IGCSE MATHEMATICS • EXERCISE SHEET

E6.6 Pythagoras' theorem and trigonometry in 3D

Name: _____ Class: _____ Date: _____ **Total: 17 marks**

KEY WORDS space diagonal 体对角线 • slant edge 侧棱 • apex 顶点 • plane 平面 • vertical height 垂直高度

Objective

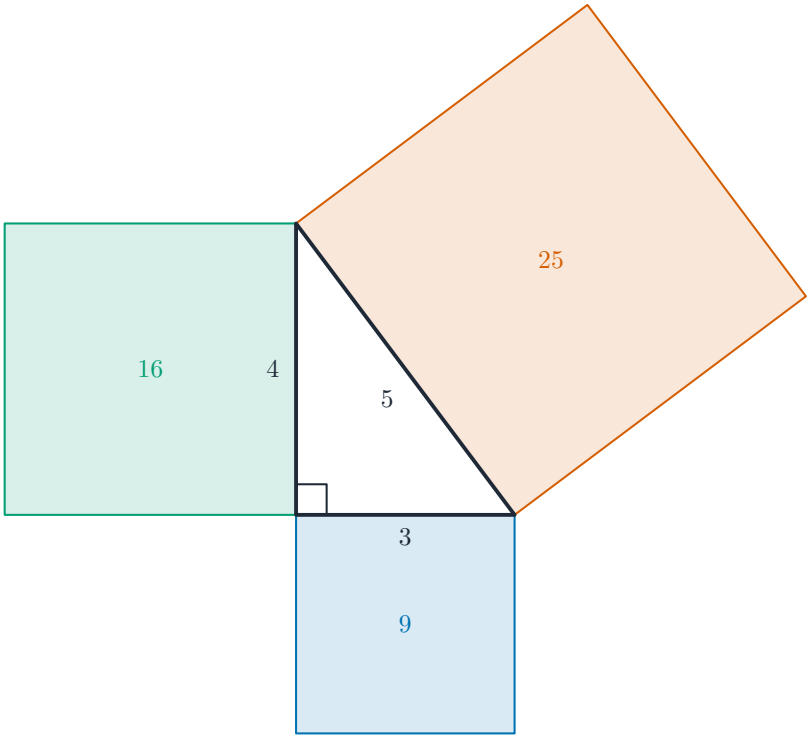
Build the skills to answer Extended exam questions on **Pythagoras' theorem and trigonometry in 3D** 三维中的勾股定理与三角学 — finding the **space diagonal** 体对角线 of a cuboid, the **slant edge** 侧棱 of a pyramid, and the **angle between a line and a plane** 直线与平面的夹角.

You must be able to:

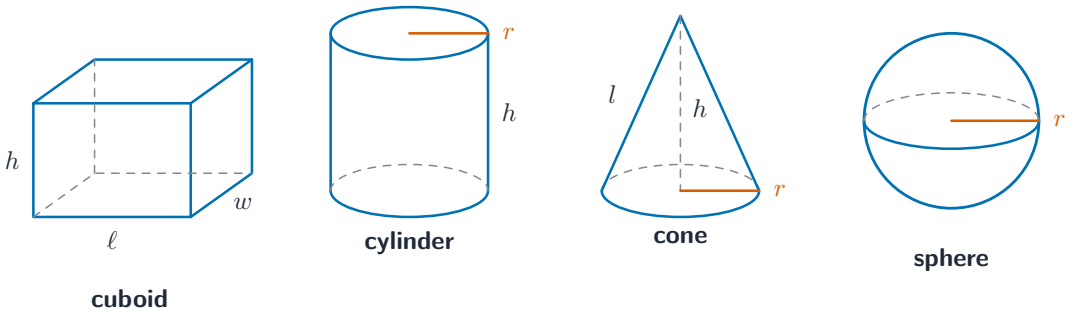
- find a length in three dimensions by using Pythagoras twice
- identify the right-angled triangle inside a 3D solid and draw it separately
- find the angle between an edge and a base

1 Worked examples

■ Draw the 2D triangle out of the 3D picture



Every 3D question comes down to a 2D right-angled triangle you have to find



Work on the base first, then stand the height up on it

- **Two steps, always:** find a length across the BASE first, then use it with the vertical height.
- **Cuboid space diagonal:** $\sqrt{l^2 + w^2 + h^2}$ — which is just Pythagoras done twice, with the base diagonal in the middle.
- **Angle with the base:** the right angle is between the vertical height and the base, so $\tan\theta = \dfrac{\text{height}}{\text{horizontal distance}}$.
- **Never round the intermediate length.** Keep it in the calculator; rounding the base diagonal first is what costs the final accuracy mark.

Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer): the first Pythagoras and the second are separate method marks, so show both lines.

2 Practice

2.1 A cuboid measures 6 cm by 8 cm by 10 cm. Find the length of the diagonal of the 6 cm by 8 cm face. [2]

2.2 Find the length of the space diagonal of that cuboid, correct to 3 significant figures. [3]

2.3 Find the angle between the space diagonal and the 6 cm by 8 cm base, correct to 1 decimal place. [3]

3 Exam-style questions

3.1 A pyramid has a square base of side 10 cm and a vertical height of 12 cm from the centre of the base to the apex.

(a) Find the distance from the centre of the base to one corner of the base, correct to 3 significant figures. [3]

(b) Find the length of a slant edge, correct to 3 significant figures. [3]

(c) Find the angle between a slant edge and the base, correct to 1 decimal place. [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

Each answer shows the steps, then the **result**.

2.1

- $\sqrt{6^2 + 8^2} = \sqrt{100}$

$$= \mathbf{10} \text{ cm}$$

2.2

- use the base diagonal 10 cm with the height 10 cm
- $\sqrt{10^2 + 10^2} = \sqrt{200}$

$$= 14.142\ldots = \mathbf{14.1} \text{ cm}$$

- equivalently $\sqrt{6^2 + 8^2 + 10^2}$ in one line

2.3

- the triangle has the base diagonal 10 cm and the height 10 cm, with the right angle between them
- $\tan\theta = \dfrac{10}{10} = 1$

$$\theta = \mathbf{45.0}^\circ$$

3.1

(a)

- the base diagonal $= \sqrt{10^2 + 10^2} = \sqrt{200} = 14.142\ldots$ cm
- the centre is halfway along it

$$14.142 \div 2 = \mathbf{7.07} \text{ cm}$$

(b)

- the slant edge, the height and the half-diagonal form a right-angled triangle
- slant $= \sqrt{12^2 + 7.071^2} = \sqrt{194}$

$$= 13.928\ldots = \mathbf{13.9} \text{ cm}$$

(c)

- $\tan\theta = \dfrac{12}{7.071}$

$$\theta = 59.49\ldots = \mathbf{59.5}^\circ$$

- the horizontal side is the HALF-diagonal, not the full one and not the side of the square