

Week 1

IGCSE Mathematics

Homework bundle for this week

本周作业合集

exercises.lol

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1. Number

Starter Quiz · 课前小测

IGCSE Mathematics

Name: _____ Score: _____

1. Which of these is a prime number?

☐ 13

☐ 1

☐ 9

☐ 15

2. $A = \{2, 4, 6, 8, 10\}$ and $B = \{3, 6, 9\}$. How many elements are in $A \cap B$?

3. Work out $5^2 \times \sqrt[3]{8}$.

4. Write 4 500 000 as $A \times 10^n$. What is the power n?

5. Simplify $\sqrt{20}$ into the form $k\sqrt{5}$. What is k?

6. Write $3/8$ as a decimal.

7. Which surd rule is correct?

☐ $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

☐ $\sqrt{a} + \sqrt{b} = \sqrt{a+b}$

☐ $\sqrt{a} \times \sqrt{b} = \sqrt{a+b}$

☐ $\sqrt{a/b} = \sqrt{a} \times \sqrt{b}$

8. The symbol $\cap$ means:

☐ intersection (in both sets)

☐ union (in either set)

☐ complement (not in the set)

☐ empty set

9. What is $2/3 \times 4/5$?

☐ $8/15$

☐ $6/8$

☐ $8/8$

☐ $2/15$

10. Depreciation of a car's value each year is an example of:

☐ exponential decay

☐ exponential growth

☐ simple interest

☐ linear growth

1. 13

13 has exactly two factors (1 and 13). 1 is not prime; $9 = 3 \times 3$; $15 = 3 \times 5$.

2. 1

Only 6 is in both sets, so $A \cap B = \{6\}$, which has 1 element.

3. 50

$5^2 = 25$ and $\sqrt[3]{8} = 2$, so $25 \times 2 = 50$.

4. 6

$4\,500\,000 = 4.5 \times 10^6$, so $n = 6$.

5. 2

$\sqrt{20} = \sqrt{(4 \times 5)} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$, so $k = 2$.

6. 0.375

$3 \div 8 = 0.375$.

7. $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$; roots do not add like $\sqrt{a} + \sqrt{b} = \sqrt{a+b}$.

8. intersection (in both sets)

$\cap$ is intersection (elements in both A and B); $\cup$ is union.

9. 8/15

Multiply tops and bottoms: $(2 \times 4)/(3 \times 5) = 8/15$.

10. exponential decay

Losing a fixed percentage each year is exponential decay (multiplier < 1 , applied repeatedly).

E1.1 Types of number

Name: _____ Class: _____ Date: _____ **Total: 12 marks**

KEY WORDS factor 因数 • integer 整数 • hcf 最大公因数 • lcm 最小公倍数
• common multiple 公倍数

Objective

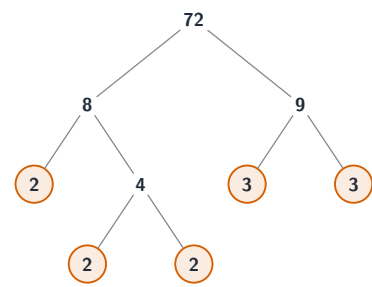
Build the skills to answer Extended exam questions on **types of number** 数的类型 — **prime factors** 质因数, **HCF and LCM** 最大公因数和最小公倍数, and **rational and irrational** 有理数和无理数 numbers.

You must be able to:

- write a number as a product of its prime factors
- find the HCF and LCM using prime factors
- identify rational and irrational numbers

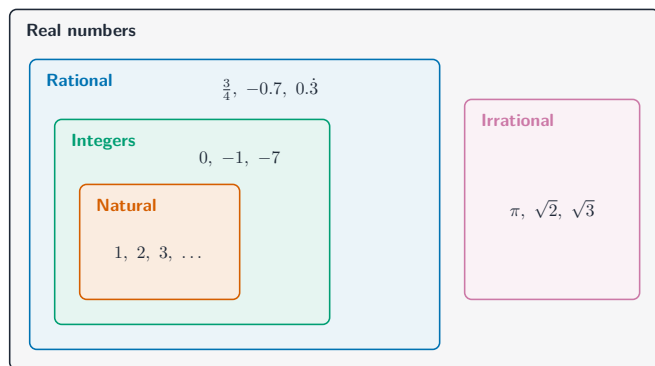
1 Worked examples

■ Prime factors, HCF and LCM



$72 = 2^3 \times 3^2$

Keep dividing by the smallest prime that fits



The number sets: natural, integer, rational and real

- **Prime factors:** $72 = 2^3 \times 3^2$ (split until every branch is prime).
- **HCF** 最大公因数: the **lowest** power of each shared prime. **LCM** 最小公倍数: the **highest** power of every prime.
- **Worked (72 and 120):** $72 = 2^3 \times 3^2$, $120 = 2^3 \times 3 \times 5 \rightarrow \text{HCF} = 2^3 \times 3 = 24$, $\text{LCM} = 2^3 \times 3^2 \times 5 = 360$.
- **Irrational** 无理数: cannot be written as a fraction (e.g. π , $\sqrt{2}$).

Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer):

2 Practice

2.1 Write 90 as a product of its prime factors. [2]

2.2 From the list $\sqrt{9}$, π , $\frac{2}{3}$, $\sqrt{5}$, write down the two **irrational** numbers. [2]

2.3 Find the HCF of 24 and 36. [2]

3 Exam-style questions

3.1 $A = 2^2 \times 3 \times 5$ and $B = 2 \times 3^2 \times 5$.

(a) Find the HCF of A and B . [2]

(b) Find the LCM of A and B . [2]

3.2 Find the lowest common multiple (LCM) of 18 and 24. [2]

Solutions

Marking: **M1** a correct method, **A1** the correct answer (allow follow-through where sensible).

2.1 $90 = 2 \times 3^2 \times 5$.

2.2 π and $\sqrt{5}$.

2.3 $24 = 2^3 \times 3$, $36 = 2^2 \times 3^2 \rightarrow \text{HCF} = 2^2 \times 3 = 12$.

3.1 (a) HCF — lowest power of each shared prime: $2^1 \times 3 \times 5 = 30$. (b) LCM — highest power of every prime: $2^2 \times 3^2 \times 5 = 180$.

3.2 $18 = 2 \times 3^2$, $24 = 2^3 \times 3 \rightarrow \text{LCM} = 2^3 \times 3^2 = 72$.

13

$$M = 2^7 \times 3^3 \times 5^2$$

- (a) Write $14M$ as a product of its prime factors.
Give your answer in index form.

..... [2]

- (b) R is an integer.
 $\frac{M}{R}$ is a cube number.

Find the smallest possible value of R .

$R =$ [2]

14 Find the value of

(a) $3^2 \div 3^{-2}$

..... [2]

(b) $16^{-\frac{3}{2}}$.

..... [2]

7 (a) The sum of all the prime numbers less than 10 is equal to 17.
Find the sum of all the prime numbers less than 16.

..... [2]

(b) x is an integer.
The sum of the prime numbers greater than 6 and less than x is equal to 18.
Find a possible value for x .

$x =$ [1]

8 Find the highest common factor (HCF) of 36 and 54.

..... [2]

10 (a) Write down **all** the factors of 18.

..... [2]

(b) Factorise.
 $3y - xy + 15 - 5x$

..... [2]

(c) $3y - xy + 15 - 5x = 18$
where x and y are positive integers.

Using your answers to **part (a)** and **part (b)**, find one possible value of x and the corresponding value of y .

$x =$, $y =$ [2]

11 A warehouse has a floor area of 800 m^2 .
The plan of the warehouse is drawn to a scale of 1 : 50.
Calculate the floor area on the plan.
Give your answer in square centimetres.

..... cm^2 [3]

E1.2 Sets

Name: _____ Class: _____ Date: _____ **Total: 10 marks**

KEY WORDS venn diagram 维恩图 • element 元素 • notation 符号 • sets 集合
• intersection 交集

Objective

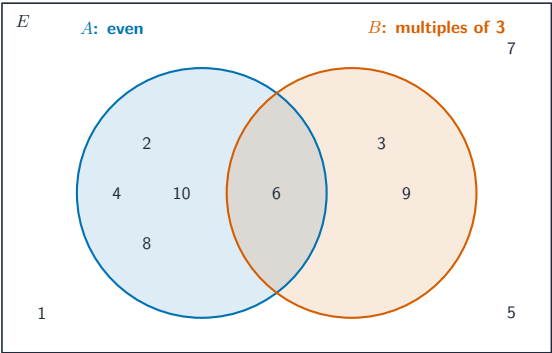
Build the skills to answer Extended exam questions on **sets** 集合 — set **notation** 符号, **Venn diagrams** 维恩图, and the **union**, **intersection** and **complement** 并集、交集、补集.

You must be able to:

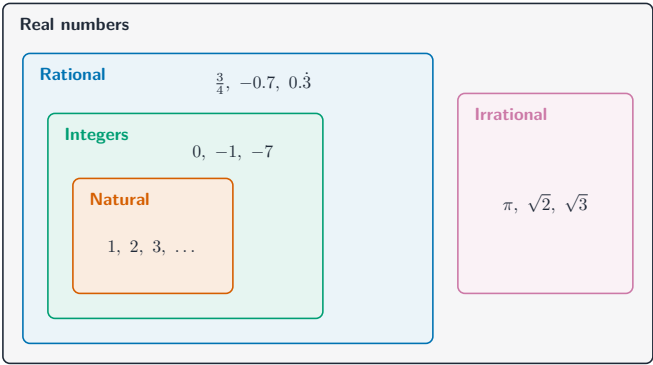
- use the symbols $\cup$, $\cap$, A' , $\in$, $n(A)$
- list elements of unions and intersections
- shade and read regions of a Venn diagram

1 Worked examples

■ Union, intersection, complement



The overlap is the intersection; everything inside either circle is the union



Number sets shown as nested regions of a Venn diagram

- $A \cup B$ **union** 并集: in A **or** B (or both). $A \cap B$ **intersection** 交集: in **both**.
- A' **complement** 补集: everything **not** in A . $n(A)$: the number of elements in A .
- **Worked:** $E = \{1-10\}$, $A = \{2, 4, 6, 8, 10\}$, $B = \{3, 6, 9\} \rightarrow A \cap B = \{6\}$, $A \cup B = \{2, 3, 4, 6, 8, 9, 10\}$, $n(A \cup B) = 7$.

Answer shapes: M1 a correct method, A1 the correct answer.

2 Practice

2.1 $E = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$. List the elements of $A \cap B$. [1]

2.2 For the same sets, list the elements of $A \cup B$. [2]

2.3 For the same sets, find $n(A')$. [2]

3 Exam-style questions

3.1 $E = \{x : 1 \leq x \leq 12, x \text{ is an integer}\}$, $P = \{\text{multiples of } 3\}$, $Q = \{\text{factors of } 12\}$.

(a) List the elements of $P \cap Q$. [2]

(b) Find $n(P \cup Q)$. [2]

3.2 On a Venn diagram with sets A and B , describe in words the region $A \cap B'$. [1]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $A \cap B = \{2, 4\}$.

2.2 $A \cup B = \{1, 2, 3, 4, 6, 8\}$.

2.3 $A' = \{5, 6, 7, 8\}$, so $n(A') = 4$.

3.1 (a) $P = \{3, 6, 9, 12\}$, $Q = \{1, 2, 3, 4, 6, 12\} \rightarrow P \cap Q = \{3, 6, 12\}$. (b) $P \cup Q = \{1, 2, 3, 4, 6, 9, 12\} \rightarrow n(P \cup Q) = 7$.

3.2 the elements that are in A but **not** in B (the part of circle A outside the overlap).

10 $\mathcal{E} = \{n: n \text{ is an integer and } 1 \leq n \leq 8\}$
 $A = \{\text{factors of } 12\}$
 $B = \{\text{odd numbers}\}$

Find

(a) $A \cap B$

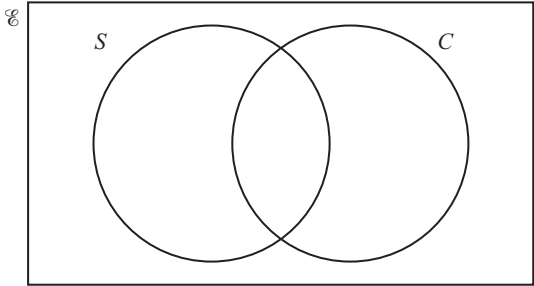
$A \cap B = \{.....\}$ [1]

(b) $n(A' \cup B)$.

..... [1]

12 A fitness club has 100 members.

60 swim (S).
70 cycle (C).
25 do not swim or cycle.



(a) Complete the Venn diagram.

[3]

(b) One member of the fitness club is chosen at random.

For this member, find

(i) $P(C)$

..... [1]

(ii) $P(S \cap C)$

..... [1]

(iii) $P(S \cup C')$.

..... [1]

E1.3 Powers and roots

Name: _____ Class: _____ Date: _____ **Total: 23 marks**

KEY WORDS power 幂 • powers and roots 幂和根 • cube root 立方根 • square root 平方根
• ratio 比

Objective

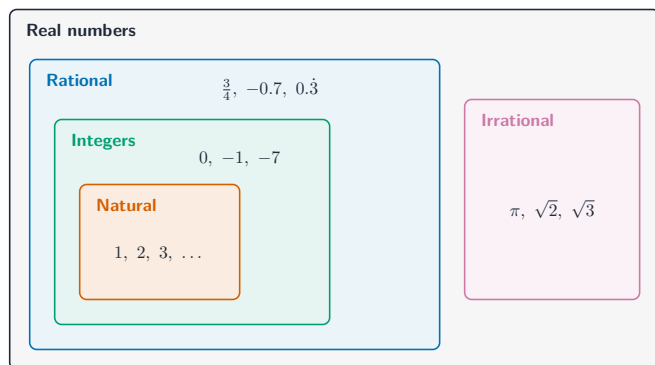
Build the skills to answer Extended exam questions on **powers and roots** 幂和根 — **squares, cubes** 平方、立方 and their roots, and evaluating expressions with powers and roots.

You must be able to:

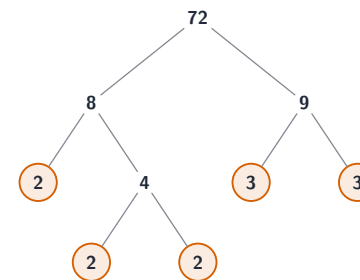
- recall squares to 15^2 and the cubes of 1–5 and 10
- find square roots and cube roots
- evaluate expressions combining powers and roots

1 Worked examples

■ Powers and roots



A power multiplies a number by itself; a root reverses it



$$72 = 2^3 \times 3^2$$

Prime factorisation underlies powers and roots

- Power** 幂: $2^5 = 32$. **Square root** 平方根: $\sqrt{169} = 13$. **Cube root** 立方根: $\sqrt[3]{8} = 2$.
- Worked:** $5^2 \times \sqrt[3]{8} = 25 \times 2 = 50$.
- Recall squares (1–15) and cubes (1, 2, 3, 4, 5, 10).

Answer shapes: **M1** a correct method, **A1** the correct answer.

2 Practice

2.1 Work out $\sqrt{196}$. [1]

2.2 Work out $\sqrt[3]{27} + 4^2$. [2]

2.3 Work out $\sqrt{144} - \sqrt[3]{64}$. [2]

3 Exam-style questions

3.1 Work out $3^3 - \sqrt{121}$. [2]

3.2 Work out $\sqrt{169} \times \sqrt[3]{125}$. [2]

3.3 Work out the value of $\sqrt{2^2 + 3^2 + 6^2}$. [2]

3.4 A cube has a volume of $2\,197\text{ cm}^3$.

(a) Find the length of one edge, and hence the total surface area of the cube. [4]

(b) A second cube has edges twice as long. Write down the ratio of the surface areas and the ratio of the volumes of the two cubes, and explain how you obtained them. [4]

(c) Without a calculator, show that $\sqrt[3]{\frac{64}{125}} \times \sqrt{\frac{81}{16}} = \frac{9}{5}$. [4]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $\sqrt{196} = 14$.

2.2 $\sqrt[3]{27} = 3$ and $4^2 = 16$, so $3 + 16 = 19$.

2.3 $\sqrt{144} = 12$, $\sqrt[3]{64} = 4$, so $12 - 4 = 8$.

3.1 $3^3 = 27$, $\sqrt{121} = 11$, so $27 - 11 = 16$.

3.2 $\sqrt{169} = 13$, $\sqrt[3]{125} = 5$, so $13 \times 5 = 65$.

3.3 $2^2 + 3^2 + 6^2 = 4 + 9 + 36 = 49$, so $\sqrt{49} = 7$.

3.4 (a) edge = $\sqrt[3]{2197} = 13$ cm; surface area = $6 \times 13^2 = 6 \times 169 = 1014$ cm².

(b) Lengths are in the ratio $1 : 2$, so areas are in the ratio $1^2 : 2^2 = 1 : 4$ and volumes in the ratio $1^3 : 2^3 = 1 : 8$. Area is a square of a length, so the length ratio is squared; volume is a cube of a length, so it is cubed.

(c) $\sqrt[3]{\frac{64}{125}} = \frac{\sqrt[3]{64}}{\sqrt[3]{125}} = \frac{4}{5}$; $\sqrt{\frac{81}{16}} = \frac{\sqrt{81}}{\sqrt{16}} = \frac{9}{4}$; $\frac{4}{5} \times \frac{9}{4} = \frac{9}{5}$.

21 (a) $(3 - \sqrt{5})(2 + 3\sqrt{5}) = a + b\sqrt{5}$

Find the value of a and the value of b .

$a =$

$b =$

[2]

(b) Rationalise the denominator.

Write your answer in its simplest form.

$$\frac{6}{\sqrt{2}}$$

..... [2]

18 Rationalise the denominator and simplify.

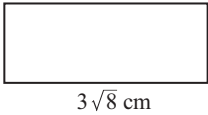
$$\frac{20}{4 + \sqrt{6}}$$

..... [3]

24 (a) Simplify.

$$\sqrt{125}-\sqrt{20}$$

(b)



$2\sqrt{2}\text{ cm}$

NOT TO
SCALE

The area of this rectangle is $k\text{ cm}^2$.

Work out the value of k .

(c) Rationalise the denominator.

$$\frac{1}{\sqrt{7}+2}$$

$k =$ [2]

..... [2]



15 (a) Expand and simplify.

$$(2-\sqrt{5})(1-3\sqrt{5})$$

(b) Rationalise the denominator.
Give your answer in its simplest form.

$$\frac{6}{\sqrt{10}}$$

16 Expand and simplify.

$$(x+4)(x-3)(3x+2)$$

..... [3]



18 (a) $\frac{7}{\sqrt{3}}$

Rationalise the denominator.
Give your answer in its simplest form.

..... [2]

(b) $(5 - \sqrt{2})(1 + 3\sqrt{2}) = c + k\sqrt{2}$

Find the value of c and the value of k .

$c =$
 $k =$ [2]

19 Write as a single fraction in its simplest form.

(a) $\frac{5a}{6} \times \frac{3b}{a}$

..... [2]

(b) $\frac{p}{2} + \frac{3t}{4}$

..... [2]

(c) $\frac{2}{x-2} - \frac{3}{x+1}$

..... [3]

E1.4 Fractions, decimals and percentages

Name: _____ Class: _____ Date: _____ **Total: 11 marks**

KEY WORDS percentage 百分比 • simplest form 最简形式 • mixed number 带分数
• recurring decimal 循环小数 • fraction 分数

Objective

Build the skills to answer Extended exam questions on **fractions, decimals and percentages** 分数、小数和百分比 — converting between them, calculating with fractions, and **recurring decimals** 循环小数 (Extended).

You must be able to:

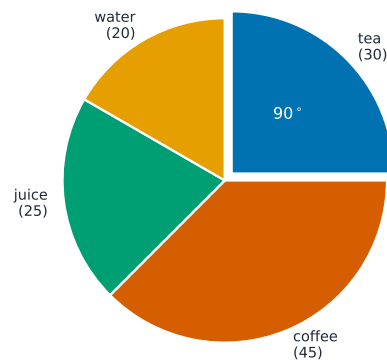
- convert between fractions, decimals and percentages
- add, subtract, multiply and divide fractions
- write a recurring decimal as a fraction

1 Worked examples

■ Working with fractions



A percentage means “out of 100”



A pie chart shows fractions and percentages of a whole

- **Convert:** $\frac{3}{8} = 0.375$; $0.07 = 7\%$; $34\% = \frac{34}{100} = 0.34$.
- **Divide fractions:** multiply by the reciprocal — $\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4} = \frac{10}{12} = \frac{5}{6}$.
- **Recurring $\rightarrow$ fraction (Extended):** for $0.1\dot{7}$, $100x = 17.7\ldots$, $10x = 1.7\ldots \rightarrow 90x = 16 \rightarrow x = \frac{16}{90} = \frac{8}{45}$.

Answer shapes: M1 a correct method, A1 the correct answer.

2 Practice

2.1 Work out $\frac{3}{4} \times \frac{2}{9}$, giving your answer in its simplest form. [2]

2.2 Write $\frac{5}{8}$ as a decimal. [1]

2.3 Work out $\frac{5}{6} - \frac{1}{4}$, giving your answer in its simplest form. [2]

3 Exam-style questions

3.1 Work out $2\frac{1}{3} \div \frac{7}{9}$, giving your answer as a mixed number. [3]

3.2 A rope of length $\frac{7}{8}$ m is cut into pieces each $\frac{7}{40}$ m long. Work out how many pieces are cut, showing your working as a division of fractions. [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $\frac{3}{4} \times \frac{2}{9} = \frac{6}{36} = \frac{1}{6}$.

2.2 $5 \div 8 = 0.625$.

2.3 common denominator 12: $\frac{10}{12} - \frac{3}{12} = \frac{7}{12}$.

3.1 $2\frac{1}{3} = \frac{7}{3}$; $\frac{7}{3} \div \frac{7}{9} = \frac{7}{3} \times \frac{9}{7} = \frac{63}{21} = 3$.

3.2 $\frac{7}{8} \div \frac{7}{40} = \frac{7}{8} \times \frac{40}{7} = \frac{280}{56} = 5$ pieces.

Name: _____

IGCSE Mathematics: **w25 22**

9 Rajid has a full bottle of juice.

He drinks $\frac{1}{3}$ of the full bottle on Monday.

He drinks $\frac{3}{7}$ of the full bottle on Tuesday.

Find the fraction of the bottle of juice remaining.

..... [3]

IGCSE Mathematics: **s25 21**

4 Work out.

$$\frac{5}{6} - \frac{2}{3} \times \frac{3}{8}$$

..... [3]

IGCSE Mathematics: **s25 22**

14 Write 0.25 as a fraction.

..... [2]

E1.7 Indices I

Name: _____ Class: _____ Date: _____ **Total: 22 marks**

KEY WORDS indices 指数 • laws of indices 指数定律 • factor 因数 • power 幂
• prime factor 质因数

Objective

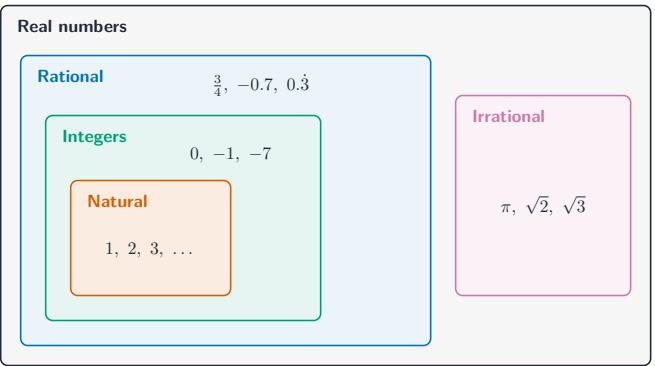
Build the skills to answer Extended exam questions on **indices** 指数 — the **laws of indices** 指数定律, and **zero, negative and fractional** 零、负、分数 powers.

You must be able to:

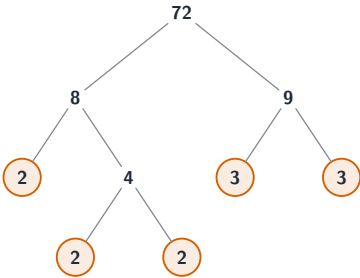
- use $a^m \times a^n = a^{m+n}$, $a^m \div a^n = a^{m-n}$, $(a^m)^n = a^{mn}$
- evaluate a^0 , a^{-n} and $a^{\frac{m}{n}}$
- simplify expressions with indices

1 Worked examples

■ Laws of indices



Add powers to multiply; subtract to divide



$$72 = 2^3 \times 3^2$$

Indices appear when a number is written in prime factors

$$a^m \times a^n = a^{m+n} \quad a^m \div a^n = a^{m-n} \quad (a^m)^n = a^{mn}$$

$$a^0 = 1 \quad a^{-n} = \frac{1}{a^n} \quad a^{\frac{m}{n}} = (\sqrt[n]{a})^m$$

- **Worked:** $2^{-3} \times 2^4 = 2^1 = 2$; $(2^3)^2 = 2^6 = 64$; $8^{-\frac{2}{3}} = \frac{1}{(\sqrt[3]{8})^2} = \frac{1}{4}$.

Answer shapes: M1 a correct method, A1 the correct answer.

2 Practice

2.1 Simplify $x^5 \times x^3$. [1]

2.2 Work out the value of 2^{-3} . [1]

2.3 Work out the value of $81^{\frac{1}{2}}$. [1]

3 Exam-style questions

3.1 Simplify $(2a^2)^3$. [2]

3.2 Work out the value of $27^{\frac{2}{3}}$. [2]

3.3 Work out the value of $16^{-\frac{1}{2}}$. [2]

3.4 (a) Simplify, leaving your answers in index form: [4]

(i) $\frac{x^7 \times x^{-3}}{x^2}$ (ii) $(2y^3)^4$

(b) Evaluate without a calculator, showing each step: [4]

(i) $27^{\frac{2}{3}}$ (ii) $\left(\frac{25}{4}\right)^{-\frac{1}{2}}$

(c) Solve $2^{x+1} = \frac{1}{32}$. [3]

(d) Explain, using the laws of indices, why $a^0 = 1$ for any non-zero a . [2]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $x^5 \times x^3 = x^8$.

2.2 $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$.

2.3 $81^{\frac{1}{2}} = \sqrt{81} = 9$.

3.1 $(2a^2)^3 = 2^3 a^6 = 8a^6$.

3.2 $27^{\frac{2}{3}} = (\sqrt[3]{27})^2 = 3^2 = 9$.

3.3 $16^{-\frac{1}{2}} = \frac{1}{16^{\frac{1}{2}}} = \frac{1}{\sqrt{16}} = \frac{1}{4}$.

3.4 (a) (i) $\frac{x^{7+(-3)}}{x^2} = \frac{x^4}{x^2} = x^2$. (ii) $(2y^3)^4 = 2^4 y^{12} = 16y^{12}$.

(b) (i) $27^{\frac{2}{3}} = (\sqrt[3]{27})^2 = 3^2 = 9$. (ii) $\left(\frac{25}{4}\right)^{-\frac{1}{2}} = \left(\frac{4}{25}\right)^{\frac{1}{2}} = \frac{2}{5}$.

(c) $\frac{1}{32} = 2^{-5}$; so $x + 1 = -5$; $x = -6$.

(d) $\frac{a^n}{a^n} = a^{n-n} = a^0$ by the division law; but any non-zero number divided by itself is 1, so $a^0 = 1$.

Name:

IGCSE Mathematics: **w25 21**

6 Find the value of

(a) $5^{-5} \times 5^5$

..... [1]

(b) $125^{\frac{2}{3}}$.

[2]

IGCSE Mathematics: **s25 21**

19 Find the value of $27^{-\frac{4}{3}}$.

..... [2]

E1.8 Standard form

Name: _____ Class: _____ Date: _____

Total: 9 marks

KEY WORDS standard form 科学记数法 • power 幂 • integer 整数

Objective

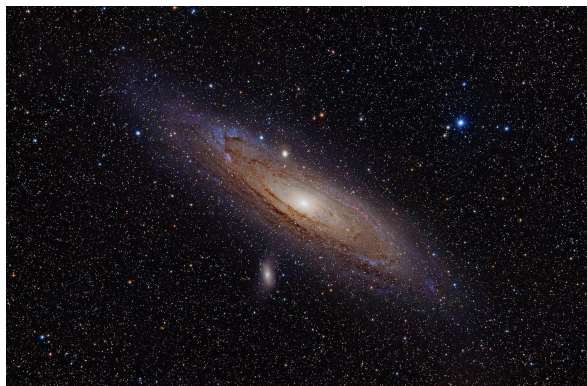
Build the skills to answer Extended exam questions on **standard form** 科学记数法 — writing numbers as $A \times 10^n$ and calculating with them.

You must be able to:

- write large and small numbers in standard form
- convert standard form back to an ordinary number
- multiply and divide numbers in standard form

1 Worked examples

■ Standard form



$A \times 10^n$ where $1 \leq A < 10$ and n is an integer

- **Convert:** $4\,500\,000 = 4.5 \times 10^6$; $0.00072 = 7.2 \times 10^{-4}$.

Standard form: $A \times 10^n$, with $1 \leq A < 10$

6 500 000 $\longrightarrow$ 6.5×10^6
point moves 6 places

0.00042 $\longrightarrow$ 4.2×10^{-4}
point moves 4 places

standard form: $a \times 10^n$ with $1 \leq a < 10$

Standard form $A \times 10^n$: the power of ten is the number of places the point moves

- **Multiply:** $(3 \times 10^5) \times (2 \times 10^{-2}) = 6 \times 10^3$ (multiply the fronts, add the powers).

Answer shapes: M1 a correct method, A1 the correct answer.

2 Practice

2.1 Write 63 000 in standard form. [1]

2.2 Write 0.0045 in standard form. [1]

2.3 Write 2.7×10^{-3} as an ordinary number. [1]

3 Exam-style questions

3.1 Work out $(4 \times 10^6) \times (2.5 \times 10^{-3})$, giving your answer in standard form. [2]

3.2 Work out $\frac{8 \times 10^5}{2 \times 10^2}$, giving your answer in standard form. [2]

3.3 Work out $(3 \times 10^4) + (5 \times 10^3)$, giving your answer in standard form. [2]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 6.3×10^4 .

2.2 4.5×10^{-3} .

2.3 0.0027.

3.1 $4 \times 2.5 = 10$ and $10^6 \times 10^{-3} = 10^3$, giving $10 \times 10^3 = 1 \times 10^4$.

3.2 $8 \div 2 = 4$ and $10^5 \div 10^2 = 10^3$, giving 4×10^3 .

3.3 $3 \times 10^4 = 30\,000$ and $5 \times 10^3 = 5\,000$, so the sum is $35\,000 = 3.5 \times 10^4$.

11

2.89×10^{-1}

1.3×10^{12}

8.3×10^1

9×10^{11}

2.03×10^{-5}

0.3×10^{-2}

Use a number from the box to complete each statement.

The number that is not written in standard form is

The largest number is

The smallest number is

[2]

12

A vase contains flowers that are red or pink or white.
Ruth picks a flower at random from the vase.
The probability that the flower is **not** red is 0.9 .
The probability that the flower is **not** pink is 0.65 .

Find the probability that the flower is white.

..... [2]

13

The point (5, 1024) lies on the curve $y = c^x$, where c is a whole number.

Find the y -coordinate of the point on the curve with x -coordinate -2 .

..... [3]

E1.10 Limits of accuracy

Name: _____ Class: _____ Date: _____

Total: 11 marks

KEY WORDS lower bound 下界 • upper bound 上界 • lower and upper bounds 下界和上界

• area 面积 • limits of accuracy 精确度的界限

Objective

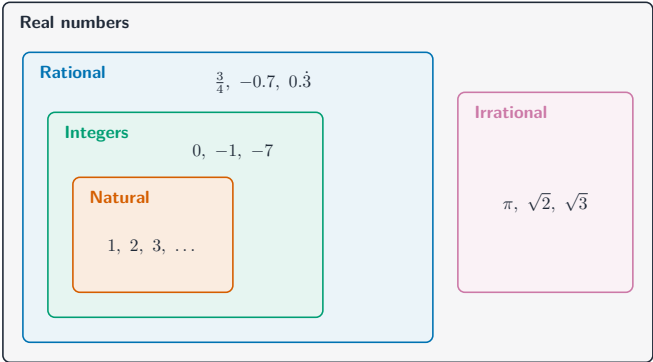
Build the skills to answer Extended exam questions on **limits of accuracy** 精确度的界限 — the **lower and upper bounds** 下界和上界 of a rounded value, and using bounds in calculations.

You must be able to:

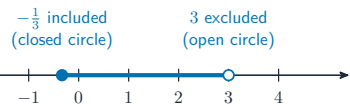
- give the upper and lower bounds of a rounded measurement
- find the largest and smallest result of a calculation
- choose the right bounds for a sum, product or quotient

1 Worked examples

■ Upper and lower bounds



A rounded value could be anything that rounds to it



Upper and lower bounds place a value within an interval

- **Bounds:** 635 m to the nearest metre $\rightarrow 634.5 \leq h < 635.5$.
- **Largest product:** use both **upper** bounds. **Smallest:** both **lower** bounds.
- **Quotient:** the **largest** $\frac{a}{b}$ uses the **largest** a and the **smallest** b .

Answer shapes: **M1** a correct method, **A1** the correct answer.

2 Practice

2.1 A length is 24 cm, to the nearest cm. Write down the lower bound. [1]

2.2 A mass is 3.5 kg, correct to 1 decimal place. Write down the upper bound. [1]

2.3 A time is 40 s, to the nearest 10 s. Write down the lower and upper bounds. [2]

3 Exam-style questions

3.1 A rectangle measures 12 cm by 7 cm, each side to the nearest cm.

(a) Calculate the upper bound of the area. [2]

(b) Calculate the lower bound of the perimeter. [2]

3.2 A car travels 90 m (to the nearest 10 m) in 8.0 s (to the nearest 0.1 s). Calculate the upper bound of its average speed. [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 23.5 cm.

2.2 3.55 kg.

2.3 lower bound 35 s, upper bound 45 s.

3.1 (a) upper bounds 12.5 and 7.5 $\rightarrow$ area = $12.5 \times 7.5 = 93.75 \text{ cm}^2$. (b) lower bounds 11.5 and 6.5 $\rightarrow$ perimeter = $2(11.5 + 6.5) = 36 \text{ cm}$.

3.2 largest speed = largest distance $\div$ smallest time. Distance upper bound = 95 m; time lower bound = 7.95 s $\rightarrow \frac{95}{7.95} = 11.9 \text{ m/s}$ (to 3 s.f.).

25 Jenna has a length of wire measuring 68 cm, correct to the nearest cm.

From this wire she cuts off two smaller pieces

- a piece of length 4.7 cm, correct to the nearest mm
- a piece of length 10.0 cm, correct to the nearest mm.

Work out the lower bound and the upper bound for the length of the wire remaining.

Lower bound = cm

Upper bound = cm [3]

E1.12 Rates

Name: _____ Class: _____ Date: _____

Total: 32 marks

KEY WORDS average speed 平均速度 • density 密度 • mass 质量 • volume 体积 • pressure 压强

Objective

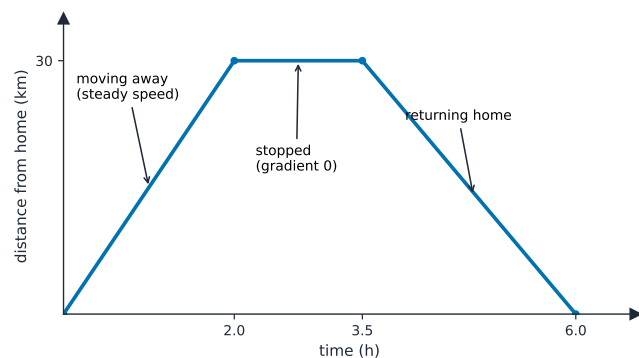
Build the skills to answer Extended exam questions on **rates** 比率 — **average speed** 平均速度, **density** 密度, and other rates that combine two quantities.

You must be able to:

- use average speed = $\frac{\text{distance}}{\text{time}}$
- use density = $\frac{\text{mass}}{\text{volume}}$
- convert times correctly and rearrange a rate formula

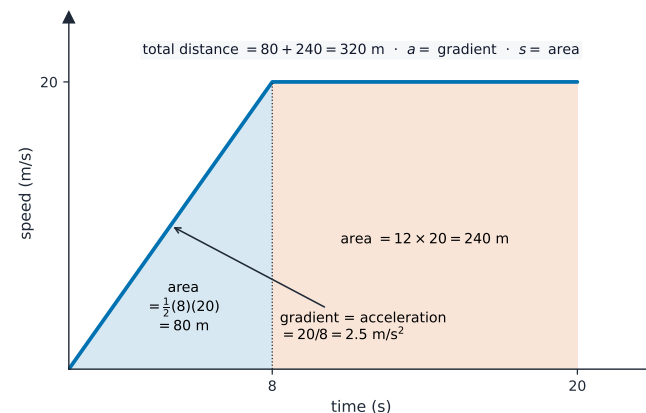
1 Worked examples

■ Speed, distance, time



speed = gradient of distance-time graph • flat $\Rightarrow$ speed = 0

A rate compares two quantities in different units



A rate also reads off a speed-time graph

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}} \quad \text{density} = \frac{\text{mass}}{\text{volume}}$$

- **Worked:** 45 km in 3 h 45 min = 3.75 h $\rightarrow$ speed = $\frac{45}{3.75} = 12$ km/h.
- Change minutes to a decimal of an hour (45 min = 0.75 h) — **not** 3.45.

Answer shapes: M1 a correct method, A1 the correct answer.

■ Three formulas with the same shape

$$\text{speed} = \frac{\text{distance}}{\text{time}} \quad \text{density} = \frac{\text{mass}}{\text{volume}} \quad \text{pressure} = \frac{\text{force}}{\text{area}}$$

Each rearranges the same way, and each is checked the same way — by the **units**. If a speed comes out in km/h but the question wanted m/s, the conversion is part of the answer.

The unit conversions worth memorising:

- 1 km/h = $\frac{1000}{3600}$ m/s, so **divide by 3.6** to go from km/h to m/s and multiply by 3.6 to come back.
- 1 m³ = 1 000 000 cm³, and 1 g/cm³ = 1000 kg/m³.
- Time given as 2 h 30 min must become 2.5 h — **not** 2.30.

△ **Average speed is total distance $\div$ total time**, never the average of the speeds. A journey of 60 km at 60 km/h then 60 km at 30 km/h takes 1 + 2 = 3 hours for 120 km, so the average speed is 40 km/h — not the 45 you get by averaging the two speeds. That difference is a favourite exam question, and averaging the speeds is the trap it is set to catch.

Worked example. A car travels 150 km in 1 h 15 min. Find its average speed in km/h and in m/s.

- 1 h 15 min = 1.25 h.
- Speed = $\frac{150}{1.25} = 120$ km/h.
- In m/s: $120 \div 3.6 = 33.3$ m/s (3 s.f.).

2 Practice

2.1 A car travels 120 km in 2 hours. Calculate its average speed. [1]

2.2 A metal block has mass 240 g and volume 30 cm³. Calculate its density. [2]

2.3 A runner's average speed is 5 m/s. How far does she run in 40 seconds? [2]

2.4 Convert 72 km/h to m/s. [2]

2.5 A block has mass 480 g and volume 60 cm³. Find its density. [2]

2.6 A force of 250 N acts on an area of 0.5 m². Find the pressure. [2]

3 Exam-style questions

3.1 A train travels 150 km in 1 hour 30 minutes. Calculate its average speed in km/h. [2]

3.2 Gold has a density of 19.3 g/cm³. Calculate the mass of a gold bar of volume 50 cm³. [2]

3.3 A cyclist travels 36 km at an average speed of 24 km/h. Calculate the time taken, in minutes. [3]

3.4 A lorry travels from town A to town B, a distance of 135 km, in 1 h 48 min.

- (a) Calculate the average speed of the lorry in km/h. [3]

- (b) Convert this speed to m/s, giving your answer to 3 significant figures. [2]

- (c) The lorry returns from B to A at an average speed of 90 km/h. Calculate the average speed for the whole journey there and back. [4]

- (d) Explain why the answer to (c) is not the mean of the two speeds. [2]

- (e) The lorry carries a steel block of volume 0.4 m^3 . Steel has a density of 7.8 g/cm^3 . Calculate the mass of the block in tonnes. [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $\frac{120}{2} = 60 \text{ km/h}$.

2.2 $\text{density} = \frac{240}{30} = 8 \text{ g/cm}^3$.

2.3 $\text{distance} = 5 \times 40 = 200 \text{ m}$.

2.4 $72 \div 3.6 = 20 \text{ m/s}$.

2.5 $\frac{480}{60} = 8 \text{ g/cm}^3$.

2.6 $\frac{250}{0.5} = 500 \text{ N/m}^2$, or 500 Pa .

3.1 $1 \text{ h } 30 \text{ min} = 1.5 \text{ h} \rightarrow \text{speed} = \frac{150}{1.5} = 100 \text{ km/h}$.

3.2 $\text{mass} = \text{density} \times \text{volume} = 19.3 \times 50 = 965 \text{ g}$.

3.3 $\text{time} = \frac{36}{24} = 1.5 \text{ h} = 1.5 \times 60 = 90 \text{ minutes}$.

3.4 (a) $1 \text{ h } 48 \text{ min} = 1.8 \text{ h}$; $\frac{135}{1.8} = 75 \text{ km/h}$.

(b) $75 \div 3.6 = 20.8 \text{ m/s}$.

(c) $\text{Return time} = \frac{135}{90} = 1.5 \text{ h}$; total distance = 270 km, total time = $1.8 + 1.5 = 3.3 \text{ h}$;
 $\text{average speed} = \frac{270}{3.3} = 81.8 \text{ km/h}$.

(d) Average speed is total distance divided by total time; the lorry spends longer travelling at the slower speed, so that speed carries more weight and the mean of 75 and 90 is not correct.

(e) $0.4 \text{ m}^3 = 0.4 \times 10^6 = 400\,000 \text{ cm}^3$; $\text{mass} = 400\,000 \times 7.8 = 3\,120\,000 \text{ g} = 3.12 \text{ tonnes}$.

4 Pedro makes cards.

- (a)** He makes cards at a rate of 9 cards every 20 minutes.

Work out the number of cards he makes in 8 hours.

..... [2]

- (b)** Each card costs 12 cents to make.
Pedro sells each card for 50 cents.

Work out his percentage profit on each card.

.....% [2]

- 5** Nuwa is buying a phone.
One website sells the phone for 953 Yuan.
A different website sells the same phone for \$141.
The exchange rate is 1 Yuan = \$0.152 .

Calculate the difference between these phone prices.
Give your answer in dollars, correct to the nearest cent.

\$ [2]

- 8** Hadi buys a dishwasher.
He can either pay a single payment of \$980 or he can pay using a monthly plan.
The monthly plan is 20% of \$980 **plus** 12 payments of \$75.25 .

- (a)** Hadi uses the monthly plan.

Calculate the total amount Hadi pays.

\$ [2]

- (b)** Find the percentage increase in the cost using the monthly plan compared to a single payment.

.....% [2]