IGCSE MATHEMATICS • EXERCISE SHEET

# E8.3 Probability of combined events

Name: ____________ Class: ________ Date: __________ **Total: 19 marks**

**KEY WORDS** tree diagram 树形图 • with replacement 有放回 • without replacement 无放回 • independent 独立的 • at least one 至少一个

## Objective

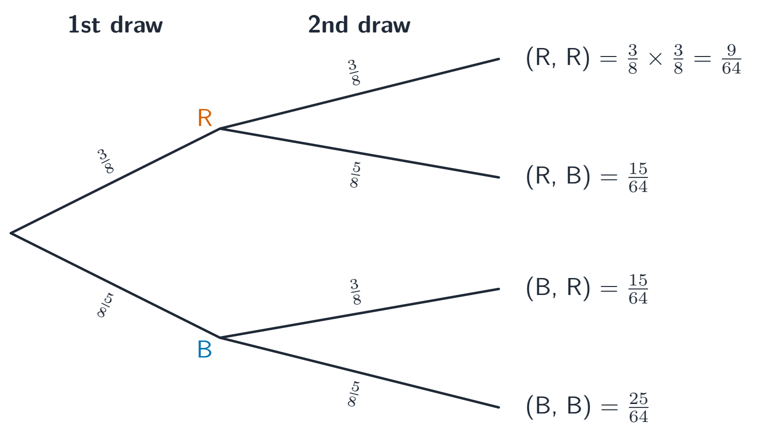
Build the skills to answer Extended exam questions on **combined events** 组合事件 — using a **tree diagram** 树形图, multiplying along branches for **AND** and adding for **OR**, and handling problems **with** and **without replacement** 有放回与无放回.

**You must be able to:**

- draw and complete a tree diagram for two events
- multiply along branches and add between branches
- change the denominators correctly when there is no replacement

## 1 Worked examples

■ **Multiply along a branch, add between branches**




*Each complete path is one outcome; its probability is the product along it*



die 2

| + | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| **1** | 2 | 3 | 4 | 5 | 6 | 7 |
| **2** | 3 | 4 | 5 | 6 | 7 | 8 |
| **3** | 4 | 5 | 6 | 7 | 8 | 9 |
| **4** | 5 | 6 | 7 | 8 | 9 | 10 |
| **5** | 6 | 7 | 8 | 9 | 10 | 11 |
| **6** | 7 | 8 | 9 | 10 | 11 | 12 |

die 1

the six highlighted cells all give a total of 7

*With small numbers of outcomes a sample space can be quicker than a tree*

- **AND ⇒ multiply, OR ⇒ add.** P(both red) is one path; P(one of each) is TWO paths added.
- **With replacement:** the second set of probabilities is the same as the first.
- **Without replacement:** BOTH the top and the bottom of the second fraction fall. From 5 red in 8, the second red is $\frac{4}{7}$, not $\frac{5}{8}$ and not $\frac{5}{7}$.
- **"At least one":** it is almost always faster to use 1− P(none).

**Answer shapes** (marked by **method marks** — M1 for a correct method, A1 for the correct answer): each correct product is a method mark, and adding the right number of them is another.

## 2 Practice

**2.1** Two fair coins are flipped. Find the probability that both land on heads. [2]

**2.2** A bag contains 5 red and 3 blue counters. Two are taken at random **with** replacement. Find the probability that both are red. [2]

**2.3** Repeat for the same bag, this time **without** replacement. [3]

## 3 Exam-style questions

**3.1** A bag contains 7 green and 5 yellow counters. Two counters are taken at random without replacement.

(a) Find the probability that both are green. [3]

(b) Find the probability that one is green and one is yellow. [4]

(c) Find the probability that at least one is yellow. [2]



**3.2** The probability that it rains on any given day is 0.3, independently of other days. Find the probability that it rains on exactly one of two days. [3]

## Solutions

Marking: **M1** a correct method, **A1** the correct answer.

Each answer shows the steps, then the **result**.

**2.1**

- $\frac{1}{2} \times \frac{1}{2}$

$$= \mathbf{\frac{1}{4}}$$

**2.2**

- with replacement the probabilities are unchanged: $\frac{5}{8} \times \frac{5}{8}$

$$= \mathbf{\frac{25}{64}}$$

**2.3**

- after one red is removed, 4 reds remain out of 7 counters
- $\frac{5}{8} \times \frac{4}{7} = \frac{20}{56}$

$$= \mathbf{\frac{5}{14}}$$

**3.1**

(a)

- $\frac{7}{12} \times \frac{6}{11}$

$$= \frac{42}{132} = \mathbf{\frac{7}{22}}$$

(b)

- green then yellow: $\frac{7}{12} \times \frac{5}{11} = \frac{35}{132}$
- yellow then green: $\frac{5}{12} \times \frac{7}{11} = \frac{35}{132}$
- there are TWO orders, so add them

$$\frac{70}{132} = \mathbf{\frac{35}{66}}$$

(c)

- "at least one yellow" is the opposite of "both green"

$$1 - \frac{7}{22} = \mathbf{\frac{15}{22}}$$

**3.2**

- rain then dry: $0.3 \times 0.7 = 0.21$
- dry then rain: $0.7 \times 0.3 = 0.21$

$$0.21 + 0.21 = \mathbf{0.42}$$

- forgetting the second order and answering 0.21 is the standard loss here