

E4.4 Similarity

Name: _____ Class: _____ Date: _____ **Total: 21 marks**

KEY WORDS similar 相似 • similar shapes 相似形 • scale factor 比例因子
• area and volume 面积和体积 • triangle 三角形

Objective

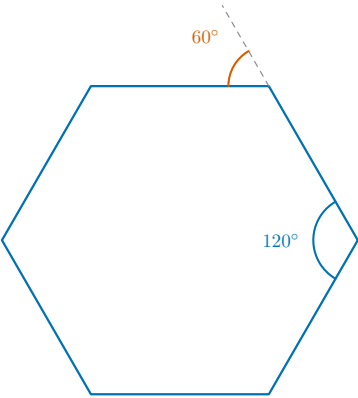
Build the skills to answer Extended exam questions on **similarity** 相似 — **similar shapes** 相似形, the **scale factor** 比例因子, and the **area and volume** 面积和体积 ratios k^2 and k^3 .

You must be able to:

- find a missing length in similar shapes
- use the area ratio k^2 for similar shapes
- use the volume ratio k^3 for similar solids

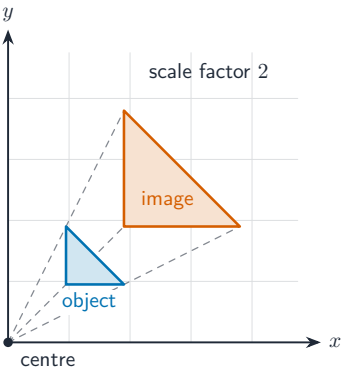
1 Worked examples

- Similar shapes and solids



interior + exterior = 180°

Similar shapes have equal angles and sides in the same ratio



Similar shapes are linked by an enlargement

- **Similar** 相似: same angles, all sides $\times$ the **scale factor** 比例因子 k .
- $\frac{\text{area } A}{\text{area } B} = k^2$; $\frac{\text{volume } A}{\text{volume } B} = k^3$.
- **Worked:** lengths 2 : 3, smaller volume 40 cm³ $\rightarrow$ ratio 8 : 27 $\rightarrow$ larger = $40 \times \frac{27}{8} = 135$ cm³.

Answer shapes: **M1** a correct method, **A1** the correct answer.

2 Practice

2.1 Two similar triangles have a scale factor of 3. A side of the small triangle is 4 cm. Find the matching side of the large triangle. [1]

2.2 Two similar shapes have lengths in the ratio 2 : 5. Write down the ratio of their areas. [1]

2.3 Two similar solids have lengths in the ratio 1 : 2. Write down the ratio of their volumes. [1]

3 Exam-style questions

3.1 Two similar shapes have lengths in the ratio 3 : 4. The area of the smaller is 18 cm². Find the area of the larger. [3]

3.2 Two similar bottles have heights 10 cm and 15 cm. The smaller holds 400 ml. Find the volume of the larger. [3]

3.3 Triangle ABC has $AB = 6$ cm, $BC = 9$ cm and area 20 cm^2 . Triangle PQR is similar to ABC , with PQ corresponding to AB and $PQ = 15$ cm.

(a) Write down the scale factor from ABC to PQR , and find QR . [3]

(b) Find the area of triangle PQR . [3]

(c) A solid cone has a volume of 54 cm^3 . A mathematically similar cone has a volume of 128 cm^3 . Find the ratio of their heights. [3]

(d) Explain why two shapes with equal angles must be similar, but two shapes with equal areas need not be. [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $4 \times 3 = 12$ cm.

2.2 $2^2 : 5^2 = 4 : 25$.

2.3 $1^3 : 2^3 = 1 : 8$.

3.1 area ratio $= 3^2 : 4^2 = 9 : 16 \rightarrow \text{larger} = 18 \times \frac{16}{9} = 32$ cm².

3.2 length ratio $10 : 15 = 2 : 3 \rightarrow \text{volume ratio } 2^3 : 3^3 = 8 : 27 \rightarrow \text{larger} = 400 \times \frac{27}{8} = 1350$ ml.

3.3 (a) scale factor $= \frac{15}{6} = 2.5$; $QR = 9 \times 2.5 = 22.5$ cm.

(b) Areas scale by the **square** of the length scale factor; $2.5^2 = 6.25$; area $= 20 \times 6.25 = 125$ cm².

(c) Volumes scale by the cube of the length scale factor, so $\left(\frac{h_2}{h_1}\right)^3 = \frac{128}{54} = \frac{64}{27}$;

$\frac{h_2}{h_1} = \sqrt[3]{\frac{64}{27}} = \frac{4}{3}$, so the ratio is 3 : 4.

(d) Equal angles fix the **shape**, so one figure is an enlargement of the other and corresponding sides are in a fixed ratio. Equal areas fix only the amount of space enclosed — a 2×8 rectangle and a 4×4 square both have area 16 but different shapes, so they are not similar.