IGCSE MATHEMATICS • EXERCISE SHEET

E8.1 Introduction to probability

Name: ____________ Class: ________ Date: __________ **Total: 21 marks**

KEY WORDS probability 概率 • probability scale 概率标度 • outcome 结果 • expected frequency 期望频数 • tree diagrams 树状图

Objective

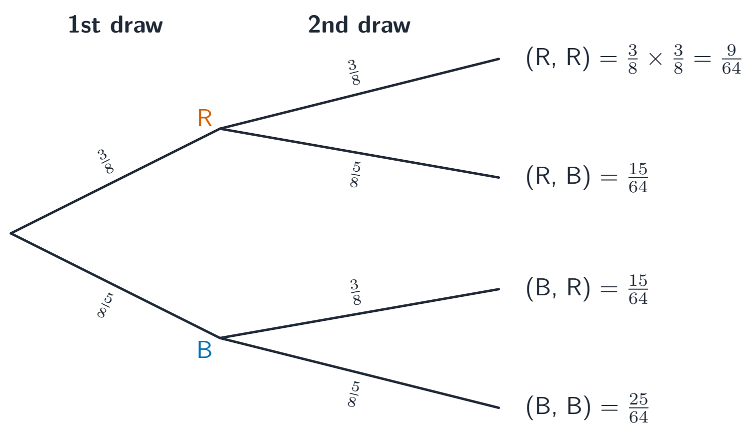
Build the skills to answer Extended exam questions on **probability 概率** — the **probability scale 概率标度**, $\mathrm{P}(A')$, **expected frequency 期望频数**, and **tree diagrams 树状图**.

You must be able to:

- find a probability and use $\mathrm{P}(A') = 1 - \mathrm{P}(A)$
- find an expected frequency
- use a tree diagram for combined events (with and without replacement)

1 Worked examples

■ **Probability and tree diagrams**



Probability runs from 0 (impossible) to 1 (certain)

die 2

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

die 1

the six highlighted cells all give a total of 7

A sample space lists all equally likely outcomes

$$\mathrm{P}(\text{event}) = \frac{\text{favourable}}{\text{total}} \qquad \mathrm{P}(A') = 1 - \mathrm{P}(A) \qquad \text{expected} = \mathrm{P} \times n$$

- **Worked (bag: 3 red, 5 blue):** $\mathrm{P}(\text{not red}) = 1 - \frac{3}{8} = \frac{5}{8}$.
- **Tree (with replacement):** $\mathrm{P}(\text{red, red}) = \frac{3}{8} \times \frac{3}{8} = \frac{9}{64}$.

Answer shapes: M1 a correct method, **A1** the correct answer.

2 Practice

2.1 A bag has 4 red and 6 green counters. Find the probability of drawing a red. [1]

2.2 The probability of rain is 0.3. Find the probability of no rain. [1]

2.3 The probability of a six is $\frac{1}{6}$. Find the expected number of sixes in 240 rolls. [2]

3 Exam-style questions

3.1 A bag has 3 red and 5 blue counters. Two are drawn **without replacement**. Find the probability that both are red. [3]

3.2 The probability that a train is late is 0.15. On 200 days, find the expected number of late days. [2]

3.3 A bag holds 5 red, 3 blue and 2 green counters. One counter is taken at random.

(a) Write down P(red) and P(not red), and state the connection between them. [3]

(b) The counter is replaced and the experiment is repeated 250 times. Find the expected number of blue counters. [2]

(c) Two counters are now taken **without** replacement. Draw a tree diagram for "green" and "not green", and use it to find the probability that exactly one is green. [5]

(d) Explain why the probabilities on the second pair of branches differ from those on the first. [2]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $\frac{4}{10} = \frac{2}{5}$.

2.2 $1 - 0.3 = 0.7$.

2.3 $\frac{1}{6} \times 240 = 40$.

3.1 first red $\frac{3}{8}$; after a red is taken, 2 reds out of 7 remain $\rightarrow \frac{2}{7}$. $\mathrm{P(red, red)} = \frac{3}{8} \times \frac{2}{7} = \frac{6}{56} = \frac{3}{28}$.

3.2 $0.15 \times 200 = 30$ days.

3.3 (a) $\mathrm{P(red)} = \dfrac{5}{10} = \dfrac{1}{2}$; $\mathrm{P(not\ red)} = \dfrac{5}{10} = \dfrac{1}{2}$; $\mathrm{P}(A') = 1 - \mathrm{P}(A)$ — the two must add to 1, because the counter is either red or not.
(b) $\mathrm{P(blue)} = \dfrac{3}{10}$; expected number $= \dfrac{3}{10} \times 250 = 75$.
(c) First pair of branches $\frac{2}{10}$ and $\frac{8}{10}$; second pair $\frac{1}{9}, \frac{8}{9}$ after green and $\frac{2}{9}, \frac{7}{9}$ after not green. Exactly one green $= \left(\frac{2}{10} \times \frac{8}{9}\right) + \left(\frac{8}{10} \times \frac{2}{9}\right) = \frac{16}{90} + \frac{16}{90} = \frac{32}{90} = \frac{16}{45}$.
(d) The first counter is not replaced, so only 9 counters remain for the second draw; and the number of greens left depends on what was drawn first, so the two second-stage pairs are different.