Solutions

Marking: **M1** a correct method, **A1** the correct answer.

Each answer shows the steps, then the **result**.

2.1

- $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$$= \mathbf{b} - \mathbf{a}$$

2.2

- $\overrightarrow{OM} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$

$$= \mathbf{\frac{1}{2}(a + b)}$$

2.3

- $\overrightarrow{RS} = 2\,(2\mathbf{a} + 3\mathbf{b}) = 2\,\overrightarrow{PQ}$
- one is a scalar multiple of the other, so they are parallel

2.4

- $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$, and P is $\frac{1}{3}$ of the way along it: $\overrightarrow{OP} = \mathbf{a} + \frac{1}{3}(\mathbf{b} - \mathbf{a})$

$$\overrightarrow{OP} = \mathbf{\frac{2}{3}a + \frac{1}{3}b}$$

2.5

- **yes**: $6\mathbf{a} - 9\mathbf{b} = 3(2\mathbf{a} - 3\mathbf{b})$, a multiple of it

3.1

(a)

- in a parallelogram AB is parallel to and equal in length to OC

$$= \mathbf{c}$$

(b)

- $\overrightarrow{OM} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$

$$= \mathbf{a} + \mathbf{\frac{1}{2}c}$$

(c)

- $\overrightarrow{OB} = \mathbf{a} + \mathbf{c}$, so $\overrightarrow{ON} = \frac{2}{3}\mathbf{a} + \frac{2}{3}\mathbf{c}$
- $\overrightarrow{MN} = \overrightarrow{ON} - \overrightarrow{OM} = \left(\frac{2}{3}\mathbf{a} + \frac{2}{3}\mathbf{c}\right) - \left(\mathbf{a} + \frac{1}{2}\mathbf{c}\right)$

$$= \mathbf{-\frac{1}{3}a + \frac{1}{6}c}$$

3.2

- $\overrightarrow{OQ} = 3\,(3\mathbf{a} - \mathbf{b}) = 3\,\overrightarrow{OP}$
- so $\overrightarrow{OQ}$ is a scalar multiple of $\overrightarrow{OP}$, i.e. the two are parallel
- they share the point O, so O, P and Q are **collinear**
- parallel alone is not enough; the common point is what makes it one straight line

3.3

- $\overrightarrow{XY} = \overrightarrow{XO} + \overrightarrow{OY} = -\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b} = \frac{1}{2}(\mathbf{b} - \mathbf{a})$
- $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$
- $\overrightarrow{XY} = \frac{1}{2}\overrightarrow{AB}$: a multiple, so **parallel**, and the factor $\frac{1}{2}$ means **half the length**

3.4

- $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$
- $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = 3\mathbf{b} - 2\mathbf{a} - \mathbf{a} = 3\mathbf{b} - 3\mathbf{a} = 3(\mathbf{b} - \mathbf{a})$
- $\overrightarrow{AC} = 3\overrightarrow{AB}$, so they are parallel, and they share the point A: A, B, C are **collinear**

3.5

(a)

- $\overrightarrow{OP} = \frac{1}{3} \times 6\mathbf{a} = 2\mathbf{a}$ and $\overrightarrow{OQ} = \frac{1}{2}(6\mathbf{a} + 6\mathbf{b}) = 3\mathbf{a} + 3\mathbf{b}$
- $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = \mathbf{a + 3b}$

(b)

- $\overrightarrow{PR} = \overrightarrow{OR} - \overrightarrow{OP} = -6\mathbf{b} - 2\mathbf{a}$
- $-2\mathbf{a} - 6\mathbf{b} = -2(\mathbf{a} + 3\mathbf{b}) = -2\overrightarrow{PQ}$
- so PR is parallel to PQ and they share the point P: P, Q, R are **collinear**