

## E3.7 Perpendicular lines

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

Total: 14 marks

**KEY WORDS** perpendicular 垂直 • negative reciprocal 负倒数  
 • perpendicular bisector 垂直平分线 • midpoint 中点 • gradient 斜率

### Objective

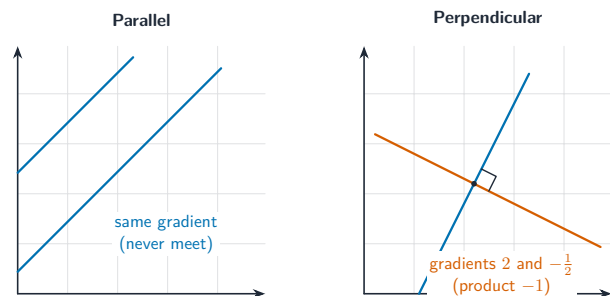
Build the skills to answer Extended exam questions on **perpendicular lines** 垂直线 — using the rule that the **product of the gradients is  $-1$**  斜率乘积为  $-1$ , finding a **perpendicular** 垂线 through a given point, and finding the **perpendicular bisector** 垂直平分线 of a line segment.

You must be able to:

- write down the gradient of a line perpendicular to a given line
- find the equation of the perpendicular through a given point
- find the perpendicular bisector of a line segment

### 1 Worked examples

■ Turn the fraction upside down and change the sign



Parallel lines share a gradient; perpendicular gradients multiply to  $-1$

- **Rule:**  $m_1 \times m_2 = -1$ , so  $m_2 = -\frac{1}{m_1}$  — the **negative reciprocal**.
- **Worked:** perpendicular to a gradient of 4 is  $-\frac{1}{4}$ ; perpendicular to  $-\frac{2}{3}$  is  $+\frac{3}{2}$ . The sign always flips.

- **Through a point:** find the new gradient first, then substitute the point to find  $c$ . Perpendicular to  $y = 2x - 1$  through  $(4, 3)$ :  $m = -\frac{1}{2}$ , so  $3 = -2 + c$  and  $y = -\frac{1}{2}x + 5$ .
- **Perpendicular bisector:** it passes through the **MIDPOINT** of the segment and is perpendicular to it — two separate facts, and both are needed.

**Answer shapes** (marked by **method marks** — M1 for a correct method, A1 for the correct answer): the negative reciprocal, and the substitution for  $c$ , are separate method marks.

### 2 Practice

**2.1** Write down the gradient of a line perpendicular to  $y = 4x + 1$ . [1]

**2.2** Write down the gradient of a line perpendicular to  $y = -\frac{2}{3}x + 5$ . [1]

**2.3** Find the equation of the line perpendicular to  $y = 2x - 1$  that passes through  $(4, 3)$ . [3]

### 3 Exam-style questions

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**3.1**  $A$  is the point  $(1, 2)$  and  $B$  is the point  $(5, 10)$ .

(a) Find the gradient of  $AB$ . [2]

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(b) Find the equation of the perpendicular bisector of  $AB$ . [4]

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**3.2** Show that the lines  $y = 3x + 2$  and  $x + 3y = 9$  are perpendicular. [3]

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## Solutions

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Marking: **M1** a correct method, **A1** the correct answer.

Each answer shows the steps, then the **result**.

### 2.1

- the negative reciprocal of 4

$$= -\frac{1}{4}$$

### 2.2

- the negative reciprocal of  $-\frac{2}{3}$ : turn it over and change the sign

$$= \frac{3}{2}$$

### 2.3

- perpendicular gradient  $= -\frac{1}{2}$
- substitute (4, 3):  $3 = -\frac{1}{2}(4) + c = -2 + c$ , so  $c = 5$

$$\mathbf{y = -\frac{1}{2}x + 5}$$

### 3.1

(a)

- $\frac{10 - 2}{5 - 1} = \frac{8}{4}$

$$= \mathbf{2}$$

(b)

- the midpoint of  $AB$  is  $\left(\frac{1+5}{2}, \frac{2+10}{2}\right) = (3, 6)$
- the perpendicular gradient is  $-\frac{1}{2}$
- substitute (3, 6):  $6 = -\frac{3}{2} + c$ , so  $c = 7.5$

$$\mathbf{y = -\frac{1}{2}x + 7.5}$$

### 3.2

- rearrange the second:  $3y = -x + 9$ , so  $y = -\frac{1}{3}x + 3$
- the gradients are 3 and  $-\frac{1}{3}$

$$3 \times \left(-\frac{1}{3}\right) = -1, \text{ so the lines are } \mathbf{\text{perpendicular}}$$