = 19$.

**3.1** $5a^2 - a^2 = 4a^2$, $-2ab + 6ab = 4ab$, so $4a^2 + 4ab + 3$.

**3.2** $\frac{2(7)+1}{7-1} = \frac{15}{6} = \frac{5}{2}$.

**3.3** $3(-2)^2 - 5(-2) = 3(4) + 10 = 22$.

**8** Kemi buys $p$ pens that each cost 40 cents.
She pays with $20.

Write an expression, in terms of $p$, for the change, in **cents**, Kemi receives from the $20.

.......................................... cents [2]

IGCSE MATHEMATICS • EXERCISE SHEET

# E2.2 Algebraic manipulation

Name: ____________ Class: ________ Date: __________ **Total: 25 marks**

**KEY WORDS** term 项 • expand 展开 • area 面积 • difference of two squares 平方差 • factorising 因式分解

## Objective

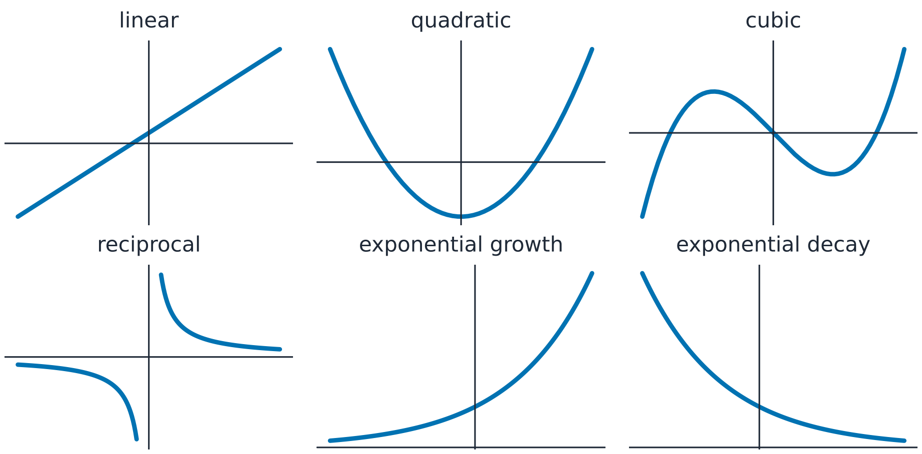
Build the skills to answer Extended exam questions on **algebraic manipulation 代数运算** — **expanding brackets 展开括号** and **factorising 因式分解** (common factors, difference of two squares, quadratics).

**You must be able to:**

- expand single and double brackets
- factorise by taking out a common factor
- factorise a difference of two squares and a quadratic

## 1 Worked examples

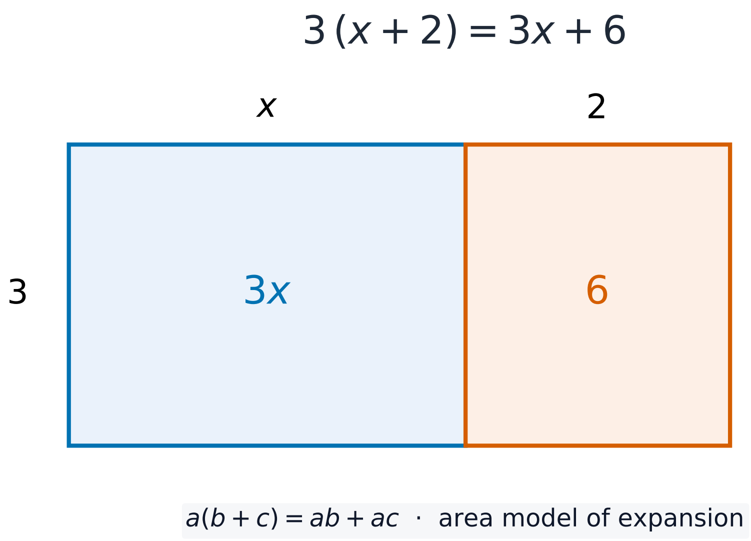
### ■ Expand and factorise




recognise $y = x^2$, $y = 1/x$, $y = 2^x$ from shape

*Factorising is the reverse of expanding*

- **Expand:** $(2x + 1)(x - 4) = 2x^2 - 7x - 4$.




*The area model of expanding a bracket:* $3(x+2) = 3x+6$

- **Common factor:** $9x^2 + 15xy = 3x(3x+5y)$.
- **Difference of two squares** 平方差: $9x^2 - 16 = (3x+4)(3x-4)$.
- **Quadratic:** $2x^2 + 7x + 3 = (x+3)(2x+1)$ (split the middle term).

**Answer shapes: M1** a correct method, **A1** the correct answer.

### ■ Choosing the factorising method

Work through these in order; the first one that applies is the right one.

1. **Common factor first, every time.** $6x^2 + 9x = 3x(2x+3)$. Missing it makes everything afterwards harder, and the mark for the common factor is lost even if the rest is right.
2. **Difference of two squares**, when there are two terms and both are squares with a minus between: $a^2 - b^2 = (a+b)(a-b)$. So $x^2 - 49 = (x+7)(x-7)$ and $9x^2 - 25 = (3x+5)(3x-5)$.
3. **A quadratic with $a = 1$:** find two numbers that multiply to the constant and add to the coefficient of $x$. For $x^2 + 3x - 10$ they are $+5$ and $-2$, so it factorises as $(x+5)(x-2)$.
4. **A quadratic with $a \neq 1$:** multiply $a$ by $c$, find two numbers multiplying to that and adding to $b$, split the middle term and factorise in pairs.
5. **Four terms:** factorise in pairs, then take out the common bracket: $xy + 2x + 3y + 6 = x(y+2) + 3(y+2) = (x+3)(y+2)$.

**Worked example (the $a \neq 1$ method).** Factorise $2x^2 + 5x - 3$.

1. $a \times c = 2 \times (-3) = -6$.
2. Two numbers multiplying to $-6$ and adding to $+5$: $+6$ and $-1$.
3. Split the middle term: $2x^2 + 6x - x - 3$.

4. Factorise in pairs: $2x(x+3) - 1(x+3)$.
5. Take out the common bracket: $(2x-1)(x+3)$.

Always **check by expanding**. It takes one line and catches a sign error, which is the commonest loss on this topic.

## 2 Practice

**2.1** Expand $3x(2x-5)$. [1]

**2.2** Factorise $6a + 9ab$. [1]

**2.3** Expand and simplify $(x+2)(x-5)$. [2]

**2.4** Factorise fully $12x^2 - 27$. [2]

**2.5** Factorise $ab + 4a + 3b + 12$. [2]

## 3 Exam-style questions

**3.1** Factorise $x^2 - 49$. [1]

**3.2** Factorise $x^2 + 3x - 10$. [2]

**3.3** Factorise $2x^2 + 5x - 3$. [2]

**3.4** (a) Factorise fully $18x^2 - 8$. [3]

(b) Factorise $3x^2 - 11x + 6$. [3]

(c) Simplify $\dfrac{x^2 - 9}{x^2 + x - 12}$, showing your factorising. [3]

(d) Hence solve $\dfrac{x^2 - 9}{x^2 + x - 12} = 2$. [3]

## Solutions

Marking: **M1** a correct method, **A1** the correct answer.

**2.1** $3x(2x-5) = 6x^2 - 15x$.

**2.2** $6a + 9ab = 3a(2 + 3b)$.

**2.3** $(x+2)(x-5) = x^2 - 5x + 2x - 10 = x^2 - 3x - 10$.

**2.4** Common factor 3: $3(4x^2 - 9)$; difference of two squares: $3(2x+3)(2x-3)$.

**2.5** $a(b+4) + 3(b+4)$; $(a+3)(b+4)$.

**3.1** $x^2 - 49 = (x+7)(x-7)$.

**3.2** two numbers multiplying to $-10$ and adding to 3 are 5 and $-2$: $(x+5)(x-2)$.

**3.3** $a \times c = -6$, middle $+5$: numbers 6 and $-1 \to 2x^2 + 6x - x - 3 = 2x(x+3) - (x+3) = (2x-1)(x+3)$.

**3.4** (a) Common factor 2: $2(9x^2 - 4)$; this is a difference of two squares; $2(3x+2)(3x-2)$.
(b) $a \times c = 18$; two numbers multiplying to 18 and adding to $-11$ are $-9$ and $-2$; $3x^2 - 9x - 2x + 6 = 3x(x-3) - 2(x-3)$; $(3x-2)(x-3)$.
(c) Numerator $= (x+3)(x-3)$; denominator $= (x+4)(x-3)$; cancelling the common $(x-3)$ gives $\dfrac{x+3}{x+4}$.
(d) $\dfrac{x+3}{x+4} = 2$, so $x + 3 = 2(x+4)$; $x + 3 = 2x + 8$, so $-5 = x$; $x = -5$.

**17** $I = M(k^2 + c^2)$

**(a)** Find the value of $I$ when $M = 7$, $k = 3$ and $c = 2$.

$I =$ .................................................. [2]

**(b)** Rearrange the formula to write $k$ in terms of $I$, $M$ and $c$.

$k =$ .................................................. [3]



**10** $b = dm + 2mk$

**(a)** $d = 3.14$, $m = 7.92$ and $k = 10.16$ .

By rounding each value correct to 1 significant figure, work out an estimate for $b$.

.................................................. [3]

**(b)** Rearrange the formula to make $m$ the subject.

$m =$ .................................................. [2]

# E2.4 Indices II

Name: ______________ Class: __________ Date: ____________ **Total: 11 marks**

**KEY WORDS** indices 指数 • equation 方程 • indices in algebra 代数中的指数 • laws of indices 指数定律 • base 底数

## Objective

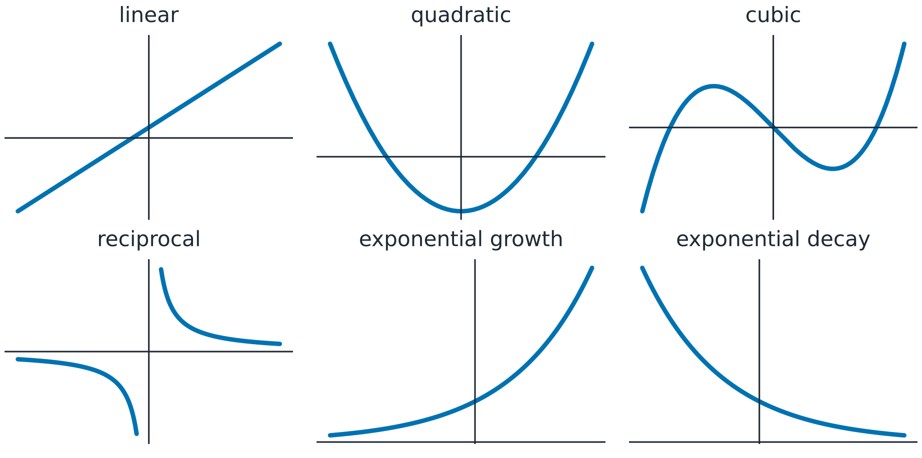
Build the skills to answer Extended exam questions on **indices in algebra 代数中的指数** — applying the **laws of indices 指数定律** to algebraic terms, and solving simple index equations.

**You must be able to:**

- simplify products, quotients and powers of algebraic terms
- handle negative powers in algebra
- solve $a^x = b$ by writing both sides with the same base

## 1 Worked examples

### Indices with letters




recognise $y = x^2$, $y = 1/x$, $y = 2^x$ from shape

*The index laws work with letters too*

$$a^m \times a^n = a^{m+n} \qquad a^m \div a^n = a^{m-n} \qquad (a^m)^n = a^{mn}$$

- **Worked:** $(5x^3)^2 = 25x^6$; $12a^5 \div 3a^{-2} = 4a^7$; $6x^7y^4 \times 5x^{-5}y = 30x^2y^5$.
- **Index equation:** $2^x = 32 = 2^5 \to x = 5$.

**Answer shapes:** **M1** a correct method, **A1** the correct answer.

## 2 Practice

**2.1** Simplify $(3a^2)^3$. [2]

**2.2** Simplify $10x^6 \div 2x^2$. [2]

**2.3** Simplify $a^4 \times a^{-7}$. [1]

## 3 Exam-style questions

**3.1** Simplify $(2x^2y)^3$. [2]

**3.2** Simplify $\dfrac{15a^7b^2}{5a^3b^2}$. [2]

**3.3** Solve $3^x = 81$. [2]

## Solutions

Marking: **M1** a correct method, **A1** the correct answer.

**2.1** $(3a^2)^3 = 3^3a^6 = 27a^6$.

**2.2** $10 \div 2 = 5$, $x^6 \div x^2 = x^4$, so $5x^4$.

**2.3** $a^4 \times a^{-7} = a^{-3}$ (or $\frac{1}{a^3}$).

**3.1** $(2x^2y)^3 = 2^3x^6y^3 = 8x^6y^3$.

**3.2** $\frac{15}{5} = 3$, $a^7 \div a^3 = a^4$, $b^2 \div b^2 = 1$, so $3a^4$.

**3.3** $81 = 3^4$, so $3^x = 3^4 \rightarrow x = 4$.

**6** Simplify.

**(a)** $\dfrac{y^5}{y^2}$

.................................................. [1]

**(b)** $3x^3 \times 5x^5$

.................................................. [2]

IGCSE MATHEMATICS • EXERCISE SHEET

# E2.5 Equations

Name: ____________ Class: ________ Date: __________ **Total: 29 marks**

**KEY WORDS** equations 方程 • simultaneous 联立 • simultaneous equations 联立方程 • linear 一次 • quadratic 二次

## Objective

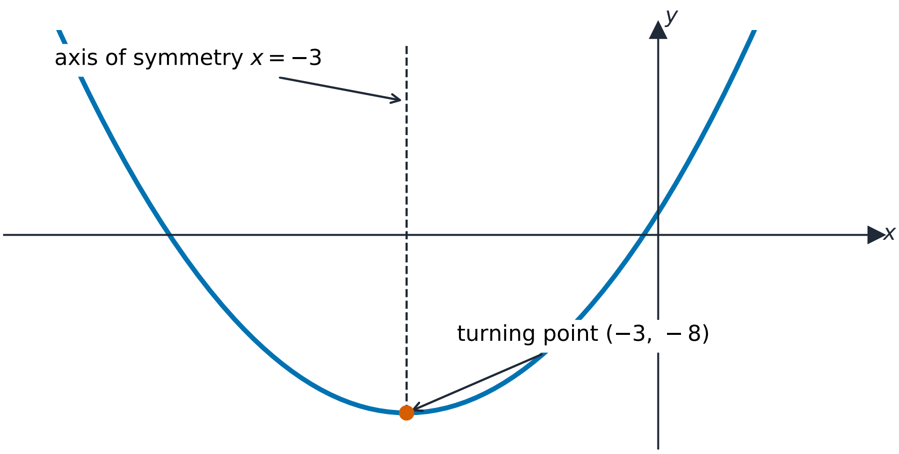
Build the skills to answer Extended exam questions on **equations 方程** — **linear 一次**, **simultaneous 联立**, and **quadratic 二次** equations (factorising and the formula).

**You must be able to:**

- solve a linear equation with brackets
- solve simultaneous linear equations
- solve a quadratic by factorising or the formula

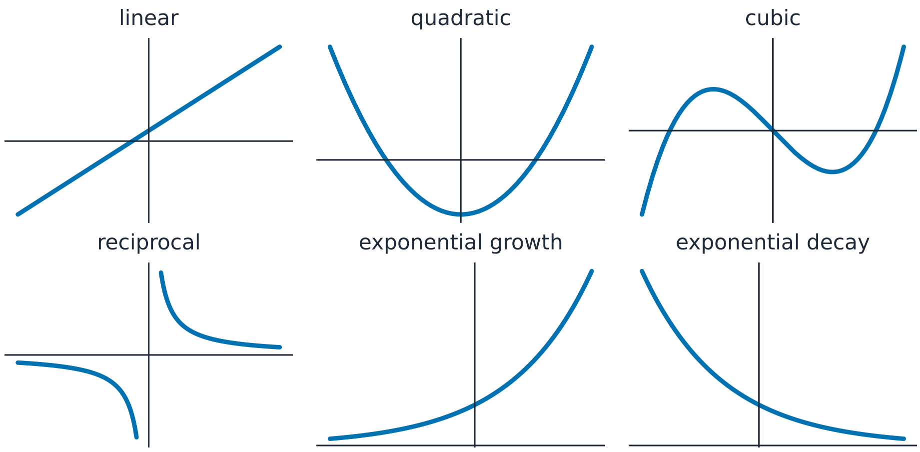
## 1 Worked examples

### ■ Solving equations




$$y = (x+3)^2 - 8$$

*Do the same to both sides until the unknown is alone*




recognise $y = x^2$, $y = 1/x$, $y = 2^x$ from shape

*An equation is solved graphically where curves cross*

- **Linear:** $5 - 2x = 3(x + 7) \to -16 = 5x \to x = -\frac{16}{5}$.
- **Simultaneous:** $2x + y = 7$, $3x - y = 8 \to$ add: $5x = 15$, $x = 3$, $y = 1$.
- **Quadratic formula:** $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

**Answer shapes: M1** a correct method, **A1** the correct answer.

## ■ The three routes through a quadratic, and how to pick one

Try them in this order.

1. **Factorising.** Find two numbers that multiply to give the constant and add to give the coefficient of $x$. For $x^2 - 2x - 8 = 0$ they are $-4$ and 2, so $(x-4)(x+2) = 0$ and $x = 4$ or $x = -2$. Fastest when the numbers are whole.
2. **The formula.** Use it whenever the question says "correct to 2 decimal places", because that is a sign the roots are not whole numbers:
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
3. **Completing the square.** Use it when the question asks for the turning point or for an exact (surd) answer.

**Worked example (formula).** Solve $2x^2 + 3x - 4 = 0$ to 2 decimal places. Here $a = 2$, $b = 3$, $c = -4$.

1. $b^2 - 4ac = 9 - 4(2)(-4) = 9 + 32 = 41$.
2. $x = \dfrac{-3 \pm \sqrt{41}}{4}$.
3. $\sqrt{41} = 6.403$, so $x = \dfrac{3.403}{4} = 0.85$ or $x = \dfrac{-9.403}{4} = -2.35$.

> Show the substituted formula before evaluating: that line earns the method mark even if the arithmetic slips.
>
> **Simultaneous equations.** With one linear and one quadratic, **substitute** the linear into the quadratic; with two linear equations, **eliminate** by adding or subtracting once the coefficients match. Always find both unknowns and pair them correctly.

## 2 Practice

**2.1** Solve $3x - 7 = 11$. [2]

**2.2** Solve $x^2 + 5x + 6 = 0$. [2]

**2.3** Solve $4(x - 1) = 2x + 6$. [2]

**2.4** Solve $x^2 - 7x + 12 = 0$ by factorising. [2]

**2.5** Write down the values of $a$, $b$ and $c$ for $3x^2 - 5x - 2 = 0$. [1]

## 3 Exam-style questions

**3.1** Solve the simultaneous equations $3x + 2y = 12$ and $x - 2y = 4$. [3]

**3.2** Solve $x^2 - 2x - 8 = 0$. [2]

**3.3** Solve $2x^2 + 3x - 4 = 0$, giving your answers correct to 2 decimal places. [3]

**3.4** (a) Solve the simultaneous equations $y = x + 3$ and $x^2 + y^2 = 29$. [5]

(b) Solve $3x^2 - 7x + 2 = 0$ by factorising. [3]

(c) Solve $x^2 + 6x + 4 = 0$, giving your answers correct to 2 decimal places. [4]

## Solutions

Marking: **M1** a correct method, **A1** the correct answer.

**2.1** $3x = 18 \to x = 6$.

**2.2** $(x+2)(x+3) = 0 \to x = -2$ or $x = -3$.

**2.3** $4x - 4 = 2x + 6 \to 2x = 10 \to x = 5$.

**2.4** $(x-3)(x-4) = 0$; $x = 3$ or $x = 4$.

**2.5** $a = 3$, $b = -5$, $c = -2$.

**3.1** add the two equations: $4x = 16 \to x = 4$; then $4 - 2y = 4 \to y = 0$.

**3.2** $(x-4)(x+2) = 0 \to x = 4$ or $x = -2$.

**3.3** $a = 2, b = 3, c = -4$: $x = \frac{-3\pm\sqrt{9+32}}{4} = \frac{-3\pm\sqrt{41}}{4} \to x = 0.85$ or $x = -2.35$.

**3.4** (a) Substitute $y = x + 3$: $x^2 + (x+3)^2 = 29$; $2x^2 + 6x + 9 = 29$, so $2x^2 + 6x - 20 = 0$ and $x^2 + 3x - 10 = 0$; $(x+5)(x-2) = 0$; $x = -5$ or $x = 2$; the matching values are $y = -2$ and $y = 5$.
(b) $(3x-1)(x-2) = 0$; $x = \frac{1}{3}$ or $x = 2$.
(c) $a = 1$, $b = 6$, $c = 4$, so $x = \dfrac{-6 \pm \sqrt{36 - 16}}{2}$; $\sqrt{20} = 4.472$; $x = -0.76$ or $x = -5.24$.

**15** Factorise.

**(a)** $x^2 - 7x + 12$

.................................................... [2]

**(b)** $5x + 10y + 6ny + 3nx$

.................................................... [2]

**16** The table shows three sequences.

| | 1st term | 2nd term | 3rd term | 4th term | 5th term | $n$th term |
|---|---|---|---|---|---|---|
| Sequence $A$ | 8 | 13 | 18 | 23 | 28 | |
| Sequence $B$ | $\frac{3}{2}$ | $\frac{4}{3}$ | $\frac{5}{4}$ | $\frac{6}{5}$ | $\frac{7}{6}$ | |
| Sequence $C$ | $\frac{1}{4}$ | $\frac{1}{2}$ | 1 | 2 | 4 | |

Complete the table to show the $n$th term of each sequence.

[5]

**17** Expand and simplify.

$$(x-2)(2x+3)(x+4)$$

.................................................. [3]

**14** These expressions are all equal in value.

$$\frac{5x-2}{3} \qquad 10-x \qquad y+11$$

Find the value of $y$.

$y =$ ............................................... [5]

**15** The population of a town is 54 000.
The population is **decreasing** exponentially at a rate of 2% per year.

**(a)** Calculate the decrease in the population at the end of 4 years.

................................................... [3]

**(b)** Find the number of complete years it takes for the population of 54 000 to first fall below 44 000.

........................................ years [2]

**16** Expand and simplify.

**(a)** $7(x+2)+4(3x-5)$

.................................................. [2]

**(b)** $(3x-y)(5x+2y)$

.................................................. [2]

**17** Make $t$ the subject of the formula.

$$x=\frac{7t}{5-t}$$

$t =$ ................................................ [3]

**6** Solve.

**(a)** $8x + 7 = 39$

$x =$ .................................................. [2]

**(b)** $2(5y - 1) = 24$

$y =$ .................................................. [3]

IGCSE Mathematics: **s25 41**

**6** Expand.

$5x^2(3x - 2)$

.................................................. [2]

IGCSE MATHEMATICS • EXERCISE SHEET

# E2.6 Inequalities

Name: ______________ Class: __________ Date: ____________ **Total: 29 marks**

**KEY WORDS** inequality 不等式 • number line 数轴 • region 区域 • term 项 • equation 方程

## Objective

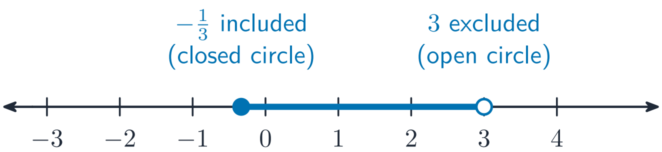
Build the skills to answer Extended exam questions on **inequalities 不等式** — solving them, the rule for **multiplying or dividing by a negative 乘除负数**, and showing a solution on a **number line 数轴**.

**You must be able to:**

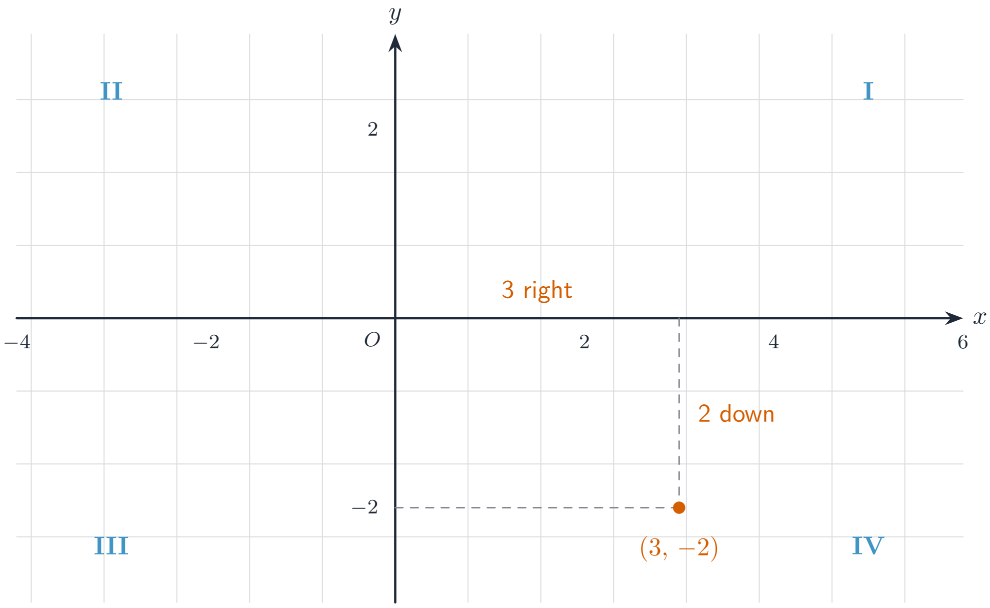
- solve a linear inequality (and reverse the sign when needed)
- solve a double inequality
- list the integers that satisfy an inequality

## 1 Worked examples

### Solving inequalities




*Solve like an equation, but reverse the sign if you ÷ or × by a negative*




*Two-variable inequalities shade regions of the plane*

- **Worked:** $-3 \leqslant 3x - 2 < 7 \to$ add 2: $-1 \leqslant 3x < 9 \to \div$ 3: $-\frac{1}{3} \leqslant x < 3$.
- **Reverse:** $-2x < 6 \to x > -3$ (sign flips when dividing by $-2$).
- **Number line:** closed circle for $\leqslant$ / $\geqslant$, open circle for $<$ / $>$.

**Answer shapes:** **M1** a correct method, **A1** the correct answer.

## ■ Inequalities: the same steps as an equation, with one extra rule

Solve an inequality exactly as you would the matching equation — **except** that multiplying or dividing both sides by a **negative** number **reverses** the inequality sign.

$$-3x > 12 \Rightarrow x < -4$$

That single rule is the difference between full marks and none on half the questions in this topic. If you would rather not risk it, move the $x$ term to the side where it is positive instead: $-3x > 12$ becomes $-12 > 3x$, so $-4 > x$, which is the same answer with no sign flip.

**A double inequality** is solved by doing the same operation to all three parts: $5 \leqslant 2x + 1 \leqslant 11$ gives $4 \leqslant 2x \leqslant 10$, then $2 \leqslant x \leqslant 5$.

**Reading the answer as integers.** For $-2 < x \leqslant 3$: the $<$ EXCLUDES $-2$ and the $\leqslant$ INCLUDES 3, so the integers are $-1, 0, 1, 2, 3$. Getting the endpoints right is usually a mark by itself.

**On a number line**, an open circle means the value is excluded ($<$ or $>$) and a filled circle means it is included ($\leqslant$ or $\geqslant$).

**Worked example.** Find the integers $n$ for which $-9 < 3n - 3 \leqslant 6$.

1. Add 3 throughout: $-6 < 3n \leqslant 9$.
2. Divide by 3 throughout: $-2 < n \leqslant 3$.
3. The endpoints: $-2$ is excluded, 3 is included.
4. So $n = -1, 0, 1, 2, 3$.

## 2 Practice

**2.1** Solve $2x + 1 > 9$. [2]

**2.2** Solve $-3x \leqslant 12$. [2]

**2.3** Solve $5x - 4 < 11$. [2]

**2.4** Solve $-4x + 3 > 19$. [2]

**2.5** Write down the inequality shown by an open circle at $-1$ and a filled circle at 4, joined by a line. [2]

## 3 Exam-style questions

**3.1** Solve the inequality $5 \leqslant 2x + 1 \leqslant 11$. [3]

**3.2** Write down the integers that satisfy $-2 < x \leqslant 3$. [2]

**3.3** Solve $7 - 2x \geqslant 1$. [2]

**3.4** (a) Solve the inequality $-11 < 4x + 5 \leqslant 13$. [3]

(b) Write down all the integers that satisfy your answer to part (a). [2]

(c) Solve $9 - 4x \geqslant 21$, and explain the step in which the inequality sign changes. [3]

(d) A rectangle has width $x$ cm and length $(x + 5)$ cm. Its perimeter is at least 30 cm and less than 50 cm. Form an inequality in $x$ and solve it. [4]

## Solutions

Marking: **M1** a correct method, **A1** the correct answer.

**2.1** $2x > 8 \rightarrow x > 4$.

**2.2** dividing by $-3$ reverses the sign: $x \geqslant -4$.

**2.3** $5x < 15 \rightarrow x < 3$.

**2.4** $-4x > 16$; dividing by $-4$ reverses the sign, so $x < -4$.

**2.5** $-1 < x \leqslant 4$.

**3.1** subtract 1: $4 \leqslant 2x \leqslant 10$; divide by 2: $2 \leqslant x \leqslant 5$.

**3.2** the integers are $-1, 0, 1, 2, 3$.

**3.3** $-2x \geqslant -6 \rightarrow$ dividing by $-2$ reverses the sign $\rightarrow x \leqslant 3$.

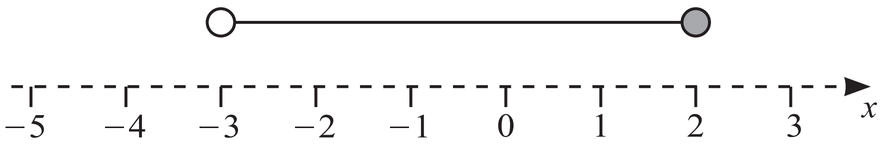
**3.4** (a) Subtract 5 throughout: $-16 < 4x \leqslant 8$; divide by 4 throughout: $-4 < x \leqslant 2$.
(b) $-3, -2, -1, 0, 1, 2$.
(c) $-4x \geqslant 12$; dividing both sides by $-4$ reverses the inequality; $x \leqslant -3$.
(d) Perimeter $= 2(x) + 2(x+5) = 4x + 10$; $30 \leqslant 4x + 10 < 50$; $20 \leqslant 4x < 40$; $5 \leqslant x < 10$.

**7** Represent the inequality $-4 < x \leqslant 3$ on the number line.


[2]

IGCSE Mathematics: **w25 41**

**7** **(a)**




Write down the inequality represented in the diagram.

.................................................. [2]

**(b)** Write down the integer values of $x$ that satisfy the inequality $-4 < 2x \leqslant 8$.

.................................................. [2]

**24** Martha walks a distance of 10 km at a speed of $x$ km/h.
She then runs a distance of 5 km at a speed of $(x+4)$ km/h.
The total time taken for the whole journey is 3.5 hours.

**(a)** Write down an expression in terms of $x$ for the time Martha is walking.

................................................ h [1]

**(b)** Show that $7x^2 - 2x - 80 = 0$.

[4]

**(c)** Solve $7x^2 - 2x - 80 = 0$, giving your answers correct to 2 decimal places.
You must show all your working.

$x =$ .................. or $x =$ .................. [3]

**(d)** Calculate the difference between the time Martha is walking and the time she is running.
Give your answer in hours and minutes correct to the nearest minute.

**18** One day, Anya runs 12 km at a speed of $x$ km/h.
The next day she walks 10 km at a speed of $(x-4)$ km/h.

**(a)** Write down an expression, in terms of $x$, for the time she spends running.

............................................ h [1]

**(b)** Write down an expression, in terms of $x$, for the time she spends walking.

............................................ h [1]

**(c)** The time Anya spends walking is 1 hour more than the time she spends running.

Write an equation in terms of $x$ and show that it simplifies to $x^2 - 2x - 48 = 0$.

[4]

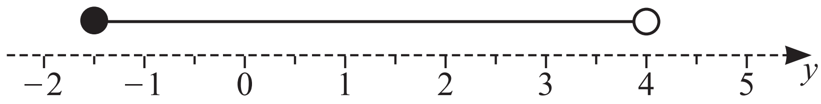
**(d)** Use factorisation to solve the equation $x^2 - 2x - 48 = 0$.

$x =$ ................. or $x =$ ................. [3]

**(e)** Find the time Anya spends running.

............................................ h [1]

**7**




Write down the inequality in $y$ shown by the number line.

................................................... [2]

IGCSE MATHEMATICS • EXERCISE SHEET

# E2.10 Graphs of functions

Name: ____________ Class: ________ Date: __________ **Total: 28 marks**

**KEY WORDS** table of values 数值表 • coordinates 坐标 • axis 坐标轴 • function 函数 • roots 根

## Objective

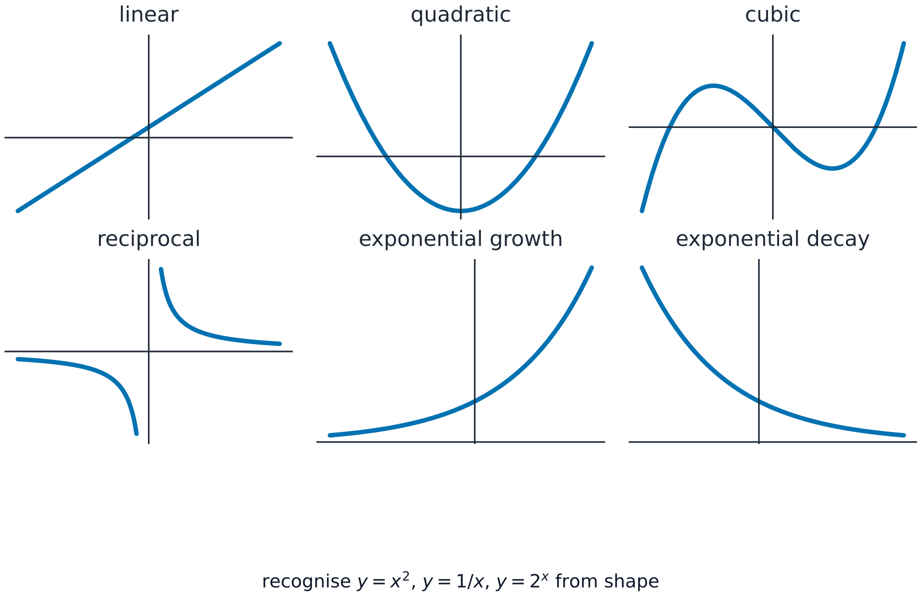
Build the skills to answer Extended exam questions on **graphs of functions** 函数图象 — making a **table of values** 数值表, finding **roots** 根 (where the graph crosses the $x$-axis), and reading solutions from a graph.

**You must be able to:**

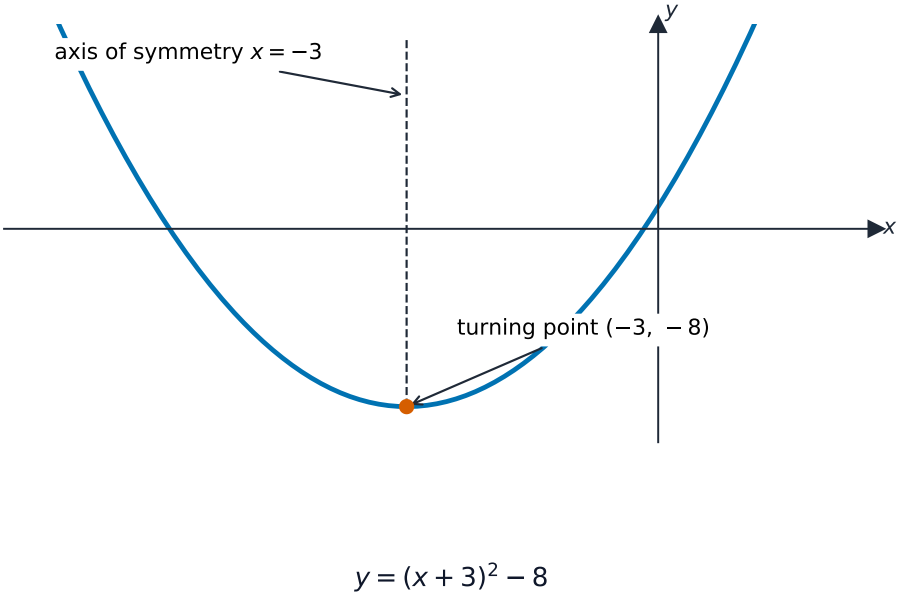
- complete a table of values for a function
- identify the roots and the $y$-intercept of a graph
- use a graph to solve an equation

## 1 Worked examples

### Tables, roots and intersections




*Plot points from a table, then join with a smooth curve*




*The quadratic is one key function graph*

- **Table:** for $y = x^2 - 3$, when $x = 2$, $y = 4 - 3 = 1$.
- **Roots** 根: where $y = 0$ (the graph crosses the $x$-axis).
- **Solve from a graph:** the **intersection** 交点 of a line and a curve solves the two together.

**Answer shapes: M1** a correct method, **A1** the correct answer.

### ■ Reading a graph, and solving with a line

The **roots** are where the curve crosses the $x$-axis, so they are the solutions of $y = 0$. The $y$**-intercept** is the value at $x = 0$. The **turning point** of $y = ax^2 + bx + c$ lies on the line of symmetry $x = -\dfrac{b}{2a}$; substitute that $x$ back to get its $y$.

**Solving with a drawn line.** To solve an equation using a drawn curve, rearrange it so that one side is exactly the curve's equation; the other side is then the line to draw, and the solutions are the $x$-coordinates where the curve and line cross.

**Worked example.** The curve $y = x^2 - 2x - 3$ is drawn. Use it to solve $x^2 - 2x - 8 = 0$.

1. Rearrange so the left side matches the drawn curve: $x^2 - 2x - 8 = 0$ becomes $x^2 - 2x - 3 = 5$.
2. So draw the horizontal line $y = 5$.
3. Read the $x$-coordinates where the curve meets it: $x = -2$ and $x = 4$.
4. Check by factorising: $(x - 4)(x + 2) = 0$, which agrees.

**Sketching.** Give the shape (a positive $x^2$ term opens upwards, a negative one opens downwards), the roots, the $y$-intercept and the turning point. For a cubic, the shape rises from bottom-left to top-right when the $x^3$ term is positive, and the roots are still where it crosses the $x$-axis.

## 2 Practice

**2.1** For $y = x^2 - 4$, find the value of $y$ when $x = 3$. [1]

**2.2** Write down the $y$-intercept of $y = 2x^2 - 5$. [1]

**2.3** Find the roots of $y = x^2 - 9$. [2]

**2.4** Find the coordinates of the turning point of $y = x^2 - 6x + 5$. [3]

**2.5** State the line you would draw on the graph of $y = x^2 + x$ to solve $x^2 + x - 4 = 0$. [1]

## 3 Exam-style questions

**3.1** $y = x^2 - 2x - 3$.

(a) Complete the value of $y$ when $x = -2$. [1]

(b) Find the roots of $x^2 - 2x - 3 = 0$. [2]

**3.2** The graph of $y = x^2 - 2x - 3$ is drawn. By solving a quadratic, find the $x$-values where the graph meets the line $y = 5$. [3]

**3.3** The curve $y = x^2 - 4x + 1$ is to be drawn for $-1 \leqslant x \leqslant 5$.

(a) Complete the table of values. [3]

| $x$ | $-1$ | **0** | **1** | **2** | **3** | **4** | **5** |
|---|---|---|---|---|---|---|---|
| $y$ | | | | | | | |

(b) Write down the coordinates of the turning point of the curve. [3]

(c) Use the curve to solve $x^2 - 4x + 1 = 0$, giving your answers correct to 1 decimal place. [2]

(d) By drawing a suitable straight line, explain how you would solve $x^2 - 4x - 2 = 0$, and give the equation of that line. [2]

(e) Solve $x^2 - 4x - 2 = 0$ algebraically, giving your answers correct to 2 decimal places. [4]

## Solutions

Marking: **M1** a correct method, **A1** the correct answer.

**2.1** $y = 3^2 - 4 = 5$.

**2.2** the $y$-intercept is at $x = 0$: $y = -5$.

**2.3** $x^2 - 9 = 0 \to (x+3)(x-3) = 0 \to x = 3$ or $x = -3$.

**2.4** $x = -\dfrac{-6}{2} = 3$; $y = 9 - 18 + 5 = -4$; the turning point is $(3, -4)$.

**2.5** $y = 4$.

**3.1** (a) $y = (-2)^2 - 2(-2) - 3 = 4 + 4 - 3 = 5$. (b) $(x-3)(x+1) = 0 \to x = 3$ or $x = -1$.

**3.2** set $x^2 - 2x - 3 = 5 \to x^2 - 2x - 8 = 0 \to (x-4)(x+2) = 0 \to x = 4$ or $x = -2$.

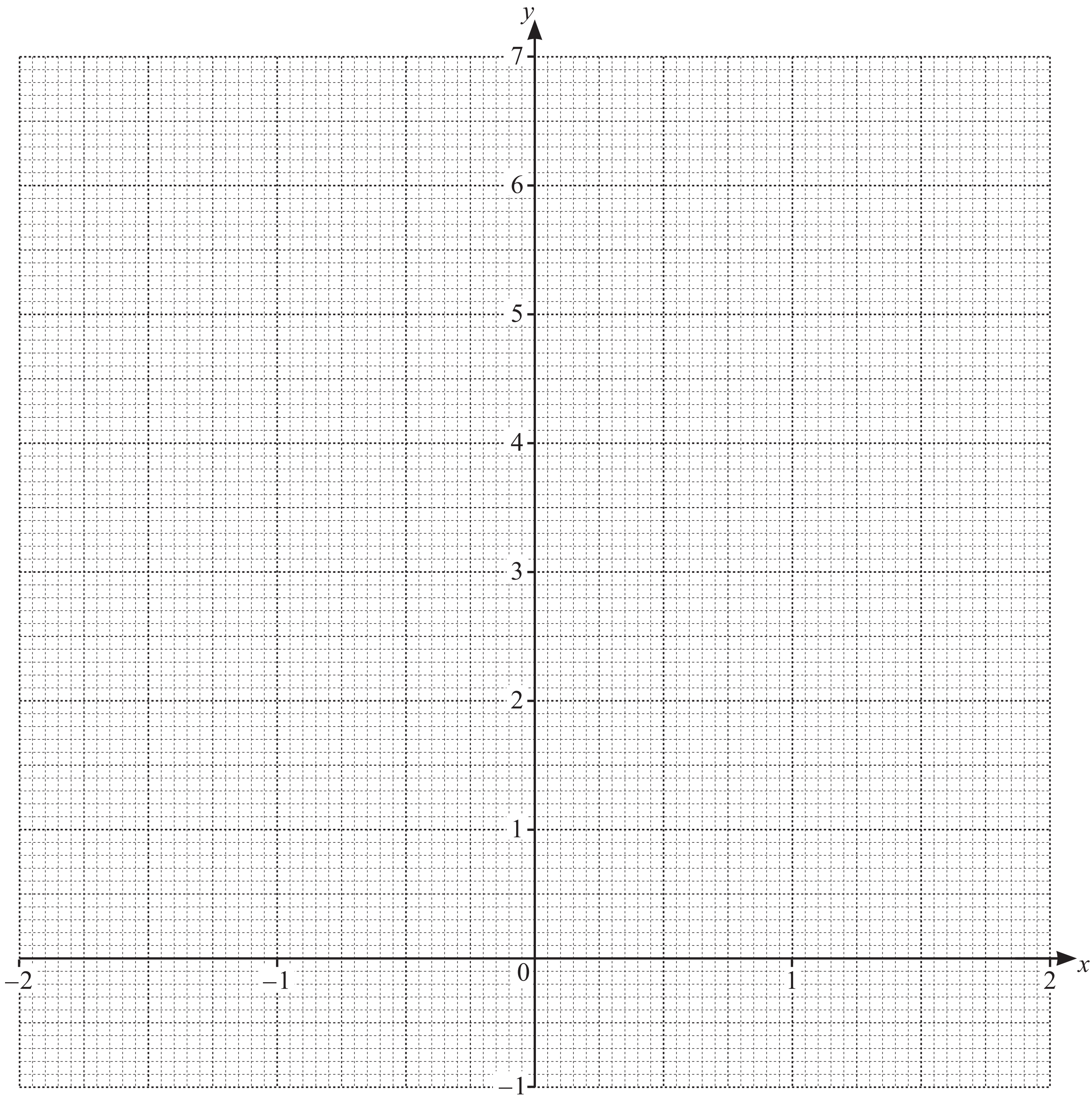
**3.3** (a) $6, 1, -2, -3, -2, 1, 6$.
(b) $x = -\dfrac{-4}{2} = 2$; $y = 4 - 8 + 1 = -3$; the turning point is $(2, -3)$.
(c) The curve crosses the $x$-axis at about $x = 0.3$ and $x = 3.7$.
(d) Rearranging, $x^2 - 4x - 2 = 0$ becomes $x^2 - 4x + 1 = 3$, so draw the line $y = 3$ and read where it meets the curve.
(e) $a = 1$, $b = -4$, $c = -2$, so $x = \dfrac{4 \pm \sqrt{16+8}}{2}$; $\sqrt{24} = 4.899$; $x = 4.45$ or $x = -0.45$.

**11** The table shows some values for $y = x^3 - 2x + 3$.
Where appropriate, values of $y$ are given correct to 2 decimal places.

| $x$ | $-2$ | $-1.5$ | $-1$ | $-0.5$ | $0$ | $0.5$ | $1$ | $1.5$ | $2$ |
|---|---|---|---|---|---|---|---|---|---|
| $y$ | $-1$ | | $4$ | $3.88$ | $3$ | $2.13$ | $2$ | | $7$ |

**(a)** Complete the table. [2]

**(b)** Draw the graph of $y = x^3 - 2x + 3$ for $-2 \leqslant x \leqslant 2$.

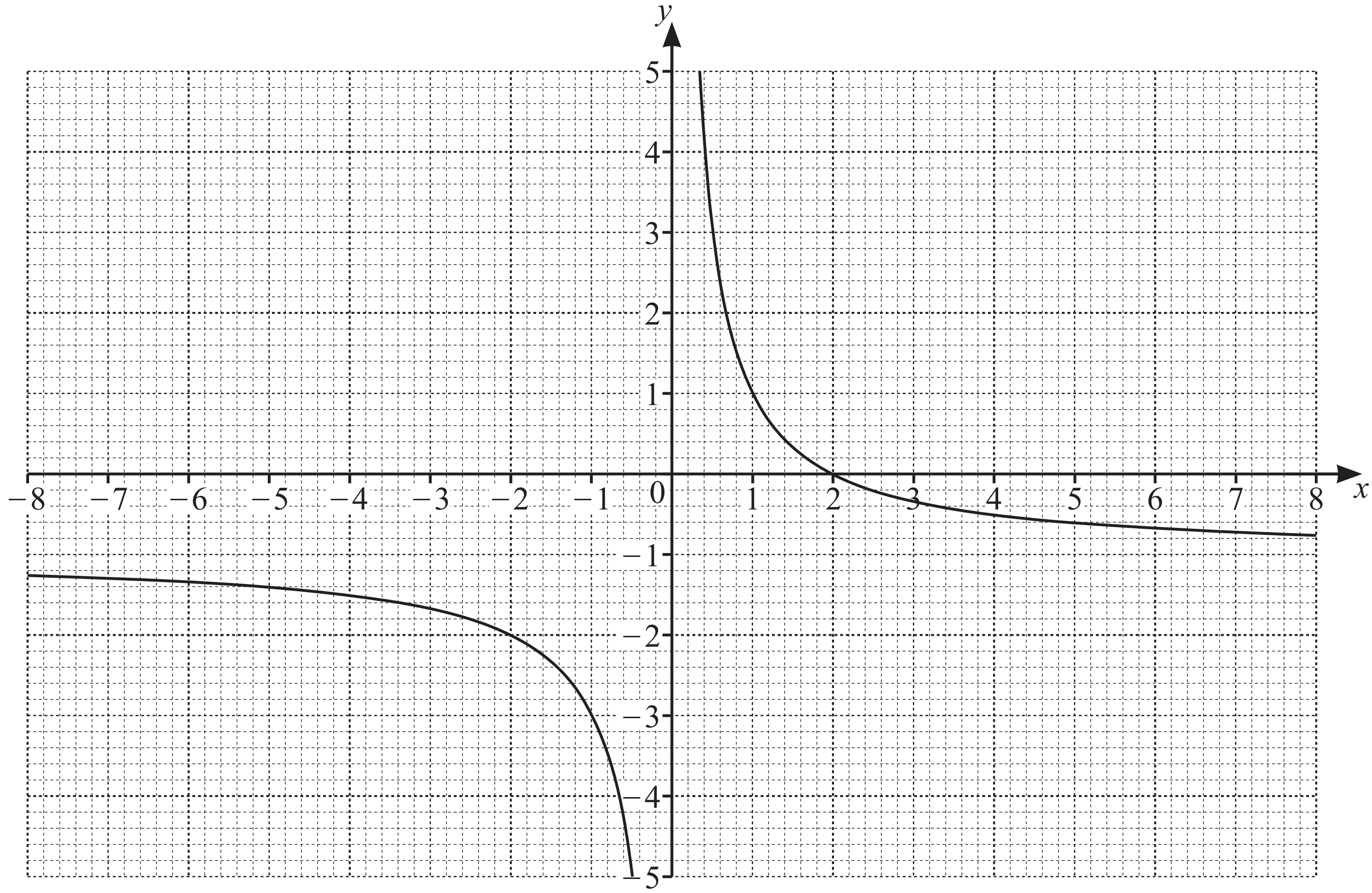



[4]

**(c)** By drawing a suitable straight line on the grid, solve the equation $x^3 - 2.5x + 1 = 0$.

$x =$ ...................... or $x =$ ...................... or $x =$ ...................... [4]

**15**




The diagram shows the graph of $y = \frac{2}{x} - 1$.

**(a)** Write down the coordinates of the point where the graph crosses the $x$-axis.

( ................. , ................. ) [1]

**(b)** Write down the equation of each asymptote.

................................................

................................................

[2]

**(c)** By drawing a suitable straight line on the grid, solve $\frac{2}{x} - x - 1 = 0$.

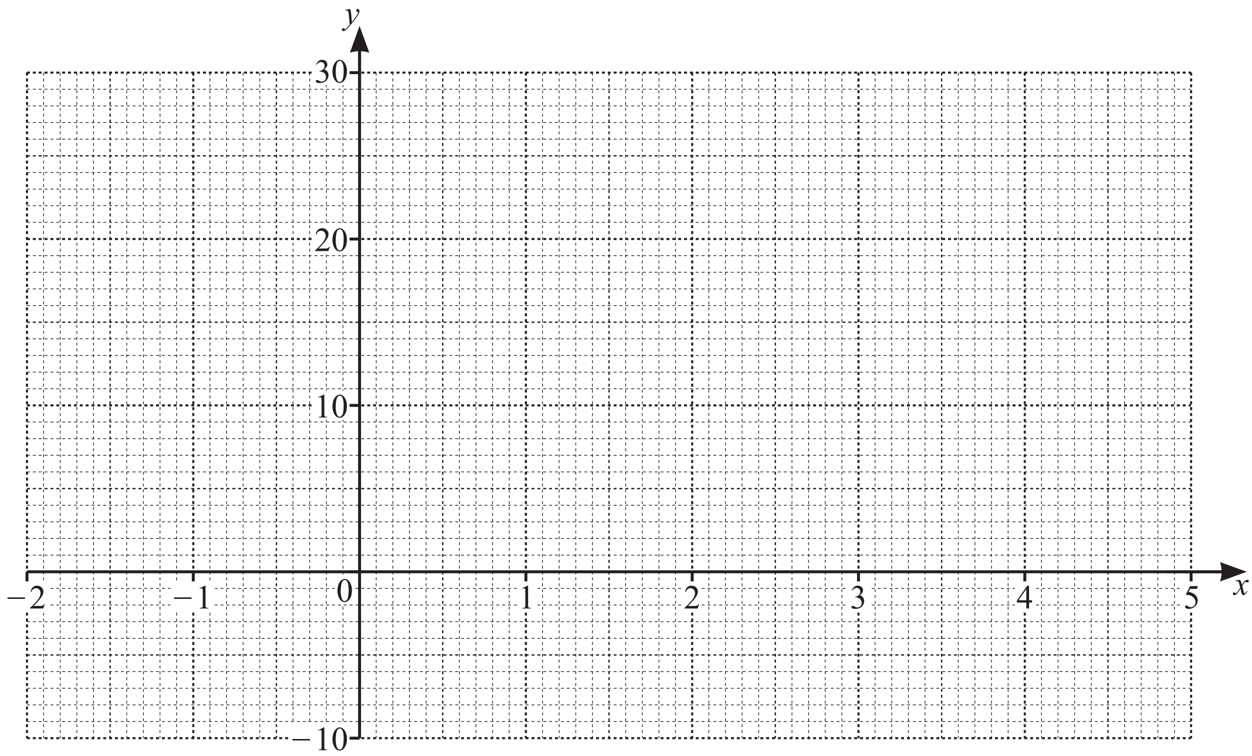
$x =$ ................. or $x =$ ................. [3]

**14** The table shows some values for $y = 5x^2 - x^3 - 4$.

| $x$ | $-2$ | $-1$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|---|---|
| $y$ | 24 | | $-4$ | | 8 | 14 | | $-4$ |

**(a)** Complete the table. [3]

**(b)** On the grid, draw the graph of $y = 5x^2 - x^3 - 4$ for $-2 \leqslant x \leqslant 5$.




[4]

**(c)** By drawing a suitable straight line on the grid, solve the equation $x^3 - 5x^2 - x + 14 = 0$.

$x =$ ................. or $x =$ ................. or $x =$ ................. [4]

IGCSE MATHEMATICS • EXERCISE SHEET

# E2.11 Sketching curves

Name: ____________ Class: ________ Date: __________ **Total: 7 marks**

**KEY WORDS** turning point 转折点 • parabola 抛物线 • shapes 形状 • line of symmetry 对称轴 • symmetry 对称

## Objective

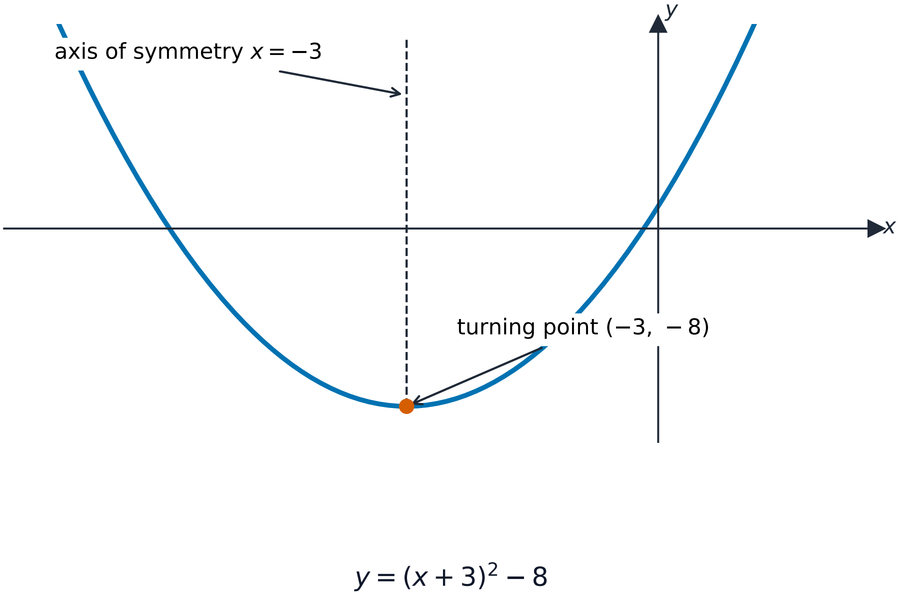
Build the skills to answer Extended exam questions on **sketching curves** 草绘曲线 — the **shapes** 形状 of common graphs, the **turning point** 转折点 of a parabola, and key features.

**You must be able to:**

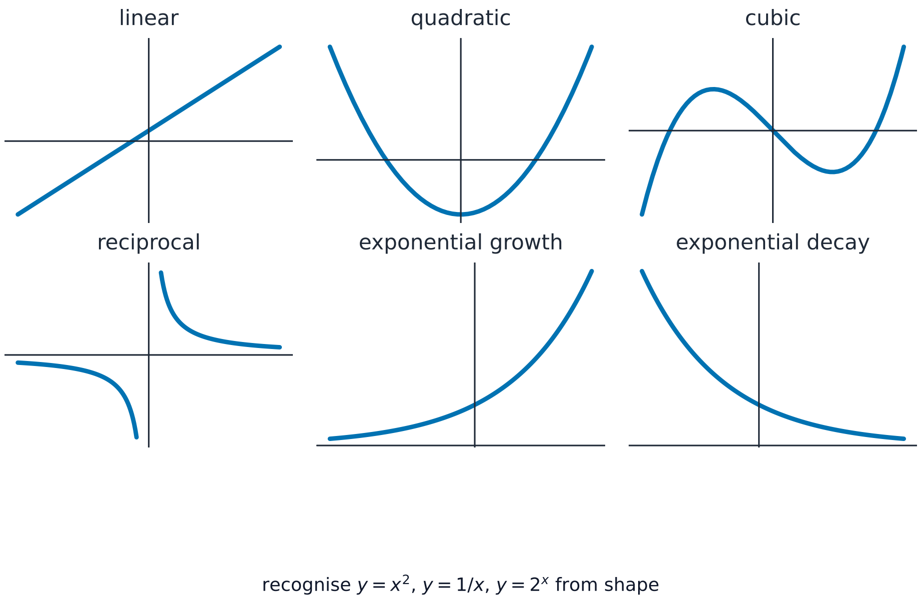
- recognise the shape of linear, quadratic, cubic and reciprocal graphs
- find the turning point of a parabola by completing the square
- state the line of symmetry of a parabola

## 1 Worked examples

### ■ Shapes and turning points




*Completing the square gives the turning point*




*The family of standard curve shapes to recognise*

- **Shapes:** linear (line), **quadratic** (a **parabola 抛物线**, U if $a > 0$), cubic (S-shape), reciprocal (two curves).
- **Turning point:** $y = (x+3)^2 - 8$ has its turning point at $(-3, -8)$.
- **Line of symmetry 对称轴:** the vertical line through the turning point ($x = -3$ here).

**Answer shapes: M1** a correct method, **A1** the correct answer.

## 2 Practice

**2.1** Write down the shape of the graph of $y = x^2 + 1$. [1]

**2.2** Write down the turning point of $y = (x-4)^2 + 2$. [1]

**2.3** Write down the $y$-intercept of $y = 3x - 5$. [1]

## 3 Exam-style questions

**3.1** Write $x^2 + 6x + 5$ in the form $(x + p)^2 + q$. [2]

**3.2** Hence write down the coordinates of the turning point of $y = x^2 + 6x + 5$. [1]

**3.3** Write down the equation of the line of symmetry of $y = x^2 + 6x + 5$. [1]

## Solutions

Marking: **M1** a correct method, **A1** the correct answer.

**2.1** a parabola (a U-shaped curve).

**2.2** $(4, 2)$.

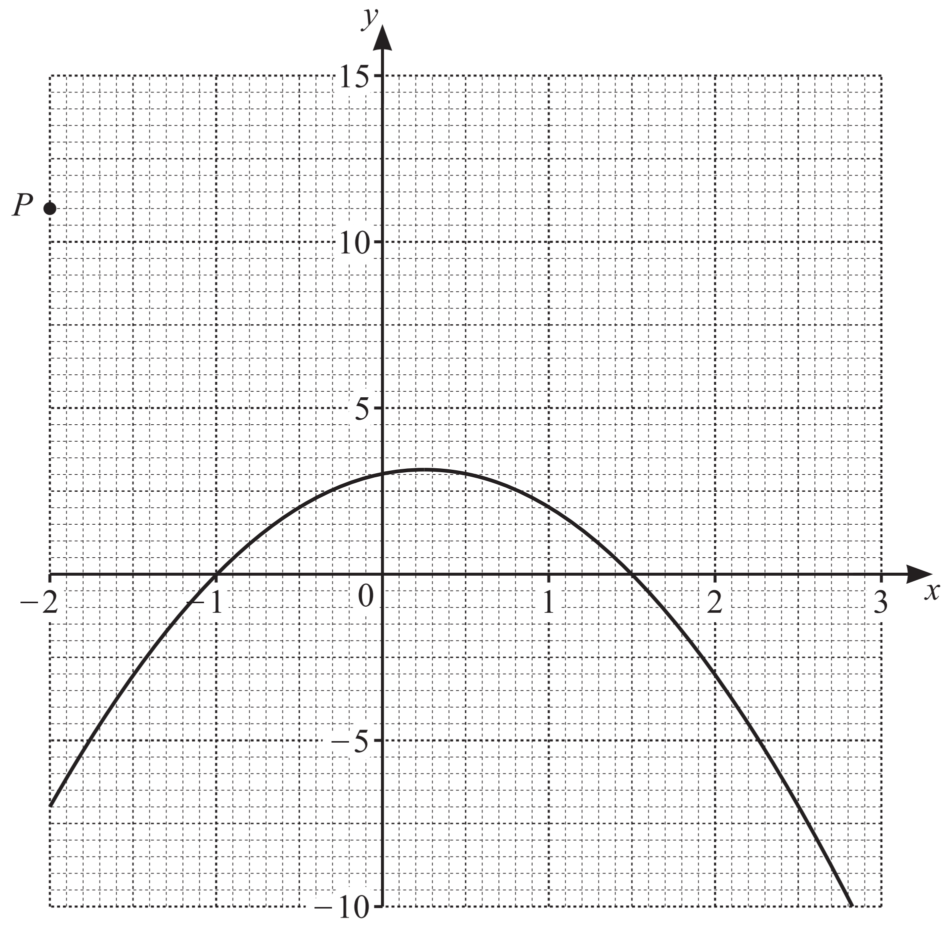
**2.3** the $y$-intercept is $-5$.

**3.1** half of 6 is 3, $3^2 = 9$: $x^2 + 6x + 5 = (x + 3)^2 - 9 + 5 = (x + 3)^2 - 4$.

**3.2** the turning point is $(-3, -4)$.

**3.3** the line of symmetry is $x = -3$.

**11** The diagram shows the graph of $y = \mathrm{f}(x)$ and the point $P(-2, 11)$.




The tangent from $P$ touches the graph of $y = \mathrm{f}(x)$ at the point $(a, b)$.
The values of $a$ and $b$ are integers.

**(a)** By drawing this tangent, find the value of $a$ and the value of $b$.

$a =$ .................. , $b =$ .................. [2]

**(b)** Find the equation of the tangent.
Give your answer in the form $y = mx + c$.

$y =$ ............................................... [3]