IGCSE MATHEMATICS • EXERCISE SHEET

E5.4 Surface area and volume

Name: ____________ Class: ________ Date: __________ **Total: 33 marks**

KEY WORDS cylinder 圆柱 • cuboid 长方体 • surface area 表面积 • sphere 球 • cross-section 横截面

Objective

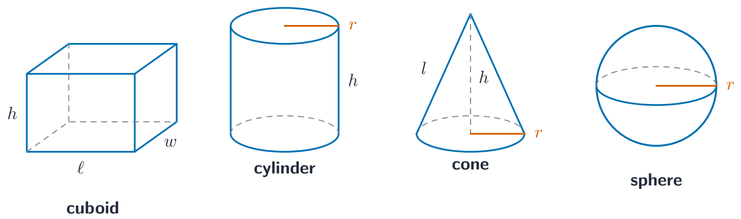
Build the skills to answer Extended exam questions on **surface area and volume 表面积和体积** — the **cylinder, cone and sphere 圆柱、圆锥和球**, and parts of solids such as a **hemisphere 半球**.

You must be able to:

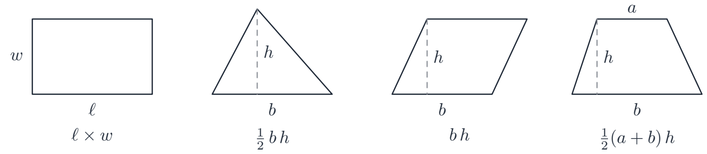
- find the volume of a cylinder, cone, sphere and pyramid
- find the surface area of a cylinder and cone
- work with parts of solids (hemispheres)

1 Worked examples

Volumes and surface areas



Learn the key formulas — some are on the formula list



Face areas build up the surface area of a solid

$$\text{cylinder: } V = \pi r^2 h,\quad \text{SA} = 2\pi r h + 2\pi r^2 \qquad \text{cone: } V = \tfrac{1}{3}\pi r^2 h$$

$$\text{sphere: } V = \tfrac{4}{3}\pi r^3,\quad \text{SA} = 4\pi r^2 \qquad \text{hemisphere: } V = \tfrac{2}{3}\pi r^3$$

- **Worked (cylinder $r = 5$, $h = 10$):** $V = \pi(25)(10) = 250\pi$; SA $= 100\pi + 50\pi = 150\pi$ cm^2.

Answer shapes: M1 a correct method, **A1** the correct answer.

The formulas, and the three traps

Solid	Volume	Curved / total surface area
cylinder	$\pi r^2 h$	curved $2\pi r h$; total $2\pi r h + 2\pi r^2$
cone	$\frac{1}{3}\pi r^2 h$	curved $\pi r l$; total $\pi r l + \pi r^2$
sphere	$\frac{4}{3}\pi r^3$	$4\pi r^2$
prism	area of cross-section × length	perimeter of cross-section × length, plus the two ends

Trap 1 — the cone uses the SLANT height. The curved surface area needs l, not the vertical height h. If you are given h and r, find l by Pythagoras: $l = \sqrt{r^2 + h^2}$. Using h in $\pi r l$ is the single most common error on this topic.

Trap 2 — read "curved" against "total". A closed cylinder has two circular ends; an open one (a pipe, a tank with no lid) has one or none. Count the faces the object actually has before adding anything.

Trap 3 — a hemisphere's total surface area is not half a sphere's. Cutting a sphere in half gives $\frac{1}{2}(4\pi r^2) = 2\pi r^2$ of curved surface **plus** the flat circular face πr^2, so the total is $3\pi r^2$.

Composite solids — split the shape, work out each part, then add or subtract. Say which you are doing, because that reasoning carries a mark.

Worked example. A solid is a cylinder of radius 5 cm and height 12 cm with a cone of the same radius and height 12 cm on top. Find its total volume and its total surface area.

1. Volume of cylinder $= \pi(5)^2(12) = 300\pi$.
2. Volume of cone $= \frac{1}{3}\pi(5)^2(12) = 100\pi$; total $= 400\pi = 1260$ cm^3 (3 s.f.).

3. Slant height $l = \sqrt{5^2 + 12^2} = 13$.
4. Surface area: base circle $\pi(5)^2 = 25\pi$, curved cylinder $2\pi(5)(12) = 120\pi$, curved cone $\pi(5)(13) = 65\pi$. The join is inside the solid, so it is **not** a surface.
5. Total $= 210\pi = 660$ cm^2 (3 s.f.).

2 Practice

2.1 Find the volume of a cuboid 4 cm by 3 cm by 5 cm. [1]

2.2 Find the volume of a cylinder of radius 3 cm and height 10 cm, in terms of π. [2]

2.3 Find the volume of a sphere of radius 3 cm, in terms of π. [2]

2.4 Find the curved surface area of a cone with radius 6 cm and slant height 10 cm. [2]

2.5 A cone has radius 8 cm and vertical height 15 cm. Find its slant height. [2]

2.6 Show that the total surface area of a solid hemisphere of radius r is $3\pi r^2$. [2]

3 Exam-style questions

3.1 A cone has radius 6 cm and height 8 cm. Find its volume, in terms of π. [2]

3.2 Find the total surface area of a cylinder of radius 4 cm and height 9 cm, in terms of π. [3]

3.3 A solid hemisphere has radius 6 cm. Find its volume, in terms of π. [2]

3.4 A grain silo is a cylinder of radius 3 m and height 8 m, with a cone of the same radius and vertical height 4 m on top. It is open at the bottom.

(a) Calculate the volume of the silo. Give your answer to 3 significant figures. [4]

(b) Calculate the slant height of the cone. [2]

(c) Calculate the total area of metal needed to make the silo, given that it has no base. [4]

(d) The silo is filled with grain of density 0.75 g/cm^3. Calculate the mass of grain, in tonnes. [3]

(e) A second silo is mathematically similar, with all lengths twice as large. State the ratio of their volumes and of their surface areas. [2]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $4 \times 3 \times 5 = 60$ cm^3.

2.2 $\pi \times 3^2 \times 10 = 90\pi$ cm^3.

2.3 $\frac{4}{3}\pi \times 3^3 = \frac{4}{3}\pi \times 27 = 36\pi$ cm^3.

2.4 $\pi r l = \pi(6)(10) = 60\pi = 188$ cm^2.

2.5 $l = \sqrt{8^2 + 15^2} = \sqrt{289} = 17$ cm.

2.6 Curved surface $= \frac{1}{2}(4\pi r^2) = 2\pi r^2$; plus the flat circular face πr^2, giving $3\pi r^2$.

3.1 $\frac{1}{3}\pi \times 6^2 \times 8 = \frac{1}{3}\pi \times 288 = 96\pi$ cm^3.

3.2 SA $= 2\pi(4)(9) + 2\pi(4)^2 = 72\pi + 32\pi = 104\pi$ cm^2.

3.3 $\frac{2}{3}\pi \times 6^3 = \frac{2}{3}\pi \times 216 = 144\pi$ cm^3.

3.4 (a) Cylinder: $\pi(3)^2(8) = 72\pi$; cone: $\frac{1}{3}\pi(3)^2(4) = 12\pi$; total $= 84\pi = 264$ m^3.
(b) $l = \sqrt{3^2 + 4^2} = 5$ m.
(c) Curved cylinder $= 2\pi(3)(8) = 48\pi$; curved cone $= \pi(3)(5) = 15\pi$; no base and the join is internal, so total $= 63\pi = 198$ m^2.
(d) 264 m^3 $= 264 \times 10^6$ cm^3; mass $= 264 \times 10^6 \times 0.75 = 1.98 \times 10^8$ g $= 198$ tonnes.
(e) Volumes 1 : 8; surface areas 1 : 4.