Solutions

Marking: **M1** a correct method, **A1** the correct answer.

Each answer shows the steps, then the **result**.

2.1

- $3x^4 \to 12x^3$, $-5x^2 \to -10x$, and the 7 vanishes

$$\frac{dy}{dx} = \mathbf{12x^3 - 10x}$$

2.2

- $\dfrac{dy}{dx} = 3x^2 - 4$
- substitute $x = 2$: $3(4) - 4$

$$= \mathbf{8}$$

2.3

- $\dfrac{dy}{dx} = 2x - 6$, and set it to zero
- $x = 3$
- substitute into the original: $9 - 18 + 5$

$$= (\mathbf{3},\ \mathbf{-4})$$

2.4

- expand first: $y = x^3 + 3x^2$

$$\frac{dy}{dx} = \mathbf{3x^2 + 6x}$$

2.5

- $\dfrac{dy}{dx} = 2x - 5 = 3$, so $x = 4$
- $y = 4^2 - 5 \times 4 = -4$: $(\mathbf{4},\ \mathbf{-4})$

3.1

(a)

- differentiate term by term

$$\frac{dy}{dx} = \mathbf{6x^2 - 18x + 12}$$

(b)

- set it to zero: $6x^2 - 18x + 12 = 0$, so $x^2 - 3x + 2 = 0$
- $(x - 1)(x - 2) = 0$, so $x = 1$ or $x = 2$
- $y(1) = 2 - 9 + 12 = 5$ and $y(2) = 16 - 36 + 24 = 4$

$$(\mathbf{1},\ \mathbf{5}) \text{ and } (\mathbf{2},\ \mathbf{4})$$

(c)

- $\dfrac{d^2y}{dx^2} = 12x - 18$
- at $x = 1$ this is -6, which is negative, so $(1,\ 5)$ is a **maximum**; at $x = 2$ it is $+6$, so $(2,\ 4)$ is a **minimum**

3.2

- $\dfrac{dy}{dx} = 3x^2 - 4x$; at $x = 2$: $3(4) - 4(2) = 4$
- the point: $y = 2^3 - 2(2^2) = 0$, so $(2,\ 0)$
- $y - 0 = 4(x - 2)$: $\boldsymbol{y = 4x - 8}$

3.3

(a)

- the side parallel to the wall is $60 - 2x$, so $A = x(60 - 2x) = 60x - 2x^2$

(b)

- $\dfrac{dA}{dx} = 60 - 4x$
- $60 - 4x = 0$, so $x = 15$
- $A = 60(15) - 2(15^2) = 900 - 450 = \mathbf{450}\ \mathrm{m}^2$

3.4

- $\dfrac{dy}{dx} = 3x^2 + 2ax$, which is 0 at $x = 2$: $12 + 4a = 0$
- $\boldsymbol{a = -3}$
- the point lies on the curve: $-1 = 8 + 4a + b = 8 - 12 + b$
- $\boldsymbol{b = 3}$