

E7.3 Magnitude of a vector

Name: _____ Class: _____ Date: _____

Total: 15 marks

KEY WORDS magnitude 模 • column vector 列向量 • component 分量 • scalar multiple 数乘
 • exact form 精确值

Objective

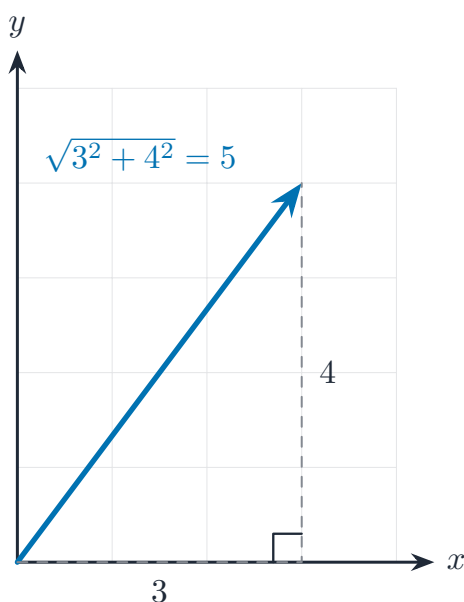
Build the skills to answer Extended exam questions on the **magnitude of a vector** 向量的模 — finding $|\mathbf{a}|$ from a **column vector** 列向量, combining vectors first and then taking the magnitude, and working backwards from a given magnitude.

You must be able to:

- find the magnitude of a column vector using Pythagoras
- add, subtract and multiply column vectors before finding a magnitude
- find a missing component when the magnitude is given

1 Worked examples

■ Magnitude is Pythagoras on the two components



The two components are the short sides; the magnitude is the hypotenuse

- **Formula:** if $\mathbf{a} = \begin{pmatrix} x \\ y \end{pmatrix}$ then $|\mathbf{a}| = \sqrt{x^2 + y^2}$.
- **The signs vanish**, because both components are squared. $\begin{pmatrix} 6 \\ -8 \end{pmatrix}$ has magnitude 10, the same as $\begin{pmatrix} 6 \\ 8 \end{pmatrix}$.
- **Combine FIRST.** $|\mathbf{a} + \mathbf{b}|$ means add the vectors and then take the magnitude; it is NOT $|\mathbf{a}| + |\mathbf{b}|$.
- **Scalar multiple:** $2\mathbf{a}$ multiplies BOTH components by 2.

Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer): the combined column vector, and the square root expression, are separate method marks.

2 Practice

2.1 $\mathbf{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$. Find $|\mathbf{a}|$. [2]

2.2 $\mathbf{b} = \begin{pmatrix} -5 \\ 12 \end{pmatrix}$. Find $|\mathbf{b}|$. [2]

2.3 $\mathbf{c} = \begin{pmatrix} 6 \\ -8 \end{pmatrix}$. Find $|\mathbf{c}|$. [2]

3 Exam-style questions

3.1 $\mathbf{a} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$.

(a) Find $\mathbf{a} + \mathbf{b}$ as a column vector. [1]

(b) Find $|\mathbf{a} + \mathbf{b}|$, giving your answer in exact form. [2]

(c) Find $|2\mathbf{a} - \mathbf{b}|$, correct to 3 significant figures. [3]

3.2 The vector $\begin{pmatrix} k \\ 8 \end{pmatrix}$ has magnitude 17. Find the positive value of k . [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

Each answer shows the steps, then the **result**.

2.1

$$\begin{aligned} \bullet \sqrt{3^2 + 4^2} &= \sqrt{25} \\ &= \mathbf{5} \end{aligned}$$

2.2

$$\begin{aligned} \bullet \sqrt{(-5)^2 + 12^2} &= \sqrt{169} \\ &= \mathbf{13} \end{aligned}$$

2.3

$$\begin{aligned} \bullet \sqrt{6^2 + (-8)^2} &= \sqrt{100} \\ &= \mathbf{10} \end{aligned}$$

3.1

(a)

$$\begin{aligned} \bullet \text{ add the components separately} \\ \begin{pmatrix} 3 \\ 2 \end{pmatrix} \end{aligned}$$

(b)

$$\begin{aligned} \bullet \sqrt{3^2 + 2^2} &= \sqrt{13} \\ &= \sqrt{\mathbf{13}} \end{aligned}$$

• 13 has no square factor, so this is already exact

(c)

$$\begin{aligned} \bullet 2\mathbf{a} &= \begin{pmatrix} 8 \\ -6 \end{pmatrix}, \text{ so } 2\mathbf{a} - \mathbf{b} = \begin{pmatrix} 9 \\ -11 \end{pmatrix} \\ \bullet \sqrt{81 + 121} &= \sqrt{202} \\ &= 14.212 \dots = \mathbf{14.2} \end{aligned}$$

3.2

$$\begin{aligned} \bullet k^2 + 8^2 &= 17^2 \\ \bullet k^2 &= 289 - 64 = 225 \\ &k = \mathbf{15} \end{aligned}$$