

E3.7 Perpendicular lines

Name: _____ Class: _____ Date: _____

Total: 14 marks

KEY WORDS perpendicular 垂直 • negative reciprocal 负倒数
 • perpendicular bisector 垂直平分线 • midpoint 中点 • gradient 斜率

Objective

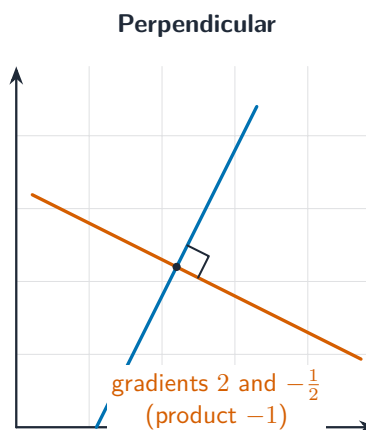
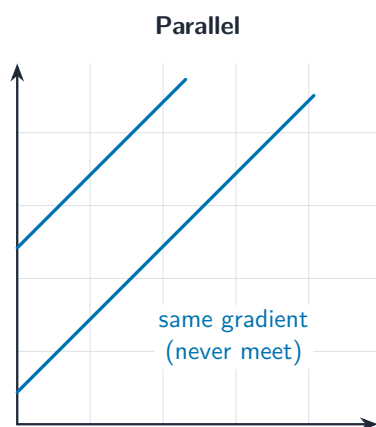
Build the skills to answer Extended exam questions on **perpendicular lines** 垂直线 — using the rule that the **product of the gradients is -1** 斜率乘积为 -1 , finding a **perpendicular** 垂线 through a given point, and finding the **perpendicular bisector** 垂直平分线 of a line segment.

You must be able to:

- write down the gradient of a line perpendicular to a given line
- find the equation of the perpendicular through a given point
- find the perpendicular bisector of a line segment

1 Worked examples

- Turn the fraction upside down and change the sign



Parallel lines share a gradient; perpendicular gradients multiply to -1

- **Rule:** $m_1 \times m_2 = -1$, so $m_2 = -\frac{1}{m_1}$ — the **negative reciprocal**.
- **Worked:** perpendicular to a gradient of 4 is $-\frac{1}{4}$; perpendicular to $-\frac{2}{3}$ is $+\frac{3}{2}$. The sign always flips.

- **Through a point:** find the new gradient first, then substitute the point to find c . Perpendicular to $y = 2x - 1$ through $(4, 3)$: $m = -\frac{1}{2}$, so $3 = -2 + c$ and $y = -\frac{1}{2}x + 5$.
- **Perpendicular bisector:** it passes through the MIDPOINT of the segment and is perpendicular to it — two separate facts, and both are needed.

Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer): the negative reciprocal, and the substitution for c , are separate method marks.

2 Practice

2.1 Write down the gradient of a line perpendicular to $y = 4x + 1$. [1]

2.2 Write down the gradient of a line perpendicular to $y = -\frac{2}{3}x + 5$. [1]

2.3 Find the equation of the line perpendicular to $y = 2x - 1$ that passes through $(4, 3)$. [3]

3 Exam-style questions

3.1 A is the point $(1, 2)$ and B is the point $(5, 10)$.

(a) Find the gradient of AB . [2]

(b) Find the equation of the perpendicular bisector of AB . [4]

3.2 Show that the lines $y = 3x + 2$ and $x + 3y = 9$ are perpendicular. [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

Each answer shows the steps, then the **result**.

2.1

- the negative reciprocal of 4

$$= -\frac{1}{4}$$

2.2

- the negative reciprocal of $-\frac{2}{3}$: turn it over and change the sign

$$= \frac{3}{2}$$

2.3

- perpendicular gradient $= -\frac{1}{2}$
- substitute (4, 3): $3 = -\frac{1}{2}(4) + c = -2 + c$, so $c = 5$

$$\mathbf{y = -\frac{1}{2}x + 5}$$

3.1

(a)

$$\bullet \frac{10 - 2}{5 - 1} = \frac{8}{4}$$

$$\mathbf{= 2}$$

(b)

- the midpoint of AB is $\left(\frac{1+5}{2}, \frac{2+10}{2}\right) = (3, 6)$
- the perpendicular gradient is $-\frac{1}{2}$
- substitute (3, 6): $6 = -\frac{3}{2} + c$, so $c = 7.5$

$$\mathbf{y = -\frac{1}{2}x + 7.5}$$

3.2

- rearrange the second: $3y = -x + 9$, so $y = -\frac{1}{3}x + 3$
- the gradients are 3 and $-\frac{1}{3}$

$$3 \times \left(-\frac{1}{3}\right) = -1, \text{ so the lines are } \mathbf{\text{perpendicular}}$$