

E8.3 Probability of combined events

Name: _____ Class: _____ Date: _____

Total: 19 marks

KEY WORDS tree diagram 树形图 • with replacement 有放回 • without replacement 无放回
• independent 独立的 • at least one 至少一个

Objective

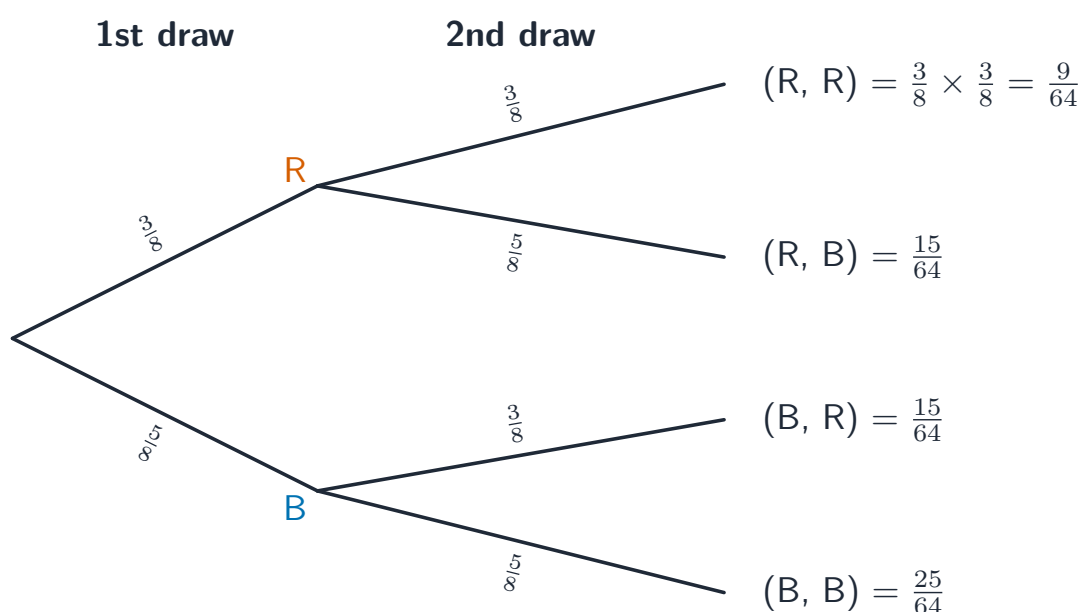
Build the skills to answer Extended exam questions on **combined events** 组合事件 — using a **tree diagram** 树形图, multiplying along branches for **AND** and adding for **OR**, and handling problems **with** and **without replacement** 有放回与无放回.

You must be able to:

- draw and complete a tree diagram for two events
- multiply along branches and add between branches
- change the denominators correctly when there is no replacement

1 Worked examples

■ Multiply along a branch, add between branches



Each complete path is one outcome; its probability is the product along it

		die 2					
+		1	2	3	4	5	6
die 1	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

the six highlighted cells all give a total of 7

With small numbers of outcomes a sample space can be quicker than a tree

- **AND** $\Rightarrow$ **multiply**, **OR** $\Rightarrow$ **add**. P(both red) is one path; P(one of each) is TWO paths added.
- **With replacement**: the second set of probabilities is the same as the first.
- **Without replacement**: BOTH the top and the bottom of the second fraction fall. From 5 red in 8, the second red is $\frac{4}{7}$, not $\frac{5}{8}$ and not $\frac{5}{7}$.
- **"At least one"**: it is almost always faster to use $1 - P(\text{none})$.

Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer): each correct product is a method mark, and adding the right number of them is another.

2 Practice

2.1 Two fair coins are flipped. Find the probability that both land on heads. [2]

2.2 A bag contains 5 red and 3 blue counters. Two are taken at random **with** replacement. Find the probability that both are red. [2]

2.3 Repeat for the same bag, this time **without** replacement. [3]

3 Exam-style questions

3.1 A bag contains 7 green and 5 yellow counters. Two counters are taken at random without replacement.

(a) Find the probability that both are green. [3]

(b) Find the probability that one is green and one is yellow. [4]

(c) Find the probability that at least one is yellow. [2]

3.2 The probability that it rains on any given day is 0.3, independently of other days. Find the probability that it rains on exactly one of two days. [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

Each answer shows the steps, then the **result**.

2.1

$$\begin{aligned} \bullet \quad \frac{1}{2} \times \frac{1}{2} \\ = \frac{1}{4} \end{aligned}$$

2.2

$$\begin{aligned} \bullet \quad \text{with replacement the probabilities are unchanged: } \frac{5}{8} \times \frac{5}{8} \\ = \frac{25}{64} \end{aligned}$$

2.3

$$\begin{aligned} \bullet \quad \text{after one red is removed, 4 reds remain out of 7 counters} \\ \bullet \quad \frac{5}{8} \times \frac{4}{7} = \frac{20}{56} \\ = \frac{5}{14} \end{aligned}$$

3.1

(a)

$$\begin{aligned} \bullet \quad \frac{7}{12} \times \frac{6}{11} \\ = \frac{42}{132} = \frac{7}{22} \end{aligned}$$

(b)

- green then yellow: $\frac{7}{12} \times \frac{5}{11} = \frac{35}{132}$
- yellow then green: $\frac{5}{12} \times \frac{7}{11} = \frac{35}{132}$
- there are TWO orders, so add them

$$\frac{70}{132} = \frac{35}{66}$$

(c)

- “at least one yellow” is the opposite of “both green”

$$1 - \frac{7}{22} = \frac{15}{22}$$

3.2

- rain then dry: $0.3 \times 0.7 = 0.21$
- dry then rain: $0.7 \times 0.3 = 0.21$

$$0.21 + 0.21 = \mathbf{0.42}$$

- forgetting the second order and answering 0.21 is the standard loss here