

Week 1

IGCSE Mathematics

Homework bundle for this week

本周作业合集

exercises.lol

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1. Which of these is a prime number?

☐ 13

☐ 1

☐ 9

☐ 15

2. $A = \{2, 4, 6, 8, 10\}$ and $B = \{3, 6, 9\}$. How many elements are in $A \cap B$?

3. Work out $5^2 \times \sqrt[3]{8}$.

4. Write 4 500 000 as $A \times 10^n$. What is the power n ?

5. Simplify $\sqrt{20}$ into the form $k\sqrt{5}$. What is k ?

6. Write $3/8$ as a decimal.

7. Which surd rule is correct?

☐ $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$

☐ $\sqrt{a} + \sqrt{b} = \sqrt{a+b}$

☐ $\sqrt{a} \times \sqrt{b} = \sqrt{a+b}$

☐ $\sqrt{a/b} = \sqrt{a} \times \sqrt{b}$

8. The symbol $\cap$ means:

☐ intersection (in both sets)

☐ union (in either set)

☐ complement (not in the set)

☐ empty set

9. What is $2/3 \times 4/5$?

☐ $8/15$

☐ $6/8$

☐ $8/8$

☐ $2/15$

10. Depreciation of a car's value each year is an example of:

☐ exponential decay

☐ exponential growth

☐ simple interest

☐ linear growth

1. **13**

13 has exactly two factors (1 and 13). 1 is not prime; $9 = 3 \times 3$; $15 = 3 \times 5$.

2. **1**

Only 6 is in both sets, so $A \cap B = \{6\}$, which has 1 element.

3. **50**

$5^2 = 25$ and $\sqrt[3]{8} = 2$, so $25 \times 2 = 50$.

4. **6**

$4\,500\,000 = 4.5 \times 10^6$, so $n = 6$.

5. **2**

$\sqrt{20} = \sqrt{(4 \times 5)} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$, so $k = 2$.

6. **0.375**

$3 \div 8 = 0.375$.

7. **$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$**

$\sqrt{a} \times \sqrt{b} = \sqrt{ab}$; roots do not add like $\sqrt{a} + \sqrt{b} = \sqrt{a+b}$.

8. **intersection (in both sets)**

$\cap$ is intersection (elements in both A and B); $\cup$ is union.

9. **8/15**

Multiply tops and bottoms: $(2 \times 4)/(3 \times 5) = 8/15$.

10. **exponential decay**

Losing a fixed percentage each year is exponential decay (multiplier < 1 , applied repeatedly).

E1.1 Types of number

Name: _____ Class: _____ Date: _____

Total: 12 marks

KEY WORDS factor 因数 • integer 整数 • hcf 最大公因数 • lcm 最小公倍数 • common multiple 公倍数

Objective

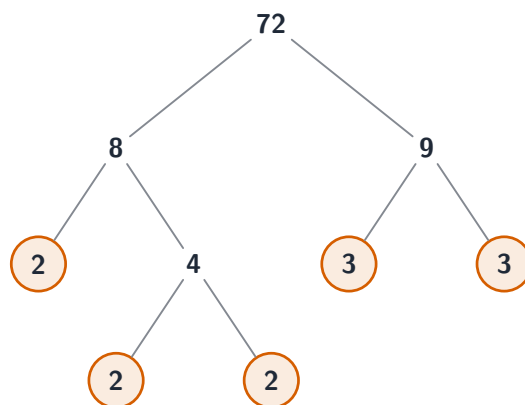
Build the skills to answer Extended exam questions on **types of number** 数的类型 — **prime factors** 质因数, **HCF and LCM** 最大公因数和最小公倍数, and **rational and irrational** 有理数和无理数 numbers.

You must be able to:

- write a number as a product of its prime factors
- find the HCF and LCM using prime factors
- identify rational and irrational numbers

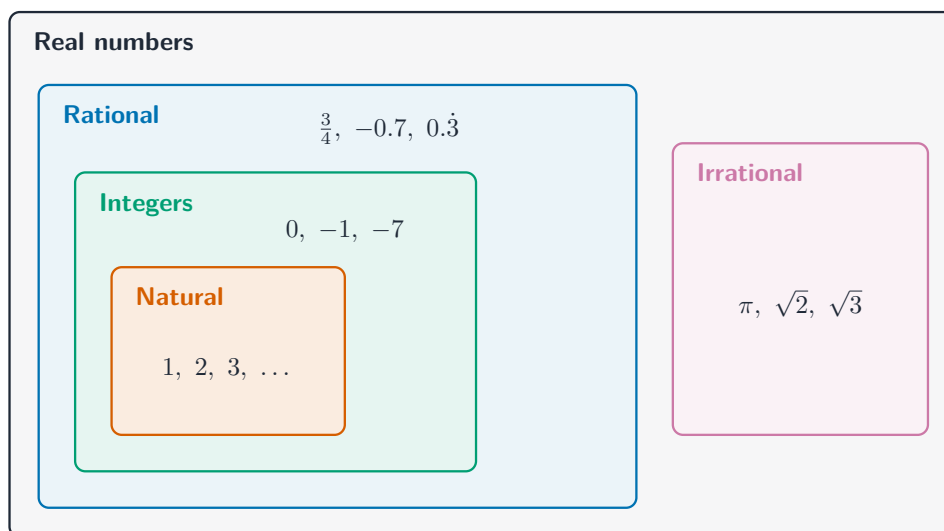
1 Worked examples

■ Prime factors, HCF and LCM



$$72 = 2^3 \times 3^2$$

Keep dividing by the smallest prime that fits



The number sets: natural, integer, rational and real

- **Prime factors:** $72 = 2^3 \times 3^2$ (split until every branch is prime).
- **HCF** 最大公因数: the **lowest** power of each shared prime. **LCM** 最小公倍数: the **highest** power of every prime.
- **Worked (72 and 120):** $72 = 2^3 \times 3^2$, $120 = 2^3 \times 3 \times 5 \rightarrow \text{HCF} = 2^3 \times 3 = 24$, $\text{LCM} = 2^3 \times 3^2 \times 5 = 360$.
- **Irrational** 无理数: cannot be written as a fraction (e.g. $\pi, \sqrt{2}$).

Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer):

2 Practice

2.1 Write 90 as a product of its prime factors. [2]

2.2 From the list $\sqrt{9}, \pi, \frac{2}{3}, \sqrt{5}$, write down the two **irrational** numbers. [2]

2.3 Find the HCF of 24 and 36. [2]

3 Exam-style questions

3.1 $A = 2^2 \times 3 \times 5$ and $B = 2 \times 3^2 \times 5$.

(a) Find the HCF of A and B . [2]

(b) Find the LCM of A and B . [2]

3.2 Find the lowest common multiple (LCM) of 18 and 24. [2]

Solutions

Marking: **M1** a correct method, **A1** the correct answer (allow follow-through where sensible).

2.1 $90 = 2 \times 3^2 \times 5$.

2.2 π and $\sqrt{5}$.

2.3 $24 = 2^3 \times 3$, $36 = 2^2 \times 3^2 \rightarrow \text{HCF} = 2^2 \times 3 = 12$.

3.1 (a) HCF — lowest power of each shared prime: $2^1 \times 3 \times 5 = 30$. (b) LCM — highest power of every prime: $2^2 \times 3^2 \times 5 = 180$.

3.2 $18 = 2 \times 3^2$, $24 = 2^3 \times 3 \rightarrow \text{LCM} = 2^3 \times 3^2 = 72$.

13

$M = 2^7 \times 3^3 \times 5^2$

- (a)
- Write $14M$ as a product of its prime factors.
Give your answer in index form.

.....

[2]

- (b)
- R is an integer.

$\frac{M}{R}$ is a cube number.

Find the smallest possible value of R .

$R =$

[2]

14

Find the value of

(a)

$3^2 \div 3^{-2}$

.....

[2]

(b)

$16^{-\frac{3}{2}}$.

.....

[2]

7 (a) The sum of all the prime numbers less than 10 is equal to 17.

Find the sum of all the prime numbers less than 16.

..... [2]

(b) x is an integer.
The sum of the prime numbers greater than 6 and less than x is equal to 18.

Find a possible value for x .

$x =$ [1]

8 Find the highest common factor (HCF) of 36 and 54.

..... [2]

10 (a) Write down **all** the factors of 18.

..... [2]

(b) Factorise.

$3y - xy + 15 - 5x$

..... [2]

(c) $3y - xy + 15 - 5x = 18$

where x and y are positive integers.

Using your answers to **part (a)** and **part (b)**, find one possible value of x and the corresponding value of y .

$x =$, $y =$ [2]

11 A warehouse has a floor area of 800 m^2 .
The plan of the warehouse is drawn to a scale of 1 : 50.

Calculate the floor area on the plan.
Give your answer in square centimetres.

E1.2 Sets

Name: _____ Class: _____ Date: _____

Total: 10 marks

KEY WORDS venn diagram 维恩图 • element 元素 • notation 符号 • sets 集合 • intersection 交集

Objective

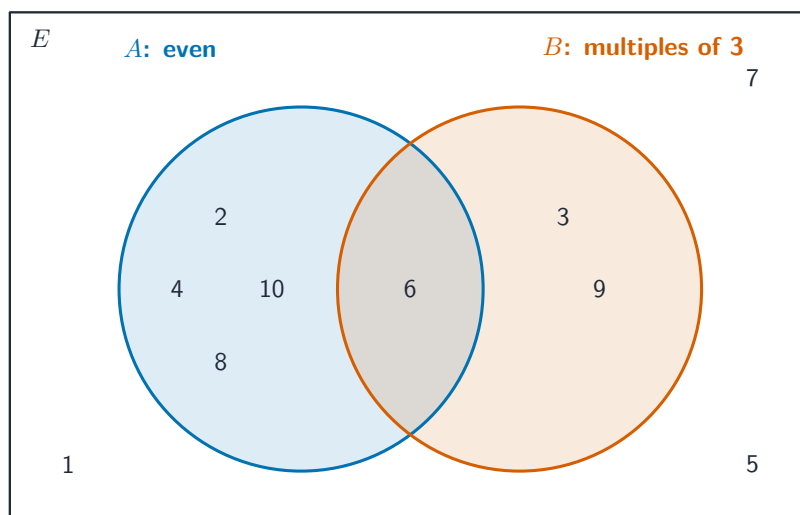
Build the skills to answer Extended exam questions on **sets** 集合 — set **notation** 符号, **Venn diagrams** 维恩图, and the **union**, **intersection** and **complement** 并集、交集、补集.

You must be able to:

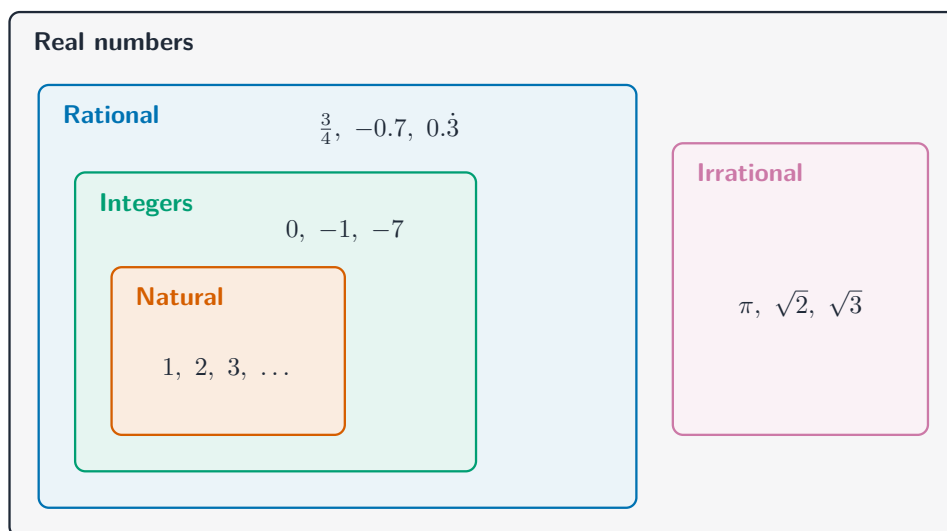
- use the symbols $\cup$, $\cap$, A' , $\in$, $n(A)$
- list elements of unions and intersections
- shade and read regions of a Venn diagram

1 Worked examples

■ Union, intersection, complement



The overlap is the intersection; everything inside either circle is the union



Number sets shown as nested regions of a Venn diagram

- $A \cup B$ **union** 并集: in A **or** B (or both). $A \cap B$ **intersection** 交集: in **both**.
- A' **complement** 补集: everything **not** in A . $n(A)$: the number of elements in A .
- **Worked:** $E = \{1-10\}$, $A = \{2, 4, 6, 8, 10\}$, $B = \{3, 6, 9\} \rightarrow A \cap B = \{6\}$, $A \cup B = \{2, 3, 4, 6, 8, 9, 10\}$, $n(A \cup B) = 7$.

Answer shapes: M1 a correct method, A1 the correct answer.

2 Practice

2.1 $E = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $A = \{1, 2, 3, 4\}$, $B = \{2, 4, 6, 8\}$. List the elements of $A \cap B$. [1]

2.2 For the same sets, list the elements of $A \cup B$. [2]

2.3 For the same sets, find $n(A')$. [2]

3 Exam-style questions

3.1 $E = \{x : 1 \leq x \leq 12, x \text{ is an integer}\}$, $P = \{\text{multiples of 3}\}$, $Q = \{\text{factors of 12}\}$.

(a) List the elements of $P \cap Q$. [2]

(b) Find $n(P \cup Q)$. [2]

3.2 On a Venn diagram with sets A and B , describe in words the region $A \cap B'$. [1]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $A \cap B = \{2, 4\}$.

2.2 $A \cup B = \{1, 2, 3, 4, 6, 8\}$.

2.3 $A' = \{5, 6, 7, 8\}$, so $n(A') = 4$.

3.1 (a) $P = \{3, 6, 9, 12\}$, $Q = \{1, 2, 3, 4, 6, 12\} \rightarrow P \cap Q = \{3, 6, 12\}$. (b) $P \cup Q = \{1, 2, 3, 4, 6, 9, 12\} \rightarrow n(P \cup Q) = 7$.

3.2 the elements that are in A but **not** in B (the part of circle A outside the overlap).

10 $\mathcal{E} = \{n: n \text{ is an integer and } 1 \leq n \leq 8\}$

$A = \{\text{factors of } 12\}$

$B = \{\text{odd numbers}\}$

Find

(a) $A \cap B$

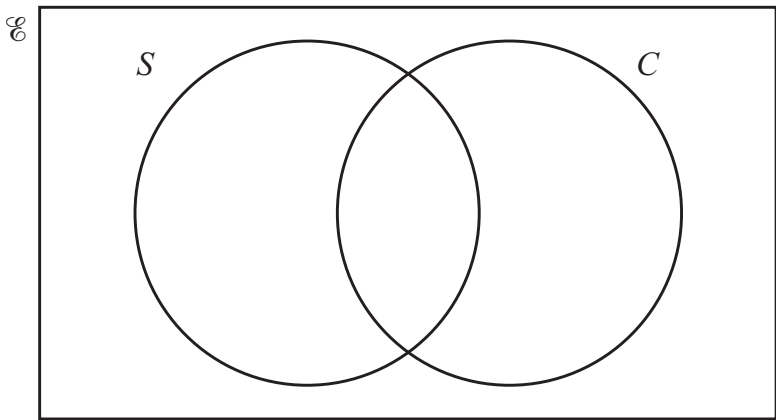
$A \cap B = \{\dots\dots\dots\}$ [1]

(b) $n(A' \cup B)$.

$\dots\dots\dots$ [1]

12 A fitness club has 100 members.

- 60 swim (S).
- 70 cycle (C).
- 25 do not swim or cycle.



(a) Complete the Venn diagram.

[3]

(b) One member of the fitness club is chosen at random.

For this member, find

(i) $P(C)$

..... [1]

(ii) $P(S \cap C)$

..... [1]

(iii) $P(S \cup C')$.

..... [1]

E1.3 Powers and roots

Name: _____ Class: _____ Date: _____

Total: 11 marks**KEY WORDS** power 幂 • powers and roots 幂和根 • cube root 立方根 • square root 平方根 • prime factor 质因数

Objective

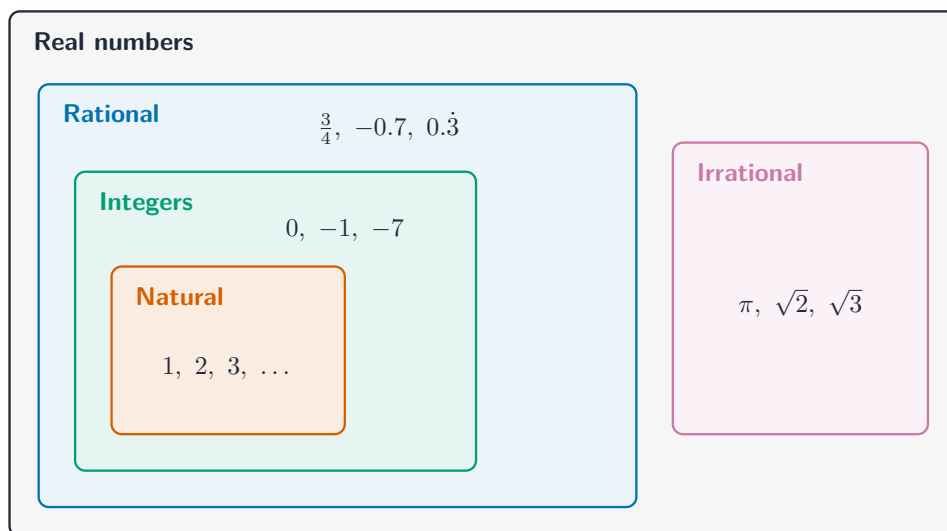
Build the skills to answer Extended exam questions on **powers and roots** 幂和根 — **squares, cubes** 平方、立方 and their roots, and evaluating expressions with powers and roots.

You must be able to:

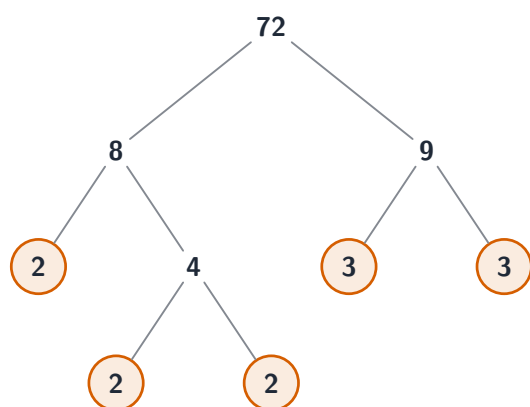
- recall squares to 15^2 and the cubes of 1–5 and 10
- find square roots and cube roots
- evaluate expressions combining powers and roots

1 Worked examples

■ Powers and roots



A power multiplies a number by itself; a root reverses it



$$72 = 2^3 \times 3^2$$

Prime factorisation underlies powers and roots

- **Power** 幂: $2^5 = 32$. **Square root** 平方根: $\sqrt{169} = 13$. **Cube root** 立方根: $\sqrt[3]{8} = 2$.
- **Worked:** $5^2 \times \sqrt[3]{8} = 25 \times 2 = 50$.
- Recall squares (1–15) and cubes (1, 2, 3, 4, 5, 10).

Answer shapes: **M1** a correct method, **A1** the correct answer.

2 Practice

2.1 Work out $\sqrt{196}$. [1]

2.2 Work out $\sqrt[3]{27} + 4^2$. [2]

2.3 Work out $\sqrt{144} - \sqrt[3]{64}$. [2]

3 Exam-style questions

3.1 Work out $3^3 - \sqrt{121}$. [2]

3.2 Work out $\sqrt{169} \times \sqrt[3]{125}$. [2]

3.3 Work out the value of $\sqrt{2^2 + 3^2 + 6^2}$. [2]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $\sqrt{196} = 14$.

2.2 $\sqrt[3]{27} = 3$ and $4^2 = 16$, so $3 + 16 = 19$.

2.3 $\sqrt{144} = 12$, $\sqrt[3]{64} = 4$, so $12 - 4 = 8$.

3.1 $3^3 = 27$, $\sqrt{121} = 11$, so $27 - 11 = 16$.

3.2 $\sqrt{169} = 13$, $\sqrt[3]{125} = 5$, so $13 \times 5 = 65$.

3.3 $2^2 + 3^2 + 6^2 = 4 + 9 + 36 = 49$, so $\sqrt{49} = 7$.

21 (a) $(3 - \sqrt{5})(2 + 3\sqrt{5}) = a + b\sqrt{5}$

Find the value of a and the value of b .

$a =$

$b =$

[2]

- (b) Rationalise the denominator.
Write your answer in its simplest form.

$$\frac{6}{\sqrt{2}}$$

..... [2]

- 18 Rationalise the denominator and simplify.

$$\frac{20}{4 + \sqrt{6}}$$

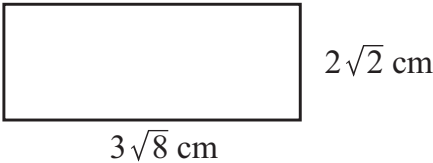
..... [3]

24 (a) Simplify.

$\sqrt{125} - \sqrt{20}$

..... [2]

(b)



NOT TO
SCALE

The area of this rectangle is $k \text{ cm}^2$.

Work out the value of k .

$k =$ [2]

(c) Rationalise the denominator.

$\frac{1}{\sqrt{7} + 2}$

..... [2]

15 (a) Expand and simplify.

$$(2 - \sqrt{5})(1 - 3\sqrt{5})$$

..... [2]

(b) Rationalise the denominator.
Give your answer in its simplest form.

$$\frac{6}{\sqrt{10}}$$

..... [2]

16 Expand and simplify.

$$(x + 4)(x - 3)(3x + 2)$$

..... [3]

18 (a) $\frac{7}{\sqrt{3}}$

Rationalise the denominator.
Give your answer in its simplest form.

..... [2]

(b) $(5 - \sqrt{2})(1 + 3\sqrt{2}) = c + k\sqrt{2}$

Find the value of c and the value of k .

$c =$

$k =$ [2]

19 Write as a single fraction in its simplest form.

(a) $\frac{5a}{6} \times \frac{3b}{a}$

..... [2]

(b) $\frac{p}{2} + \frac{3t}{4}$

..... [2]

(c) $\frac{2}{x-2} - \frac{3}{x+1}$

E1.4 Fractions, decimals and percentages

Name: _____ Class: _____ Date: _____

Total: 11 marks

KEY WORDS percentage 百分比 • simplest form 最简形式 • mixed number 带分数 • recurring decimal 循环小数
• fraction 分数

Objective

Build the skills to answer Extended exam questions on **fractions, decimals and percentages** 分数、小数和百分比 — converting between them, calculating with fractions, and **recurring decimals** 循环小数 (Extended).

You must be able to:

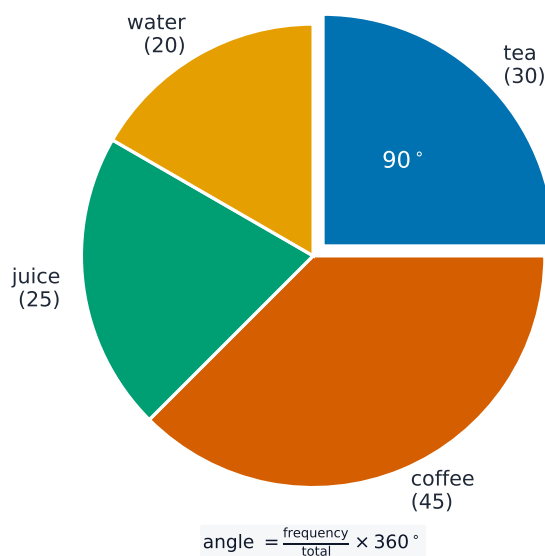
- convert between fractions, decimals and percentages
- add, subtract, multiply and divide fractions
- write a recurring decimal as a fraction

1 Worked examples

■ Working with fractions



A percentage means “out of 100”



A pie chart shows fractions and percentages of a whole

- **Convert:** $\frac{3}{8} = 0.375$; $0.07 = 7\%$; $34\% = \frac{34}{100} = 0.34$.
- **Divide fractions:** multiply by the reciprocal — $\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4} = \frac{10}{12} = \frac{5}{6}$.
- **Recurring $\rightarrow$ fraction (Extended):** for $0.1\dot{7}$, $100x = 17.7\ldots$, $10x = 1.7\ldots \rightarrow 90x = 16 \rightarrow x = \frac{16}{90} = \frac{8}{45}$.

Answer shapes: **M1** a correct method, **A1** the correct answer.

2 Practice

2.1 Work out $\frac{3}{4} \times \frac{2}{9}$, giving your answer in its simplest form. [2]

2.2 Write $\frac{5}{8}$ as a decimal. [1]

2.3 Work out $\frac{5}{6} - \frac{1}{4}$, giving your answer in its simplest form. [2]

3 Exam-style questions

3.1 Work out $2\frac{1}{3} \div \frac{7}{9}$, giving your answer as a mixed number. [3]

3.2 Write the recurring decimal $0.\dot{2}\dot{7}$ as a fraction in its simplest form. [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $\frac{3}{4} \times \frac{2}{9} = \frac{6}{36} = \frac{1}{6}$.

2.2 $5 \div 8 = 0.625$.

2.3 common denominator 12: $\frac{10}{12} - \frac{3}{12} = \frac{7}{12}$.

3.1 $2\frac{1}{3} = \frac{7}{3}$; $\frac{7}{3} \div \frac{7}{9} = \frac{7}{3} \times \frac{9}{7} = \frac{63}{21} = 3$.

3.2 let $x = 0.2727 \dots \rightarrow 100x = 27.2727 \dots \rightarrow 100x - x = 27 \rightarrow 99x = 27 \rightarrow x = \frac{27}{99} = \frac{3}{11}$.

9 Rajid has a full bottle of juice.
He drinks $\frac{1}{3}$ of the full bottle on Monday.
He drinks $\frac{3}{7}$ of the full bottle on Tuesday.
Find the fraction of the bottle of juice remaining.

..... [3]

4 Work out.

$$\frac{5}{6} - \frac{2}{3} \times \frac{3}{8}$$

..... [3]

14 Write 0.25 as a fraction.

..... [2]

E1.7 Indices I

Name: _____ Class: _____ Date: _____

Total: 9 marks**KEY WORDS** indices 指数 • laws of indices 指数定律 • power 幂 • prime factor 质因数 • factor 因数

Objective

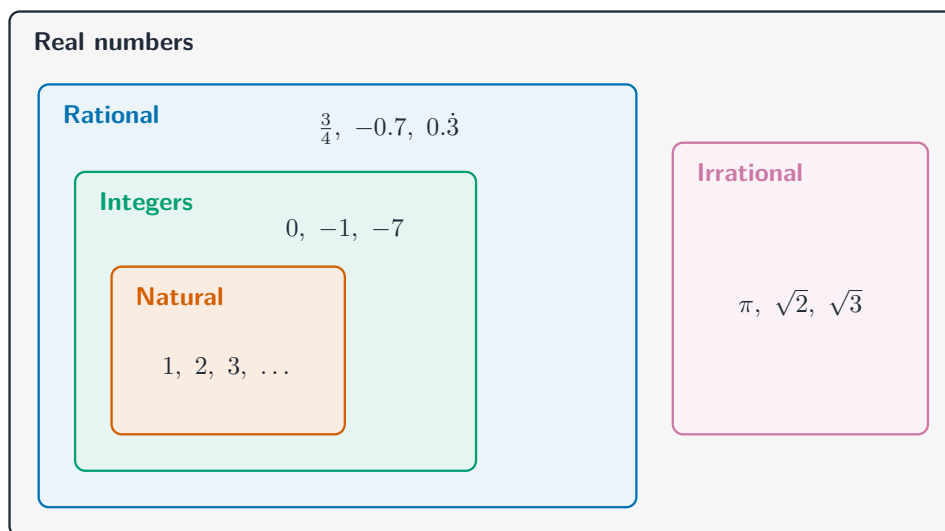
Build the skills to answer Extended exam questions on **indices** 指数 — the **laws of indices** 指数定律, and **zero, negative and fractional** 零、负、分数 powers.

You must be able to:

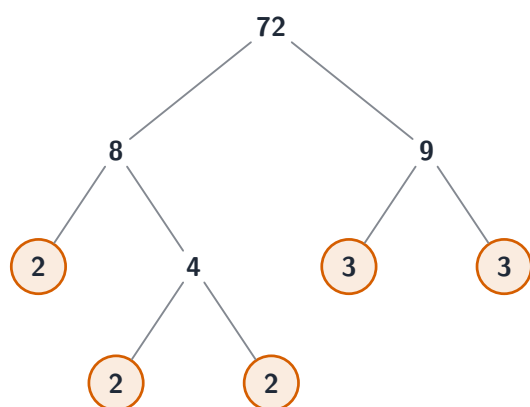
- use $a^m \times a^n = a^{m+n}$, $a^m \div a^n = a^{m-n}$, $(a^m)^n = a^{mn}$
- evaluate a^0 , a^{-n} and $a^{\frac{m}{n}}$
- simplify expressions with indices

1 Worked examples

■ Laws of indices



Add powers to multiply; subtract to divide



$$72 = 2^3 \times 3^2$$

Indices appear when a number is written in prime factors

$$a^m \times a^n = a^{m+n} \quad a^m \div a^n = a^{m-n} \quad (a^m)^n = a^{mn}$$

$$a^0 = 1 \quad a^{-n} = \frac{1}{a^n} \quad a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$$

- **Worked:** $2^{-3} \times 2^4 = 2^1 = 2$; $(2^3)^2 = 2^6 = 64$; $8^{-\frac{2}{3}} = \frac{1}{(\sqrt[3]{8})^2} = \frac{1}{4}$.

Answer shapes: **M1** a correct method, **A1** the correct answer.

2 Practice

2.1 Simplify $x^5 \times x^3$. [1]

2.2 Work out the value of 2^{-3} . [1]

2.3 Work out the value of $81^{\frac{1}{2}}$. [1]

3 Exam-style questions

3.1 Simplify $(2a^2)^3$. [2]

3.2 Work out the value of $27^{\frac{2}{3}}$. [2]

3.3 Work out the value of $16^{-\frac{1}{2}}$. [2]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $x^5 \times x^3 = x^8$.

2.2 $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$.

2.3 $81^{\frac{1}{2}} = \sqrt{81} = 9$.

3.1 $(2a^2)^3 = 2^3 a^6 = 8a^6$.

3.2 $27^{\frac{2}{3}} = (\sqrt[3]{27})^2 = 3^2 = 9$.

3.3 $16^{-\frac{1}{2}} = \frac{1}{16^{\frac{1}{2}}} = \frac{1}{\sqrt{16}} = \frac{1}{4}$.

6 Find the value of

(a) $5^{-5} \times 5^5$

..... [1]

(b) $125^{\frac{2}{3}}$.

[2]

19 Find the value of $27^{-\frac{2}{3}}$.

..... [2]

E1.8 Standard form

Name: _____ Class: _____ Date: _____

Total: 9 marks

KEY WORDS standard form 科学记数法 • power 幂 • integer 整数

Objective

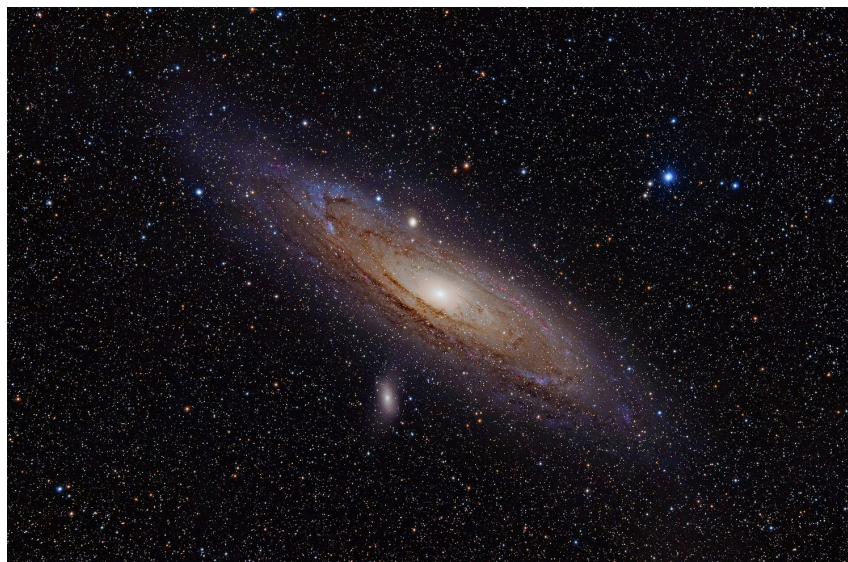
Build the skills to answer Extended exam questions on **standard form** 科学记数法 — writing numbers as $A \times 10^n$ and calculating with them.

You must be able to:

- write large and small numbers in standard form
- convert standard form back to an ordinary number
- multiply and divide numbers in standard form

1 Worked examples

■ Standard form



$A \times 10^n$ where $1 \leq A < 10$ and n is an integer

- **Convert:** $4\,500\,000 = 4.5 \times 10^6$; $0.00072 = 7.2 \times 10^{-4}$.

Standard form: $A \times 10^n$, **with** $1 \leq A < 10$

$$\boxed{6\,500\,000} \longrightarrow \boxed{6.5 \times 10^6}$$

point moves 6 places

$$\boxed{0.00042} \longrightarrow \boxed{4.2 \times 10^{-4}}$$

point moves 4 places

standard form: $a \times 10^n$ with $1 \leq a < 10$

Standard form $A \times 10^n$: the power of ten is the number of places the point moves

- **Multiply:** $(3 \times 10^5) \times (2 \times 10^{-2}) = 6 \times 10^3$ (multiply the fronts, add the powers).

Answer shapes: **M1** a correct method, **A1** the correct answer.

2 Practice

2.1 Write 63 000 in standard form. [1]

2.2 Write 0.0045 in standard form. [1]

2.3 Write 2.7×10^{-3} as an ordinary number. [1]

3 Exam-style questions

3.1 Work out $(4 \times 10^6) \times (2.5 \times 10^{-3})$, giving your answer in standard form. [2]

3.2 Work out $\frac{8 \times 10^5}{2 \times 10^2}$, giving your answer in standard form. [2]

3.3 Work out $(3 \times 10^4) + (5 \times 10^3)$, giving your answer in standard form. [2]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 6.3×10^4 .

2.2 4.5×10^{-3} .

2.3 0.0027.

3.1 $4 \times 2.5 = 10$ and $10^6 \times 10^{-3} = 10^3$, giving $10 \times 10^3 = 1 \times 10^4$.

3.2 $8 \div 2 = 4$ and $10^5 \div 10^2 = 10^3$, giving 4×10^3 .

3.3 $3 \times 10^4 = 30\,000$ and $5 \times 10^3 = 5\,000$, so the sum is $35\,000 = 3.5 \times 10^4$.

11

2.89×10^{-1}	1.3×10^{12}	8.3×10^1	9×10^{11}	2.03×10^{-5}	0.3×10^{-2}
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Use a number from the box to complete each statement.

The number that is not written in standard form is

The largest number is

The smallest number is

[2]

- 12 A vase contains flowers that are red or pink or white.
Ruth picks a flower at random from the vase.
The probability that the flower is **not** red is 0.9 .
The probability that the flower is **not** pink is 0.65 .

Find the probability that the flower is white.

..... [2]

- 13 The point (5, 1024) lies on the curve $y = c^x$, where c is a whole number.
Find the y -coordinate of the point on the curve with x -coordinate -2 .

..... [3]

E1.10 Limits of accuracy

Name: _____ Class: _____ Date: _____

Total: 11 marks

KEY WORDS lower bound 下界 • upper bound 上界 • lower and upper bounds 下界和上界 • area 面积
• limits of accuracy 精确度的界限

Objective

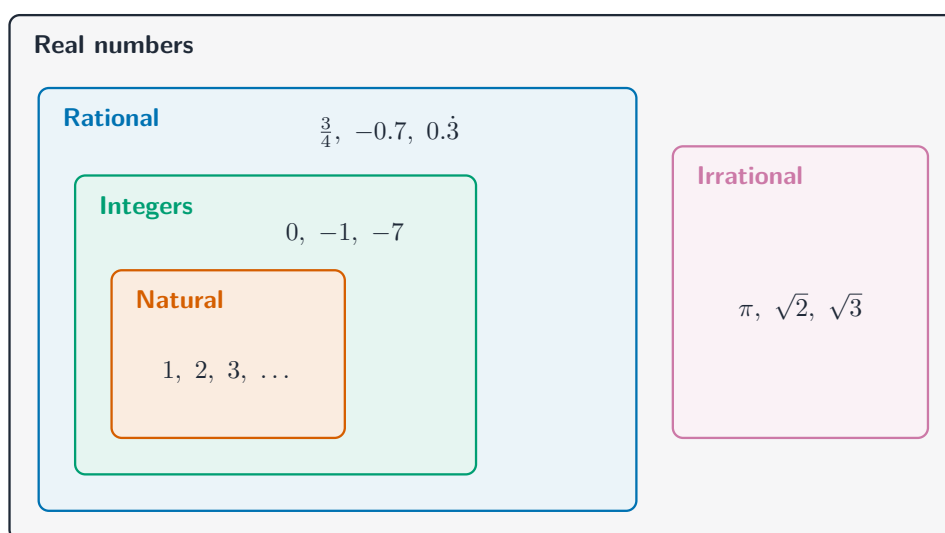
Build the skills to answer Extended exam questions on **limits of accuracy** 精确度的界限 — the **lower and upper bounds** 下界和上界 of a rounded value, and using bounds in calculations.

You must be able to:

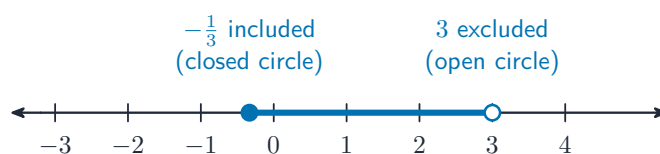
- give the upper and lower bounds of a rounded measurement
- find the largest and smallest result of a calculation
- choose the right bounds for a sum, product or quotient

1 Worked examples

■ Upper and lower bounds



A rounded value could be anything that rounds to it



Upper and lower bounds place a value within an interval

- **Bounds:** 635 m to the nearest metre $\rightarrow 634.5 \leq h < 635.5$.
- **Largest product:** use both **upper** bounds. **Smallest:** both **lower** bounds.
- **Quotient:** the **largest** $\frac{a}{b}$ uses the **largest** a and the **smallest** b .

Answer shapes: **M1** a correct method, **A1** the correct answer.

2 Practice

2.1 A length is 24 cm, to the nearest cm. Write down the lower bound. [1]

2.2 A mass is 3.5 kg, correct to 1 decimal place. Write down the upper bound. [1]

2.3 A time is 40 s, to the nearest 10 s. Write down the lower and upper bounds. [2]

3 Exam-style questions

3.1 A rectangle measures 12 cm by 7 cm, each side to the nearest cm.

(a) Calculate the upper bound of the area. [2]

(b) Calculate the lower bound of the perimeter. [2]

3.2 A car travels 90 m (to the nearest 10 m) in 8.0 s (to the nearest 0.1 s). Calculate the

upper bound of its average speed.

[3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 23.5 cm.

2.2 3.55 kg.

2.3 lower bound 35 s, upper bound 45 s.

3.1 (a) upper bounds 12.5 and 7.5 $\rightarrow$ area = $12.5 \times 7.5 = 93.75 \text{ cm}^2$. (b) lower bounds 11.5 and 6.5 $\rightarrow$ perimeter = $2(11.5 + 6.5) = 36 \text{ cm}$.

3.2 largest speed = largest distance $\div$ smallest time. Distance upper bound = 95 m; time lower bound = 7.95 s $\rightarrow \frac{95}{7.95} = 11.9 \text{ m/s}$ (to 3 s.f.).

25 Jenna has a length of wire measuring 68 cm, correct to the nearest cm.

From this wire she cuts off two smaller pieces

- a piece of length 4.7 cm, correct to the nearest mm
- a piece of length 10.0 cm, correct to the nearest mm.

Work out the lower bound and the upper bound for the length of the wire remaining.

Lower bound = cm

Upper bound = cm [3]

E1.12 Rates

Name: _____ Class: _____ Date: _____

Total: 12 marks

KEY WORDS average speed 平均速度 • density 密度 • mass 质量 • volume 体积 • rates 比率

Objective

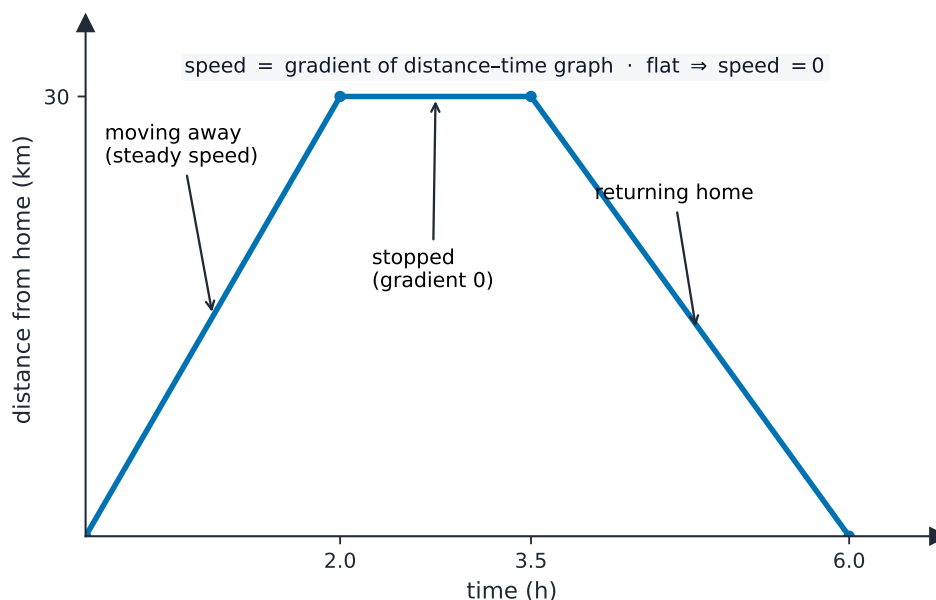
Build the skills to answer Extended exam questions on **rates** 比率 — **average speed** 平均速度, **density** 密度, and other rates that combine two quantities.

You must be able to:

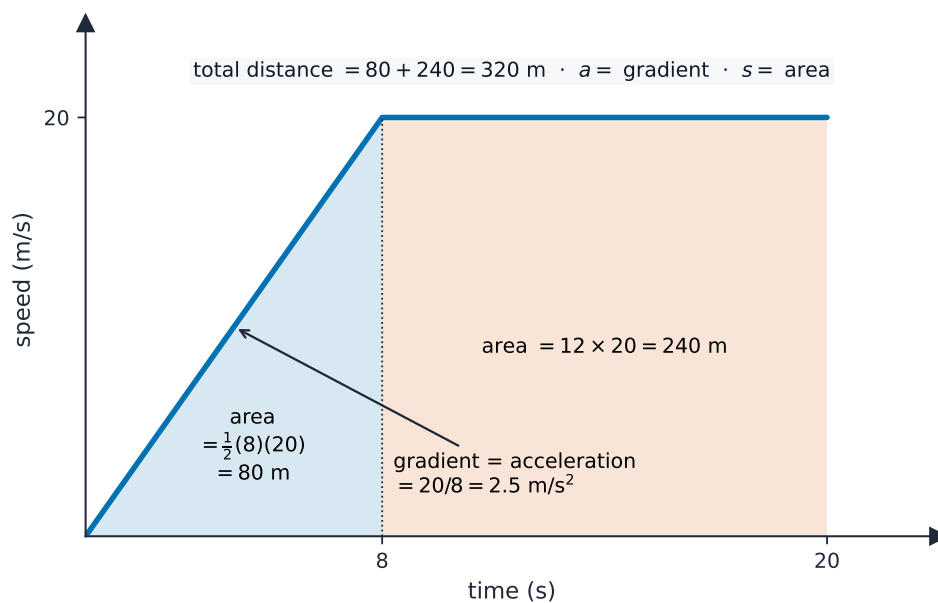
- use average speed $= \frac{\text{distance}}{\text{time}}$
- use density $= \frac{\text{mass}}{\text{volume}}$
- convert times correctly and rearrange a rate formula

1 Worked examples

■ Speed, distance, time



A rate compares two quantities in different units



A rate also reads off a speed-time graph

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}} \quad \text{density} = \frac{\text{mass}}{\text{volume}}$$

- **Worked:** 45 km in 3 h 45 min = 3.75 h $\rightarrow$ speed = $\frac{45}{3.75} = 12$ km/h.
- Change minutes to a decimal of an hour (45 min = 0.75 h) — **not** 3.45.

Answer shapes: M1 a correct method, A1 the correct answer.

2 Practice

2.1 A car travels 120 km in 2 hours. Calculate its average speed. [1]

2.2 A metal block has mass 240 g and volume 30 cm³. Calculate its density. [2]

2.3 A runner's average speed is 5 m/s. How far does she run in 40 seconds? [2]

3 Exam-style questions

3.1 A train travels 150 km in 1 hour 30 minutes. Calculate its average speed in km/h. [2]

3.2 Gold has a density of 19.3 g/cm^3 . Calculate the mass of a gold bar of volume 50 cm^3 . [2]

3.3 A cyclist travels 36 km at an average speed of 24 km/h. Calculate the time taken, in minutes. [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $\frac{120}{2} = 60$ km/h.

2.2 density = $\frac{240}{30} = 8$ g/cm³.

2.3 distance = $5 \times 40 = 200$ m.

3.1 1 h 30 min = 1.5 h $\rightarrow$ speed = $\frac{150}{1.5} = 100$ km/h.

3.2 mass = density $\times$ volume = $19.3 \times 50 = 965$ g.

3.3 time = $\frac{36}{24} = 1.5$ h = $1.5 \times 60 = 90$ minutes.

4 Pedro makes cards.

- (a)** He makes cards at a rate of 9 cards every 20 minutes.

Work out the number of cards he makes in 8 hours.

..... [2]

- (b)** Each card costs 12 cents to make.
Pedro sells each card for 50 cents.

Work out his percentage profit on each card.

..... % [2]

5 Nuwa is buying a phone.

One website sells the phone for 953 Yuan.

A different website sells the same phone for \$141.

The exchange rate is 1 Yuan = \$0.152 .

Calculate the difference between these phone prices.

Give your answer in dollars, correct to the nearest cent.

\$ [2]

- 8** Hadi buys a dishwasher.
 He can either pay a single payment of \$980 or he can pay using a monthly plan.
 The monthly plan is 20% of \$980 **plus** 12 payments of \$75.25 .

(a) Hadi uses the monthly plan.

Calculate the total amount Hadi pays.

\$ [2]

(b) Find the percentage increase in the cost using the monthly plan compared to a single payment.

..... % [2]