

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

Each answer shows the steps, then the **result**.

2.1

$$\begin{aligned}\bullet \quad \overrightarrow{AB} &= \overrightarrow{AO} + \overrightarrow{OB} \\ &= \mathbf{b} - \mathbf{a}\end{aligned}$$

2.2

$$\begin{aligned}\bullet \quad \overrightarrow{OM} &= \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) \\ &= \frac{1}{2}(\mathbf{a} + \mathbf{b})\end{aligned}$$

2.3

- $\overrightarrow{RS} = 2(2\mathbf{a} + 3\mathbf{b}) = 2\overrightarrow{PQ}$
- one is a scalar multiple of the other, so they are parallel

3.1

(a)

- in a parallelogram AB is parallel to and equal in length to OC
$$= \mathbf{c}$$

(b)

$$\begin{aligned}\bullet \quad \overrightarrow{OM} &= \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB} \\ &= \mathbf{a} + \frac{1}{2}\mathbf{c}\end{aligned}$$

(c)

$$\begin{aligned}\bullet \quad \overrightarrow{OB} &= \mathbf{a} + \mathbf{c}, \text{ so } \overrightarrow{ON} = \frac{2}{3}\mathbf{a} + \frac{2}{3}\mathbf{c} \\ \bullet \quad \overrightarrow{MN} &= \overrightarrow{ON} - \overrightarrow{OM} = \left(\frac{2}{3}\mathbf{a} + \frac{2}{3}\mathbf{c}\right) - \left(\mathbf{a} + \frac{1}{2}\mathbf{c}\right) \\ &= -\frac{1}{3}\mathbf{a} + \frac{1}{6}\mathbf{c}\end{aligned}$$

3.2

- $\overrightarrow{OQ} = 3(3\mathbf{a} - \mathbf{b}) = 3\overrightarrow{OP}$
- so $\overrightarrow{OQ}$ is a scalar multiple of $\overrightarrow{OP}$, i.e. the two are parallel
- they share the point O , so O , P and Q are **collinear**
- parallel alone is not enough; the common point is what makes it one straight line