IGCSE MATHEMATICS • EXERCISE SHEET

E5.5 Compound shapes and parts of shapes

Name: _____ Class: _____ Date: _____ **Total: 20 marks**

KEY WORDS sector 扇形 • arc length 弧长 • compound shape 组合图形 • semicircle 半圆 • segment 弓形

Objective

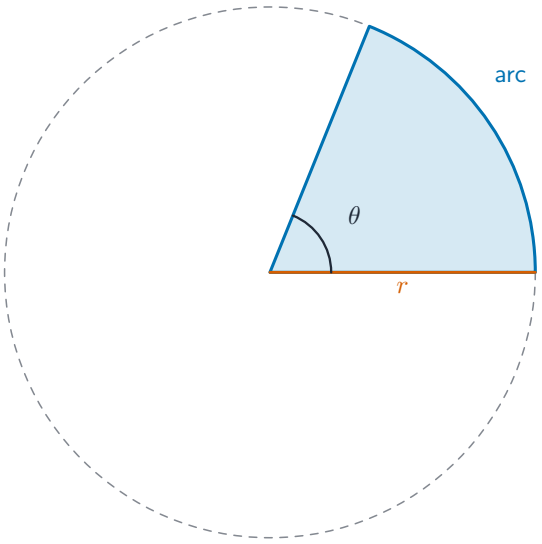
Build the skills to answer Extended exam questions on **compound shapes and parts of shapes** 组合图形与图形的一部分 — finding the **arc length** 弧长 and **sector area** 扇形面积, splitting a **compound shape** 组合图形 into pieces, and working backwards to find a sector angle.

You must be able to:

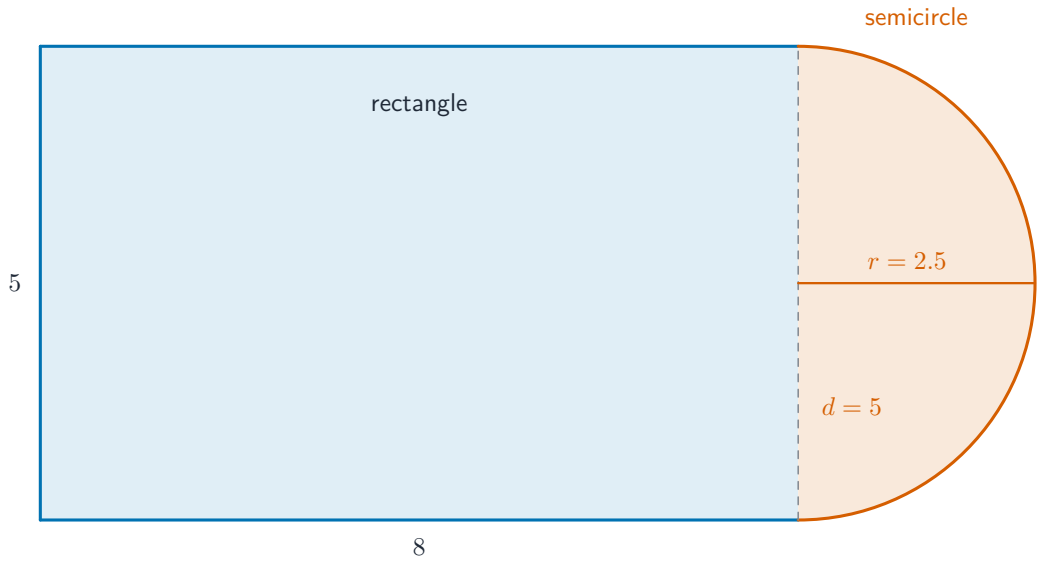
- find an arc length and a sector area using the fraction $\frac{\theta}{360}$
- split a compound shape into rectangles, triangles and parts of circles
- work backwards from a given arc length or sector area to the angle or the radius

1 Worked examples

■ A sector is a fraction of the whole circle



The fraction $\theta/360$ scales both the circumference and the area



Split it, work out each piece, then add — or subtract if a piece is cut out

- **Arc length** $= \dfrac{\theta}{360} \times 2\pi r$. For $r = 10$ and $\theta = 72°$ that is 12.6 cm.
- **Sector area** $= \dfrac{\theta}{360} \times \pi r^2$. Same fraction, different formula.
- **Compound shapes:** draw the split lines on the diagram and label each piece. A perimeter follows only the OUTSIDE edge, so the internal split lines are not counted.
- **Backwards:** from a sector area, $\theta = \dfrac{A \times 360}{\pi r^2}$.

Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer): the fraction $\theta/360$ and the circle formula are separate method marks, so show both.

2 Practice

2.1 A sector of a circle has radius 10 cm and angle 72°. Find its arc length, correct to 3 significant figures. [3]

2.2 Find the area of the same sector, correct to 3 significant figures. [3]

2.3 A shape is made from a rectangle 10 cm by 6 cm with a semicircle of diameter 6 cm joined to one short end. Find the total area, correct to 3 significant figures. [3]

3 Exam-style questions

3.1 A running track is made from a rectangle 84 m long and 70 m wide, with a semicircle of diameter 70 m at each end.

(a) Find the perimeter of the track, correct to 3 significant figures. [4]

(b) Find the area enclosed, correct to 3 significant figures. [4]

3.2 A sector of a circle of radius 8 cm has an area of 33.5 cm^2. Find the angle of the sector, correct to the nearest degree. [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

Each answer shows the steps, then the **result**.

2.1

- fraction $= \dfrac{72}{360} = \dfrac{1}{5}$
- arc $= \dfrac{1}{5} \times 2 \times \pi \times 10$

$$= 12.566\ldots = \mathbf{12.6}\text{ cm}$$

2.2

- area $= \dfrac{1}{5} \times \pi \times 10^2$

$$= 62.83\ldots = \mathbf{62.8}\text{ cm}^\mathbf{2}$$

2.3

- rectangle $= 10 \times 6 = 60$ cm^2
- semicircle of RADIUS 3: $\frac{1}{2} \times \pi \times 3^2 = 14.137\ldots$

$$60 + 14.137 = \mathbf{74.1}\text{ cm}^\mathbf{2}$$

- the diameter is 6, so the radius is 3 — halving it is the step most often missed

3.1

(a)

- the two semicircles make one whole circle of diameter 70 m
- circle circumference $= \pi \times 70 = 219.91\ldots$ m
- the two straight sides are 84 m each; the ends are curved, so the 70 m widths are NOT part of the perimeter

$$2 \times 84 + 219.91 = 387.9\ldots = \mathbf{388}\text{ m}$$

(b)

- rectangle $= 84 \times 70 = 5880$ m^2
- the two semicircles make a circle of radius 35 m
- circle area $= \pi \times 35^2 = 3848.45\ldots$ m^2

$$5880 + 3848.45 = 9728.4\ldots = \mathbf{9730}\text{ m}^\mathbf{2}$$

3.2

- $33.5 = \dfrac{\theta}{360} \times \pi \times 8^2$
- $\theta = \dfrac{33.5 \times 360}{\pi \times 64}$

$$= 59.98\ldots = \mathbf{60}^\circ$$