

E5.4 Surface area and volume

Name: _____ Class: _____ Date: _____ Total: 33 marks

KEY WORDS cylinder 圆柱 • cuboid 长方体 • surface area 表面积 • sphere 球
• cross-section 横截面

Objective

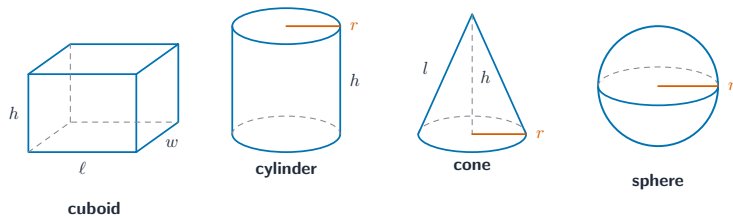
Build the skills to answer Extended exam questions on **surface area and volume** 表面积和体积 — the **cylinder, cone and sphere** 圆柱、圆锥和球, and parts of solids such as a **hemisphere** 半球.

You must be able to:

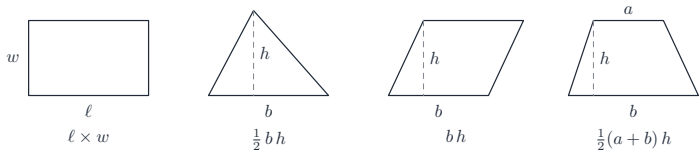
- find the volume of a cylinder, cone, sphere and pyramid
- find the surface area of a cylinder and cone
- work with parts of solids (hemispheres)

1 Worked examples

■ Volumes and surface areas



Learn the key formulas — some are on the formula list



Face areas build up the surface area of a solid

cylinder: $V = \pi r^2 h$, $SA = 2\pi r h + 2\pi r^2$ cone: $V = \frac{1}{3}\pi r^2 h$

sphere: $V = \frac{4}{3}\pi r^3$, $SA = 4\pi r^2$ hemisphere: $V = \frac{2}{3}\pi r^3$

- **Worked (cylinder $r = 5$, $h = 10$):** $V = \pi(25)(10) = 250\pi$; $SA = 100\pi + 50\pi = 150\pi \text{ cm}^2$.

Answer shapes: M1 a correct method, A1 the correct answer.

■ The formulas, and the three traps

Solid	Volume	Curved / total surface area
cylinder	$\pi r^2 h$	curved $2\pi r h$; total $2\pi r h + 2\pi r^2$
cone	$\frac{1}{3}\pi r^2 h$	curved $\pi r l$; total $\pi r l + \pi r^2$
sphere	$\frac{4}{3}\pi r^3$	$4\pi r^2$
prism	area of cross-section $\times$ length	perimeter of cross-section $\times$ length, plus the two ends

Trap 1 — the cone uses the SLANT height. The curved surface area needs l , not the vertical height h . If you are given h and r , find l by Pythagoras: $l = \sqrt{r^2 + h^2}$. Using h in $\pi r l$ is the single most common error on this topic.

Trap 2 — read “curved” against “total”. A closed cylinder has two circular ends; an open one (a pipe, a tank with no lid) has one or none. Count the faces the object actually has before adding anything.

Trap 3 — a hemisphere’s total surface area is not half a sphere’s. Cutting a sphere in half gives $\frac{1}{2}(4\pi r^2) = 2\pi r^2$ of curved surface **plus** the flat circular face πr^2 , so the total is $3\pi r^2$.

Composite solids — split the shape, work out each part, then add or subtract. Say which you are doing, because that reasoning carries a mark.

Worked example. A solid is a cylinder of radius 5 cm and height 12 cm with a cone of the same radius and height 12 cm on top. Find its total volume and its total surface area.

1. Volume of cylinder $= \pi(5)^2(12) = 300\pi$.
2. Volume of cone $= \frac{1}{3}\pi(5)^2(12) = 100\pi$; total $= 400\pi = 1260 \text{ cm}^3$ (3 s.f.).

3. Slant height $l = \sqrt{5^2 + 12^2} = 13$.
4. Surface area: base circle $\pi(5)^2 = 25\pi$, curved cylinder $2\pi(5)(12) = 120\pi$, curved cone $\pi(5)(13) = 65\pi$. The join is inside the solid, so it is **not** a surface.
5. Total $= 210\pi = 660 \text{ cm}^2$ (3 s.f.).

2 Practice

2.1 Find the volume of a cuboid 4 cm by 3 cm by 5 cm. [1]

2.2 Find the volume of a cylinder of radius 3 cm and height 10 cm, in terms of π . [2]

2.3 Find the volume of a sphere of radius 3 cm, in terms of π . [2]

2.4 Find the curved surface area of a cone with radius 6 cm and slant height 10 cm. [2]

2.5 A cone has radius 8 cm and vertical height 15 cm. Find its slant height. [2]

2.6 Show that the total surface area of a solid hemisphere of radius r is $3\pi r^2$. [2]

3 Exam-style questions

3.1 A cone has radius 6 cm and height 8 cm. Find its volume, in terms of π . [2]

3.2 Find the total surface area of a cylinder of radius 4 cm and height 9 cm, in terms of π . [3]

3.3 A solid hemisphere has radius 6 cm. Find its volume, in terms of π . [2]

3.4 A grain silo is a cylinder of radius 3 m and height 8 m, with a cone of the same radius and vertical height 4 m on top. It is open at the bottom.

(a) Calculate the volume of the silo. Give your answer to 3 significant figures. [4]

(b) Calculate the slant height of the cone. [2]

(c) Calculate the total area of metal needed to make the silo, given that it has no base.[4]

(d) The silo is filled with grain of density 0.75 g/cm^3 . Calculate the mass of grain, in tonnes. [3]

(e) A second silo is mathematically similar, with all lengths twice as large. State the ratio of their volumes and of their surface areas. [2]