

E6.3 Exact trigonometric values

Name: _____ Class: _____ Date: _____

Total: 11 marks

KEY WORDS exact value 精确值 • surd 根式 • hypotenuse 斜边
 • equilateral triangle 等边三角形 • quadrant 象限

Objective

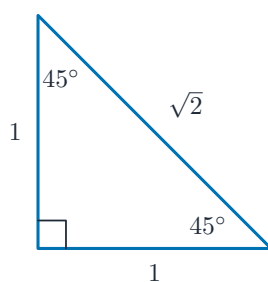
Build the skills to answer Extended exam questions on **exact trigonometric values** 三角函数的精确值 — the **sine** 正弦, **cosine** 余弦 and **tangent** 正切 of 0° , 30° , 45° , 60° and 90° , and using them to leave an answer as a **surd** 根式 rather than a decimal.

You must be able to:

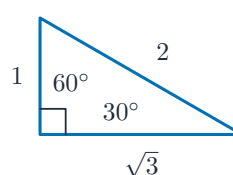
- write down the exact value of $\sin$, $\cos$ and $\tan$ for the standard angles
- use an exact value to give a length or an area in surd form
- solve a simple trigonometric equation over a given range

1 Worked examples

■ Two triangles give every exact value you need



45° triangle



30°–60° triangle

A square cut in half, and an equilateral triangle cut in half

θ	$\sin \theta$	$\cos \theta$	$\tan \theta$
0°	0	1	0
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
90°	1	0	undefined

- **Notice the symmetry:** $\sin \theta$ and $\cos \theta$ swap as θ goes from 30° to 60° , and $\tan 45^\circ = 1$ because the two short sides are equal.
- **”Exact” means no decimals.** $\frac{\sqrt{3}}{2}$, not 0.866.
- **Two answers:** $\sin$ is positive in the first AND second quadrant, so $\sin x = \frac{\sqrt{2}}{2}$ has solutions 45° and 135° in $0^\circ \leq x \leq 180^\circ$.

Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer): quoting the exact value is a method mark, and keeping the surd to the end is the answer mark.

2 Practice

2.1 Write down the exact value of $\sin 60^\circ$. [1]

2.2 Write down the exact value of $\tan 45^\circ$. [1]

2.3 Write down the exact value of $\cos 30^\circ$. [1]

3 Exam-style questions

3.1 A right-angled triangle has a hypotenuse of 12 cm and an angle of 30° . Find the exact length of the side opposite the 30° angle. [2]

3.2 Find the exact area of an equilateral triangle of side 8 cm, in the form $a\sqrt{3}$ cm². [4]

3.3 Solve $\sin x = \frac{\sqrt{2}}{2}$ for $0^\circ \leq x \leq 180^\circ$. [2]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

Each answer shows the steps, then the **result**.

2.1

$$\frac{\sqrt{3}}{2}$$

2.2

$$1$$

2.3

$$\frac{\sqrt{3}}{2}$$

3.1

- opposite = hypotenuse $\times \sin 30^\circ = 12 \times \frac{1}{2}$
 $= 6 \text{ cm}$

3.2

- split the triangle down the middle to make a 30–60–90 triangle
- the height = $8 \sin 60^\circ = 8 \times \frac{\sqrt{3}}{2} = 4\sqrt{3} \text{ cm}$
- area = $\frac{1}{2} \times 8 \times 4\sqrt{3}$
 $= 16\sqrt{3} \text{ cm}^2$

3.3

- $\sin 45^\circ = \frac{\sqrt{2}}{2}$, so $x = 45^\circ$
- sin is also positive in the second quadrant, so $x = 180 - 45$

$$x = 45^\circ \text{ or } 135^\circ$$