

E4.8 Circle theorems II

Name: _____ Class: _____ Date: _____

Total: 11 marks

KEY WORDS tangent 切线 • point of contact 切点 • alternate segment 弦切角所对的弓形
• chord 弦 • perpendicular 垂直

Objective

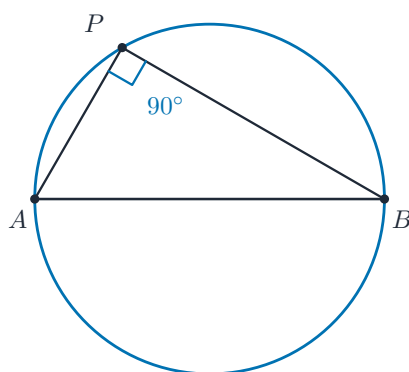
Build the skills to answer Extended exam questions on the second group of **circle theorems** 圆的定理 — the **tangent–radius** 切线与半径 right angle, **equal tangents** 切线等长 from an external point, the **alternate segment theorem** 弦切角定理, and the chord symmetry properties.

You must be able to:

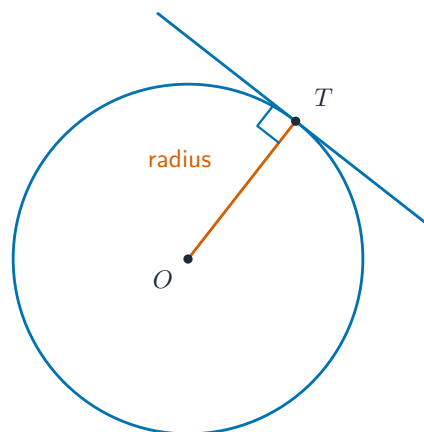
- use the fact that a tangent meets a radius at 90°
- use the fact that two tangents from the same external point are equal
- use the alternate segment theorem, and the perpendicular-from-centre property of a chord

1 Worked examples

■ Name the theorem, then do the arithmetic

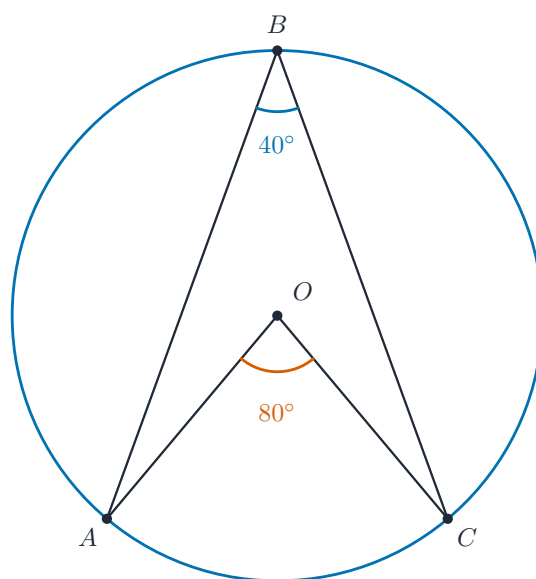


angle in a
semicircle = 90°



tangent meets
radius = 90°

A tangent is perpendicular to the radius at the point of contact



angle at centre = $2 \times$ angle at circumference

Every circle answer must quote the theorem it used

- **Tangent and radius:** the angle between them at the point of contact is 90° .
- **Equal tangents:** if TA and TB are tangents from T , then $TA = TB$, and OT bisects angle ATB .
- **Alternate segment:** the angle between a tangent and a chord equals the angle in the alternate segment — the angle “on the other side” of the chord.
- **Chord:** the perpendicular from the centre bisects the chord, which turns a chord question into Pythagoras.

Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer): the NAME of the theorem is a mark. “Angle $OAT = 90^\circ$ (tangent meets radius)” earns more than the number alone.

2 Practice

2.1 TA and TB are tangents from T to a circle. Given that $TA = 7$ cm, write down the length of TB and name the property used. [2]

2.2 Write down the size of the angle between a tangent and the radius drawn to the point of contact. [1]

2.3 The angle between a tangent at A and a chord AB is 58° . Write down the angle in the alternate segment. [1]

3 Exam-style questions

3.1 TA and TB are tangents from T to a circle with centre O , and angle $ATB = 46^\circ$.

(a) Write down the size of angle OAT , giving a reason. [1]

(b) Find the size of angle AOB . [3]

3.2 A chord PQ of length 24 cm is drawn in a circle of radius 13 cm. Find the distance from the centre to the chord. [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

Each answer shows the steps, then the **result**.

2.1

- tangents from an external point are equal in length

$$TB = \mathbf{7 \text{ cm}}$$

2.2

-

$$\mathbf{90^\circ}$$

2.3

- the alternate segment theorem gives the same angle

$$= \mathbf{58^\circ}$$

3.1

(a)

- the tangent meets the radius at the point of contact

$$\mathbf{90^\circ}$$

(b)

- $OATB$ is a quadrilateral, so its angles add to 360°
- angles OAT and OBT are both 90°

$$360 - 90 - 90 - 46 = \mathbf{134^\circ}$$

3.2

- the perpendicular from the centre bisects the chord, so half of it is 12 cm
- Pythagoras with the radius as the hypotenuse: $d = \sqrt{13^2 - 12^2}$

$$= \sqrt{25} = \mathbf{5 \text{ cm}}$$