

E2.6 Inequalities

Name: _____ Class: _____ Date: _____

Total: 29 marks

KEY WORDS inequality 不等式 • number line 数轴 • region 区域 • term 项 • equation 方程

Objective

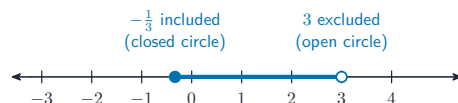
Build the skills to answer Extended exam questions on **inequalities** 不等式 — solving them, the rule for **multiplying or dividing by a negative** 乘除负数, and showing a solution on a **number line** 数轴.

You must be able to:

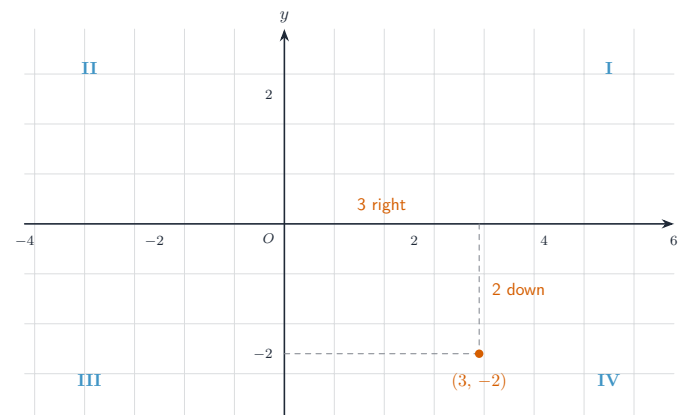
- solve a linear inequality (and reverse the sign when needed)
- solve a double inequality
- list the integers that satisfy an inequality

1 Worked examples

■ Solving inequalities



Solve like an equation, but reverse the sign if you $\div$ or $\times$ by a negative



Two-variable inequalities shade regions of the plane

- **Worked:** $-3 \leq 3x - 2 < 7 \rightarrow$ add 2: $-1 \leq 3x < 9 \rightarrow \div 3$: $-\frac{1}{3} \leq x < 3$.
- **Reverse:** $-2x < 6 \rightarrow x > -3$ (sign flips when dividing by -2).
- **Number line:** closed circle for $\leq / \geq$, open circle for $< / >$.

Answer shapes: M1 a correct method, A1 the correct answer.

■ Inequalities: the same steps as an equation, with one extra rule

Solve an inequality exactly as you would the matching equation — **except** that multiplying or dividing both sides by a **negative** number **reverses** the inequality sign.

$$-3x > 12 \Rightarrow x < -4$$

That single rule is the difference between full marks and none on half the questions in this topic. If you would rather not risk it, move the x term to the side where it is positive instead: $-3x > 12$ becomes $-12 > 3x$, so $-4 > x$, which is the same answer with no sign flip.

A **double inequality** is solved by doing the same operation to all three parts: $5 \leq 2x + 1 \leq 11$ gives $4 \leq 2x \leq 10$, then $2 \leq x \leq 5$.

Reading the answer as integers. For $-2 < x \leq 3$: the $<$ EXCLUDES -2 and the $\leq$ INCLUDES 3 , so the integers are $-1, 0, 1, 2, 3$. Getting the endpoints right is usually a mark by itself.

On a number line, an open circle means the value is excluded ($<$ or $>$) and a filled circle means it is included ($\leq$ or $\geq$).

Worked example. Find the integers n for which $-9 < 3n - 3 \leq 6$.

1. Add 3 throughout: $-6 < 3n \leq 9$.
2. Divide by 3 throughout: $-2 < n \leq 3$.
3. The endpoints: -2 is excluded, 3 is included.
4. So $n = -1, 0, 1, 2, 3$.

2 Practice

2.1 Solve $2x + 1 > 9$. [2]

2.2 Solve $-3x \leq 12$. [2]

2.3 Solve $5x - 4 < 11$. [2]

2.4 Solve $-4x + 3 > 19$. [2]

2.5 Write down the inequality shown by an open circle at -1 and a filled circle at 4 , joined by a line. [2]

3 Exam-style questions

3.1 Solve the inequality $5 \leq 2x + 1 \leq 11$. [3]

3.2 Write down the integers that satisfy $-2 < x \leq 3$. [2]

3.3 Solve $7 - 2x \geq 1$. [2]

3.4 (a) Solve the inequality $-11 < 4x + 5 \leq 13$. [3]

(b) Write down all the integers that satisfy your answer to part (a). [2]

(c) Solve $9 - 4x \geq 21$, and explain the step in which the inequality sign changes. [3]

(d) A rectangle has width x cm and length $(x + 5)$ cm. Its perimeter is at least 30 cm and less than 50 cm. Form an inequality in x and solve it. [4]