

E7.2 Vectors in two dimensions

Name: _____ Class: _____ Date: _____ **Total: 21 marks**

KEY WORDS magnitude 模 • column vectors 列向量 • position vectors 位置向量
• vectors in two dimensions 二维向量 • scalar 标量

Objective

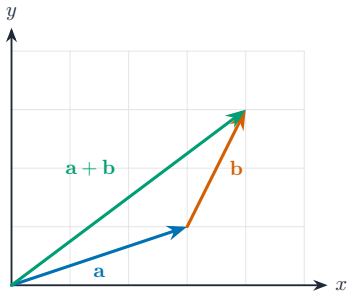
Build the skills to answer Extended exam questions on **vectors in two dimensions** 二维向量 — adding, subtracting and scaling **column vectors** 列向量, the **magnitude** 模, and **position vectors** 位置向量.

You must be able to:

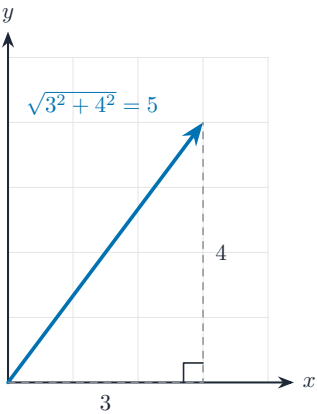
- add, subtract and multiply column vectors by a scalar
- find the magnitude of a vector
- use $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$

1 Worked examples

■ Working with vectors



Work on the top and bottom numbers separately



The magnitude (length) of a vector

- **Add/scale:** $\mathbf{a} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 2 \\ -4 \end{pmatrix} \rightarrow \mathbf{a} + \mathbf{b} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$; $3\mathbf{a} = \begin{pmatrix} 9 \\ 3 \end{pmatrix}$.
- **Magnitude 模:** $\left| \begin{pmatrix} x \\ y \end{pmatrix} \right| = \sqrt{x^2 + y^2} \rightarrow \left| \begin{pmatrix} 3 \\ 4 \end{pmatrix} \right| = 5$.
- **Position vectors:** $\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$.

Answer shapes: M1 a correct method, A1 the correct answer.

2 Practice

2.1 $\mathbf{a} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$. Find $\mathbf{a} + \mathbf{b}$. [1]

2.2 Find $2\mathbf{a}$ for $\mathbf{a} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$. [1]

2.3 Find the magnitude of $\begin{pmatrix} 6 \\ 8 \end{pmatrix}$. [2]

3 Exam-style questions

3.1 $\mathbf{a} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 1 \\ 6 \end{pmatrix}$. Find $\mathbf{a} - \mathbf{b}$. [2]

3.2 The position vector of A is $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ and of B is $\begin{pmatrix} 7 \\ 11 \end{pmatrix}$. Find $\overrightarrow{AB}$ and its magnitude. [3]

3.3 $\mathbf{a} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$.

(a) Work out $2\mathbf{a} - \mathbf{b}$ as a column vector. [2]

(b) Find $|\mathbf{a}|$, giving your answer in surd form and to 2 decimal places. [3]

(c) $OABC$ is a parallelogram with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{b}$. Write $\overrightarrow{AC}$ and $\overrightarrow{OB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [3]

(d) M is the midpoint of AB . Show that $\overrightarrow{OM} = \mathbf{a} + \frac{1}{2}\mathbf{b}$, and hence write $\overrightarrow{OM}$ as a column vector. [4]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $\begin{pmatrix} 4+1 \\ 2-3 \end{pmatrix} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}.$

2.2 $\begin{pmatrix} 6 \\ -2 \end{pmatrix}.$

2.3 $\sqrt{6^2 + 8^2} = \sqrt{100} = 10.$

3.1 $\begin{pmatrix} 5-1 \\ 2-6 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \end{pmatrix}.$

3.2 $\overrightarrow{AB} = \mathbf{b} - \mathbf{a} = \begin{pmatrix} 6 \\ 8 \end{pmatrix};$ magnitude $= \sqrt{6^2 + 8^2} = 10.$

3.3 (a) $2\mathbf{a} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}; 2\mathbf{a} - \mathbf{b} = \begin{pmatrix} 6 - (-2) \\ -2 - 4 \end{pmatrix} = \begin{pmatrix} 8 \\ -6 \end{pmatrix}.$

(b) $|\mathbf{a}| = \sqrt{3^2 + (-1)^2} = \sqrt{10} = 3.16$ to 2 d.p..

(c) $\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC} = -\mathbf{a} + \mathbf{b};$ in a parallelogram $\overrightarrow{OB} = \mathbf{a} + \mathbf{b}.$

(d) $\overrightarrow{AB} = \overrightarrow{OC} = \mathbf{b},$ so $\overrightarrow{AM} = \frac{1}{2}\mathbf{b}; \overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM} = \mathbf{a} + \frac{1}{2}\mathbf{b}; = \begin{pmatrix} 3 \\ -1 \end{pmatrix} + \begin{pmatrix} -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$