IGCSE MATHEMATICS • EXERCISE SHEET

E6.4 Trigonometric functions

Name: ____________ Class: ________ Date: __________ **Total: 24 marks**

KEY WORDS exact values 精确值 • sine and cosine rules 正弦和余弦定理 • trigonometric equation 三角方程 • trigonometric functions 三角函数 • sine 正弦

Objective

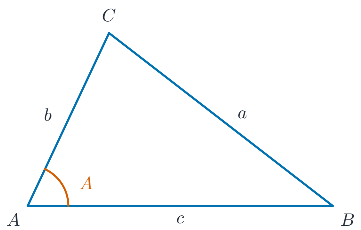
Build the skills to answer Extended exam questions on **trigonometric functions** 三角函数 — **exact values** 精确值, solving **trigonometric equations** 三角方程, and the **sine and cosine rules** 正弦和余弦定理.

You must be able to:

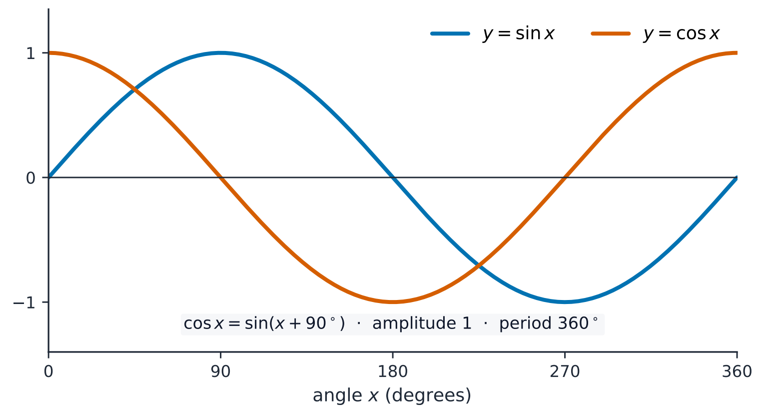
- recall exact values of sin, cos, tan for 0–90°
- use the cosine rule and the sine rule in any triangle
- find the area of a triangle with $\frac{1}{2}ab\sin C$

1 Worked examples

■ The sine and cosine rules



For any triangle, not just right-angled ones



The sine and cosine function graphs

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc\cos A \qquad \text{area} = \tfrac{1}{2}ab\sin C$$

- **Cosine rule (two sides + included angle):** $b = 7$, $c = 8$, $A = 40^\circ \to a^2 = 49 + 64 - 112\cos 40^\circ = 27.2 \to a = 5.2$ cm.
- **Exact:** $\sin 60^\circ = \frac{\sqrt{3}}{2}$; solve $\sin x = \frac{\sqrt{3}}{2}$ (0–360°) $\to x = 60^\circ$ or 120°.

Answer shapes: M1 a correct method, **A1** the correct answer.

2 Practice

2.1 Write down the exact value of $\cos 30^\circ$. [1]

2.2 Solve $\sin x = \frac{1}{2}$ for $0^\circ \leqslant x \leqslant 360^\circ$. [2]

2.3 Find the area of a triangle with sides $a = 6$ cm, $b = 9$ cm and included angle $C = 50^\circ$. [2]

3 Exam-style questions

3.1 A triangle has $b = 5$ cm, $c = 7$ cm and the angle between them $A = 60°$. Find side a. [3]

3.2 In a triangle, $a = 8$ cm, angle $A = 40°$ and angle $B = 75°$. Use the sine rule to find side b. [3]

3.3 (a) Write down the exact values of $\sin 30°$, $\cos 45°$ and $\tan 60°$. [3]

(b) Solve $2\sin x = 1$ for $0° \le x \le 360°$. [3]

(c) Solve $\cos x = -0.5$ for $0° \le x \le 360°$. [3]

(d) Sketch $y = \sin x$ for $0° \le x \le 360°$, mark its maximum and minimum, and use the sketch to explain why $\sin x = 1.4$ has no solution. [4]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $\cos 30° = \dfrac{\sqrt{3}}{2}$.

2.2 $x = 30°$ or $x = 180 - 30 = 150°$.

2.3 area $= \frac{1}{2}(6)(9)\sin 50° = 27 \times 0.766 = 20.7$ cm^2.

3.1 $a^2 = 5^2 + 7^2 - 2(5)(7)\cos 60° = 25 + 49 - 70(0.5) = 39$; $a = \sqrt{39} = 6.24$ cm.

3.2 $\dfrac{b}{\sin 75°} = \dfrac{8}{\sin 40°} \to b = \dfrac{8\sin 75°}{\sin 40°} = \dfrac{8 \times 0.966}{0.643} = 12.0$ cm.

3.3 (a) $\sin 30° = \dfrac{1}{2}$, $\cos 45° = \dfrac{\sqrt{2}}{2}$ (or $\dfrac{1}{\sqrt{2}}$), $\tan 60° = \sqrt{3}$.

(b) $\sin x = \dfrac{1}{2}$; the first solution is $x = 30°$; sine is also positive in the second quadrant, so $x = 180 - 30 = 150°$.

(c) The related acute angle is $\cos^{-1} 0.5 = 60°$; cosine is negative in the second and third quadrants; $x = 180 - 60 = 120°$ and $x = 180 + 60 = 240°$.

(d) A correct sine curve starting at $(0, 0)$, rising to $(90, 1)$, back through $(180, 0)$, down to $(270, -1)$ and back to $(360, 0)$; maximum and minimum marked. The curve never rises above $y = 1$, so no value of x gives $\sin x = 1.4$ — the sine of an angle always lies between -1 and 1.