IGCSE MATHEMATICS • EXERCISE SHEET

E7.4 Vector geometry

Name: ____________ Class: ________ Date: __________ **Total: 14 marks**

KEY WORDS position vector 位置向量 • collinear 共线 • scalar multiple 数乘 • parallelogram 平行四边形 • midpoint 中点

Objective

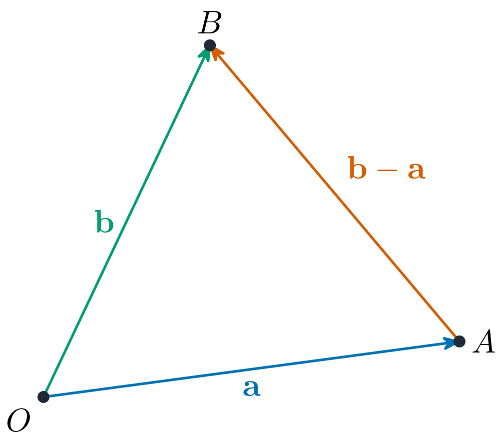
Build the skills to answer Extended exam questions on **vector geometry** 向量几何 — writing a journey in terms of given vectors, using **position vectors** 位置向量, and proving that lines are **parallel** 平行 or that points are **collinear** 共线.

You must be able to:

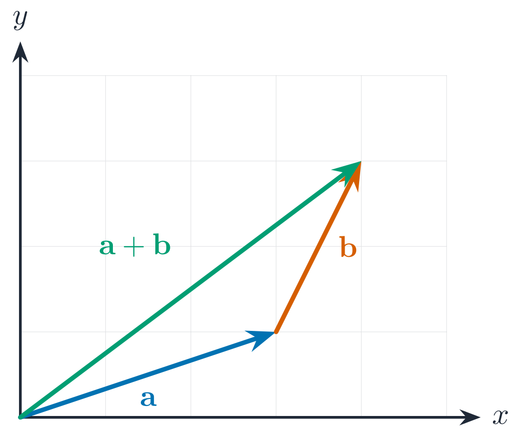
- write any journey in a diagram in terms of the given vectors
- find the position vector of a midpoint or of a point dividing a line in a given ratio
- prove two vectors are parallel, and prove three points are collinear

1 Worked examples

Travel there by any route — the answer is the same



Go backwards along a vector to reverse its sign



$$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\mathbf{a} + \mathbf{b}$$

- **The basic move:** $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$ when $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. "Destination minus start".
- **Midpoint:** $\overrightarrow{OM} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a}) = \frac{1}{2}(\mathbf{a} + \mathbf{b})$.
- **Parallel:** two vectors are parallel when one is a **scalar multiple** of the other. $4\mathbf{a} + 6\mathbf{b} = 2(2\mathbf{a} + 3\mathbf{b})$, so those two are parallel.
- **Collinear:** parallel AND sharing a point. $\overrightarrow{OQ} = 3\overrightarrow{OP}$ with the common point O means O, P and Q lie on one straight line.

Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer): a valid route written down is the method mark, even before it is simplified. For a proof, the words "scalar multiple" and "common point" are the marks.

2 Practice

2.1 $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. Write $\overrightarrow{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [1]

2.2 M is the midpoint of AB. Write $\overrightarrow{OM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$, in its simplest form. [2]

2.3 $\overrightarrow{PQ} = 2\mathbf{a} + 3\mathbf{b}$ and $\overrightarrow{RS} = 4\mathbf{a} + 6\mathbf{b}$. Show that PQ and RS are parallel. [2]

3 Exam-style questions

3.1 $OABC$ is a parallelogram with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. M is the midpoint of AB.

(a) Write down $\overrightarrow{AB}$ in terms of $\mathbf{c}$. [1]

(b) Find $\overrightarrow{OM}$ in terms of $\mathbf{a}$ and $\mathbf{c}$. [2]

(c) N lies on OB with $\overrightarrow{ON} = \frac{2}{3}\overrightarrow{OB}$. Find $\overrightarrow{MN}$ in terms of $\mathbf{a}$ and $\mathbf{c}$. [3]

3.2 $\overrightarrow{OP} = 3\mathbf{a} - \mathbf{b}$ and $\overrightarrow{OQ} = 9\mathbf{a} - 3\mathbf{b}$. Show that O, P and Q are collinear. [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

Each answer shows the steps, then the **result**.

2.1

- $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$$= \mathbf{b} - \mathbf{a}$$

2.2

- $\overrightarrow{OM} = \mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$

$$= \boldsymbol{\frac{1}{2}}(\mathbf{a} + \mathbf{b})$$

2.3

- $\overrightarrow{RS} = 2\,(2\mathbf{a} + 3\mathbf{b}) = 2\,\overrightarrow{PQ}$
- one is a scalar multiple of the other, so they are parallel

3.1

(a)

- in a parallelogram AB is parallel to and equal in length to OC

$$= \mathbf{c}$$

(b)

- $\overrightarrow{OM} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$

$$= \mathbf{a} + \boldsymbol{\frac{1}{2}}\mathbf{c}$$

(c)

- $\overrightarrow{OB} = \mathbf{a} + \mathbf{c}$, so $\overrightarrow{ON} = \frac{2}{3}\mathbf{a} + \frac{2}{3}\mathbf{c}$
- $\overrightarrow{MN} = \overrightarrow{ON} - \overrightarrow{OM} = \left(\frac{2}{3}\mathbf{a} + \frac{2}{3}\mathbf{c}\right) - \left(\mathbf{a} + \frac{1}{2}\mathbf{c}\right)$

$$= -\boldsymbol{\frac{1}{3}}\mathbf{a} + \boldsymbol{\frac{1}{6}}\mathbf{c}$$

3.2

- $\overrightarrow{OQ} = 3\,(3\mathbf{a} - \mathbf{b}) = 3\,\overrightarrow{OP}$
- so $\overrightarrow{OQ}$ is a scalar multiple of $\overrightarrow{OP}$, i.e. the two are parallel
- they share the point O, so O, P and Q are **collinear**
- parallel alone is not enough; the common point is what makes it one straight line