

E7.1 Transformations

Name: _____ Class: _____ Date: _____ Total: 26 marks

KEY WORDS reflection 反射 • translation 平移 • rotation 旋转 • centre 中心
• scale factor 比例因子

Objective

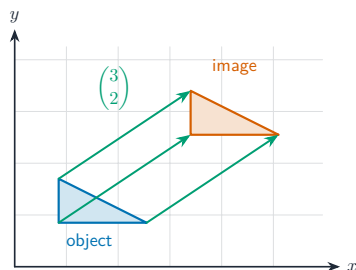
Build the skills to answer Extended exam questions on **transformations** 变换 — **reflection, rotation, enlargement and translation** 反射、旋转、放大、平移 — and describing them fully.

You must be able to:

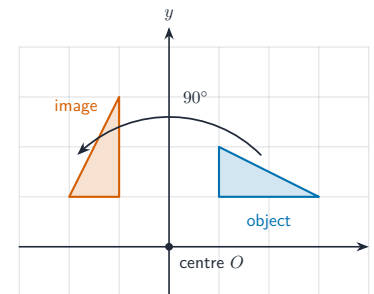
- find the image of a point under each transformation
- describe a transformation with all the required details
- use a column vector for a translation

1 Worked examples

■ The four transformations



Describe fully: name the type + all its details



Rotation is one of the four transformations

- **Reflection** 反射: give the mirror line. $(3, 2)$ in the y -axis $\rightarrow (-3, 2)$.
- **Rotation** 旋转: give centre, angle, direction. $(3, 1)$, 90° anticlockwise about $O \rightarrow (-1, 3)$.
- **Enlargement** 放大: give centre and scale factor k . $(1, 2)$ from O , $k = 2 \rightarrow (2, 4)$.
- **Translation** 平移: give the column vector. $(5, 3)$ by $\begin{pmatrix} -2 \\ 4 \end{pmatrix} \rightarrow (3, 7)$.

Answer shapes: M1 a correct method, A1 the correct answer.

■ Describing a transformation fully

“Describe fully” means give the **name** and every piece of information that fixes it. Miss one and the marks go.

Transformation	What must also be given
reflection	the equation of the mirror line, for example $y = x$ or $x = 2$
rotation	the angle, the direction (clockwise or anticlockwise) and the centre
translation	the column vector $\begin{pmatrix} a \\ b \end{pmatrix}$
enlargement	the scale factor and the centre of enlargement

Only **one** transformation may be named: writing “a reflection and then a translation” scores nothing.

How to identify it. If the shape is the same size and the same way round, it is a translation or a rotation; if it is the same size but reversed, it is a reflection; if it is a different size, it is an enlargement. For a rotation, join a point to its image and to a candidate centre and check the angle. For an enlargement, the centre is where the lines through matching pairs of vertices meet, and the scale factor is $\frac{\text{image length}}{\text{object length}}$.

Worked example. A has vertices $(1, 2)$, $(4, 2)$, $(1, 4)$ and B has $(2, 4)$, $(8, 4)$, $(2, 8)$. Describe the transformation fully.

1. The image is bigger, so it is an enlargement.
2. Scale factor: the base goes from 3 to 6, so it is 2.
3. Centre: each image coordinate is exactly twice the object coordinate, so the centre is the origin $(0, 0)$.
4. Answer: an enlargement, scale factor 2, centre $(0, 0)$.

A **negative scale factor** puts the image on the other side of the centre and turns it upside down; a **fractional scale factor** makes it smaller.

2 Practice

2.1 Reflect the point $(4, 3)$ in the x -axis. [1]

2.2 Translate the point $(2, 1)$ by the vector $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$. [1]

2.3 Enlarge the point $(2, 3)$ from the origin by scale factor 2. [1]

2.4 Describe fully the transformation that maps $(3, 2)$ onto $(-3, 2)$. [2]

2.5 State the two things you must give to describe a rotation fully, besides its name. [2]

3 Exam-style questions

3.1 The point $(4, 1)$ is rotated 90° anticlockwise about the origin. Find the image. [2]

3.2 A quadrilateral has vertices $(2, 1)$, $(5, 1)$, $(5, 3)$ and $(2, 3)$. After one transformation its image has vertices $(6, 3)$, $(15, 3)$, $(15, 9)$ and $(6, 9)$. State the name of this transformation and give the two further pieces of information needed to describe it fully. [3]

3.3 A shape is enlarged from the origin by scale factor $\frac{1}{2}$. The point $(6, 4)$ is on the shape. Find its image. [2]

3.4 Triangle P has vertices $(2, 1)$, $(6, 1)$ and $(2, 3)$.

(a) Triangle P is reflected in the line $x = 1$ to give triangle Q . Write the coordinates of the vertices of Q . [3]

(b) Triangle P is enlarged with centre $(0, 0)$ and scale factor $\frac{3}{2}$ to give triangle R . Write the coordinates of the vertices of R . [3]

(c) Triangle P is rotated 90° clockwise about the origin to give triangle S . Write the coordinates of the vertices of S . [3]

(d) Triangle T has vertices $(-2, -1)$, $(-6, -1)$ and $(-2, -3)$. Describe fully the single transformation that maps P onto T . [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $(4, -3)$.

2.2 $(2 + 3, 1 - 2) = (5, -1)$.

2.3 $(2 \times 2, 3 \times 2) = (4, 6)$.

2.4 A reflection in the y -axis, the line $x = 0$.

2.5 The angle of turn and the direction, and the centre of rotation.

3.1 the rule is $(x, y) \rightarrow (-y, x)$, so $(4, 1) \rightarrow (-1, 4)$.

3.2 An enlargement, scale factor 3, centre $(0, 0)$. (Each image coordinate is three times the object coordinate, so the centre is the origin.)

3.3 $(6 \times \frac{1}{2}, 4 \times \frac{1}{2}) = (3, 2)$.

3.4 (a) Reflecting in $x = 1$ sends (x, y) to $(2 - x, y)$; $(0, 1)$, $(-4, 1)$ and $(0, 3)$.

(b) Multiply each coordinate by $\frac{3}{2}$; $(3, 1.5)$, $(9, 1.5)$ and $(3, 4.5)$.

(c) A 90° clockwise rotation about the origin sends (x, y) to $(y, -x)$; $(1, -2)$, $(1, -6)$ and $(3, -2)$.

(d) A rotation of 180° about the origin $(0, 0)$. (An enlargement of scale factor -1 , centre the origin, is also accepted.)