IGCSE MATHEMATICS • EXERCISE SHEET

E4.7 Circle theorems I

Name: ____________ Class: ________ Date: __________ **Total: 20 marks**

KEY WORDS cyclic quadrilateral 圆内接四边形 • circle theorems 圆定理 • alternate segment theorem 弦切角定理 • circle 圆 • centre 圆心

Objective

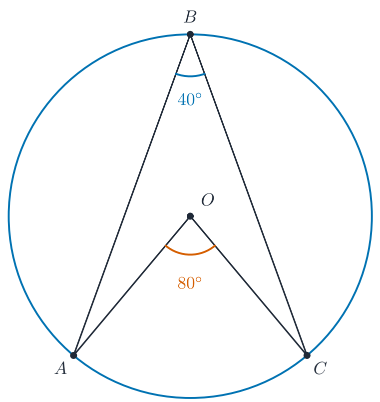
Build the skills to answer Extended exam questions on **circle theorems** 圆定理 — the angle at the **centre**, angles in the **same segment**, the **cyclic quadrilateral** 圆内接四边形, and the semicircle/tangent facts. Always **give a reason**.

You must be able to:

- use "angle at the centre = twice the angle at the circumference"
- use "angles in the same segment are equal" and the cyclic-quadrilateral rule
- use the semicircle and tangent–radius right angles

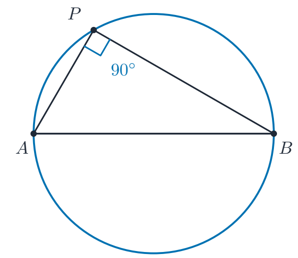
1 Worked examples

■ Circle theorems

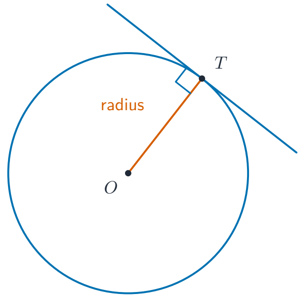


angle at centre = $2\times$ angle at circumference

Standing on the same arc



angle in a semicircle $= 90°$



tangent meets radius $= 90°$

Further circle theorems: angle in a semicircle and the tangent

- Angle at **centre** $= 2\times$ angle at **circumference** (same arc).
- Angles in the **same segment** are equal; opposite angles of a **cyclic quadrilateral** add to 180°.
- Angle in a **semicircle** $= 90°$; tangent meets radius at 90°.
- **Worked:** circumference angle $40° \to$ centre angle $= 2 \times 40 = 80°$.

Answer shapes: M1 a correct method, **A1** the correct answer.

2 Practice

2.1 The angle at the circumference standing on an arc is 35°. Find the angle at the centre on the same arc. [1]

2.2 A triangle is drawn with one side a diameter. State the angle at the point on the circle. [1]

2.3 In a cyclic quadrilateral, one angle is 85°. Find the opposite angle. [2]

3 Exam-style questions

3.1 A, B, C lie on a circle, centre O. The angle AOC at the centre is $130°$. Find the angle ABC at the circumference, giving a reason. [2]

3.2 $PQRS$ is a cyclic quadrilateral. Angle $PQR = 95°$ and angle $QRS = 70°$. Find angles PSR and SPQ. [3]

3.3 A, B, C and D lie on a circle with centre O. BD is a diameter. Angle $BAC = 32°$ and angle $ADB = 54°$.

(a) Write down, with a reason, the size of angle BCD. [2]

(b) Find angle BDC, giving your reason. [2]

(c) Find angle ABD, showing your working. [2]

(d) Find angle AOC, giving a reason. [2]

(e) TC is a tangent to the circle at C. Find the angle between TC and the chord CD, naming the theorem you use. [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $2 \times 35 = 70°$.

2.2 90° (angle in a semicircle).

2.3 opposite angles of a cyclic quadrilateral add to 180°: $180 - 85 = 95°$.

3.1 angle $ABC = \frac{1}{2} \times 130 = 65°$ (reason: angle at the centre is twice the angle at the circumference).

3.2 opposite angles add to 180°: $PSR = 180 - 95 = 85°$; $SPQ = 180 - 70 = 110°$.

3.3 (a) 90° — the angle in a semicircle is a right angle, since BD is a diameter.
(b) Angle BDC and angle BAC are angles in the **same segment**, standing on the chord BC, so they are equal: 32°.
(c) In triangle ABD, angle $BAD = 90°$ (angle in a semicircle) and angle $ADB = 54°$; so angle $ABD = 180 - 90 - 54 = 36°$.
(d) Angle ABC is the angle at the circumference standing on arc AC, and angle AOC is the angle at the centre on the same arc, so $AOC = 2 \times ABC$. $ABC = ABD + DBC$; in triangle BCD, $DBC = 180 - 90 - 32 = 58°$, so $ABC = 36 + 58 = 94°$ and $AOC = 188°$ — the reflex angle; the angle marked at the centre on the other side is $360 - 188 = 172°$.
(e) By the **alternate segment theorem**, the angle between the tangent TC and the chord CD equals the angle in the alternate segment, angle CBD; $CBD = 58°$ from part (d), so the required angle is 58°.