

E6.5 Non-right-angled triangles

Name: _____ Class: _____ Date: _____

Total: 16 marks

KEY WORDS sine rule 正弦定理 • cosine rule 余弦定理 • included angle 夹角
 • ambiguous case 两解情形 • obtuse 钝角

Objective

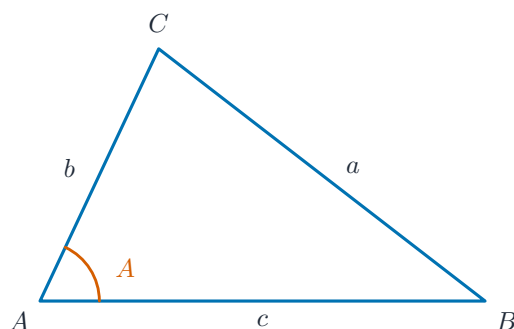
Build the skills to answer Extended exam questions on **non-right-angled triangles** 非直角三角形 — choosing between the **sine rule** 正弦定理 and the **cosine rule** 余弦定理, and using the **area formula** $\frac{1}{2}ab \sin C$ 面积公式.

You must be able to:

- choose the correct rule from what the question gives you
- use the sine rule to find a side or an angle
- use the cosine rule to find a third side, and use $\frac{1}{2}ab \sin C$ for an area

1 Worked examples

■ Which rule? Count what you are given



The angle in the cosine rule sits between the two known sides

- **Sine rule:** $\frac{a}{\sin A} = \frac{b}{\sin B}$. Use it when you have a side with its OPPOSITE angle, plus one more piece.
- **Cosine rule:** $a^2 = b^2 + c^2 - 2bc \cos A$. Use it for two sides and the angle BETWEEN them, or for all three sides.
- **Area:** $\frac{1}{2}ab \sin C$, where C is the angle between the two sides a and b .
- **The ambiguous case:** when the sine rule gives an angle, 180° minus that angle also has the same sine. Check which fits the triangle.

Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer): quoting the rule with the numbers substituted is the method mark, and the rearranged line is a second one.

2 Practice

2.1 In triangle ABC , $a = 9$ cm, $A = 40^\circ$ and $B = 65^\circ$. Find b , correct to 3 significant figures. [3]

2.2 In triangle PQR , $p = 7$ cm, $q = 9$ cm and $R = 52^\circ$. Find r , correct to 3 significant figures. [3]

2.3 Find the area of a triangle with sides 10 cm and 14 cm and an included angle of 38° , correct to 3 significant figures. [2]

3 Exam-style questions

3.1 In triangle ABC , $AB = 8$ cm, $BC = 11$ cm and angle $ABC = 73^\circ$.

(a) Find AC , correct to 3 significant figures. [3]

(b) Find the area of the triangle, correct to 3 significant figures. [2]

3.2 In triangle XYZ , $x = 6.2$ cm, $y = 9.4$ cm and angle $X = 35^\circ$. Find the acute value of angle Y , correct to 1 decimal place. [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

Each answer shows the steps, then the **result**.

2.1

- sine rule: $\frac{b}{\sin 65^\circ} = \frac{9}{\sin 40^\circ}$
- $b = \frac{9 \sin 65^\circ}{\sin 40^\circ}$
 $= 12.689 \dots = \mathbf{12.7 \text{ cm}}$

2.2

- cosine rule, since the angle lies between the two known sides
- $r^2 = 7^2 + 9^2 - 2(7)(9) \cos 52^\circ = 130 - 126 \cos 52^\circ$
- $r^2 = 52.43 \dots$
 $r = \mathbf{7.24 \text{ cm}}$
- take the square root at the very end, never of the separate terms

2.3

- area $= \frac{1}{2} \times 10 \times 14 \times \sin 38^\circ$
 $= 43.096 \dots = \mathbf{43.1 \text{ cm}^2}$

3.1

(a)

- the angle is between the two known sides, so use the cosine rule
- $AC^2 = 8^2 + 11^2 - 2(8)(11) \cos 73^\circ = 185 - 176 \cos 73^\circ$
- $AC^2 = 133.54 \dots$
 $AC = \mathbf{11.6 \text{ cm}}$

(b)

- area $= \frac{1}{2} \times 8 \times 11 \times \sin 73^\circ$
 $= 42.077 \dots = \mathbf{42.1 \text{ cm}^2}$

3.2

- sine rule: $\frac{\sin Y}{9.4} = \frac{\sin 35^\circ}{6.2}$
- $\sin Y = \frac{9.4 \sin 35^\circ}{6.2} = 0.8696 \dots$
 $Y = \mathbf{60.4^\circ}$
- the obtuse solution $180 - 60.4 = 119.6^\circ$ also has this sine; the question asked for the acute value