

E9.3 Averages and measures of spread

Name: _____ Class: _____ Date: _____

Total: 17 marks

KEY WORDS median 中位数 • mode 众数 • interquartile range 四分位距 • modal class 众数组
 • estimated mean 估计平均数

Objective

Build the skills to answer Extended exam questions on **averages and measures of spread** 平均数与离散程度 — the **mean** 平均数, **median** 中位数, **mode** 众数 and **range** 极差, the **interquartile range** 四分位距, and an **estimated mean** 估计平均数 from a grouped frequency table.

You must be able to:

- find the mean, median, mode and range of a list of values
- find the quartiles and the interquartile range
- estimate the mean from a grouped frequency table, and say why it is only an estimate

1 Worked examples

■ The median needs the list in order, every time



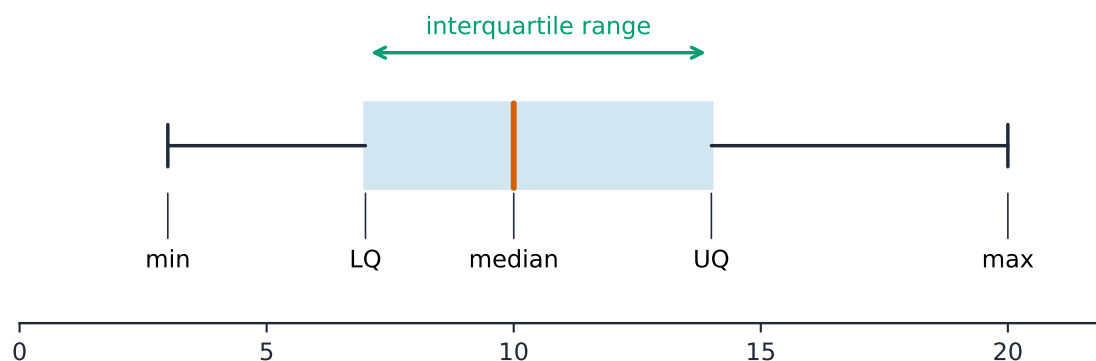
median = 3

$$\text{mean} = \frac{2+3+3+5+7}{5} = 4$$

mode = 3 (most common)

$$\text{range} = 7 - 2 = 5$$

Three different averages, three different answers



box-and-whisker: min · Q_1 · median · Q_3 · max · IQR = $Q_3 - Q_1$

The box spans the interquartile range; the whiskers reach the extremes

- **Mean** = total $\div$ how many. **Median** is the middle value of the ORDERED list. **Mode** is the most common value.
- **Range** = largest – smallest. **IQR** = $Q_3 - Q_1$, which ignores the extremes and so is less affected by an outlier.
- **Estimated mean from a grouped table:** use the MIDPOINT of each class, then $\frac{\sum fx}{\sum f}$.
- **Why it is an estimate:** the exact values inside each class are unknown, and the midpoint assumes they are evenly spread.

Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer): the midpoints row, and the $\sum fx$ total, are separate method marks in an estimated-mean question.

2 Practice

2.1 Find the mean, median, mode and range of 4, 7, 7, 9, 13.

[4]

2.2 Find the median and the interquartile range of 3, 5, 8, 8, 11, 14, 17, 21. [3]

2.3 The mean of six numbers is 14. Find their total. [1]

3 Exam-style questions

3.1 The table shows the times t minutes taken by 30 students.

Time (t minutes)	Frequency
$0 < t \leq 10$	4
$10 < t \leq 20$	11
$20 < t \leq 30$	9
$30 < t \leq 40$	6

(a) Calculate an estimate of the mean time, correct to 3 significant figures. [4]

(b) Write down the modal class. [1]

(c) Explain why the mean can only be an estimate. [1]

3.2 The mean of five numbers is 12. A sixth number is added and the mean becomes 13. Find the sixth number. [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

Each answer shows the steps, then the **result**.

2.1

- $\text{mean} = \frac{4 + 7 + 7 + 9 + 13}{5} = \frac{40}{5} = \mathbf{8}$
- the list is already in order, so the median is the third value, **7**
- $\text{mode} = \mathbf{7}$, the only repeated value
- $\text{range} = 13 - 4 = \mathbf{9}$

2.2

- eight values, so the median is the mean of the 4th and 5th: $\frac{8 + 11}{2} = \mathbf{9.5}$
- $Q_1 = \frac{5 + 8}{2} = 6.5$ and $Q_3 = \frac{14 + 17}{2} = 15.5$
 $\text{IQR} = 15.5 - 6.5 = \mathbf{9}$

2.3

- $\text{total} = 14 \times 6$
 $\qquad\qquad\qquad = \mathbf{84}$

3.1

(a)

- midpoints 5, 15, 25, 35
- $\sum fx = 4(5) + 11(15) + 9(25) + 6(35) = 20 + 165 + 225 + 210 = 620$
- $\sum f = 30$
 $\frac{620}{30} = 20.67 \dots = \mathbf{20.7}$ minutes

(b)

- the class with the highest frequency, 11

$$\mathbf{10 < t \leq 20}$$

- all four classes have the same width here, so the highest frequency is also the highest frequency density

(c)

- the individual times inside each class are not known; the midpoint assumes they are spread evenly through the class

3.2

- the first five total $12 \times 5 = 60$
- all six total $13 \times 6 = 78$

$$78 - 60 = \mathbf{18}$$