

E3.4 Length and midpoint

Name: _____ Class: _____ Date: _____

Total: 14 marks

KEY WORDS midpoint 中点 • line segment 线段 • distance between two points 两点间的距离
• exact form 精确值 • coordinates 坐标

Objective

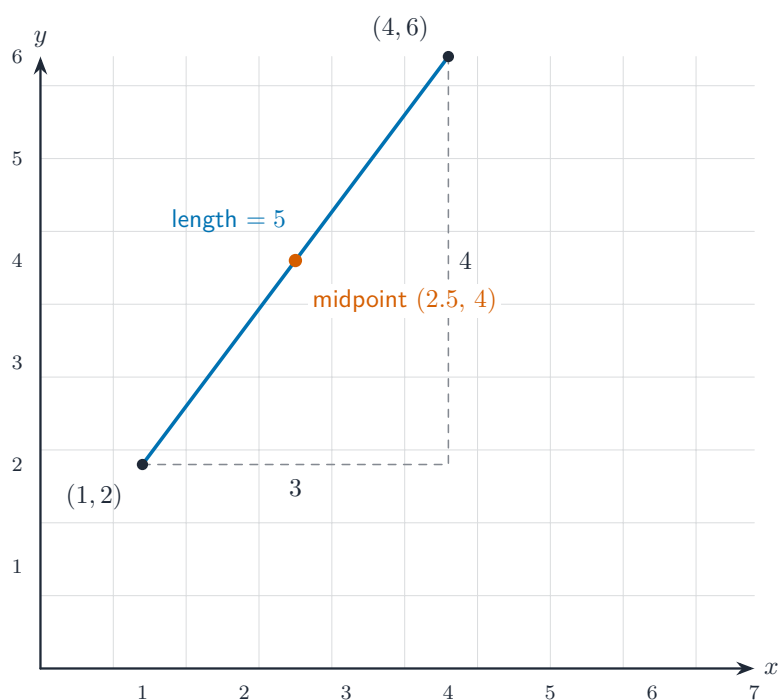
Build the skills to answer Extended exam questions on **length and midpoint** 长度与中点 — finding the **distance between two points** 两点间的距离 with Pythagoras, finding the **midpoint** 中点 of a line segment, and working backwards from a midpoint to a missing endpoint.

You must be able to:

- find the length of a line segment from the coordinates of its ends
- find the midpoint of a line segment
- find one endpoint when the midpoint and the other endpoint are known

1 Worked examples

- Length is Pythagoras; midpoint is an average



The horizontal and vertical differences are the two short sides of a right-angled triangle

- **Length:** $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$. Both differences are SQUARED, so their signs do not matter.
- **Midpoint:** $M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$ — the average of the x 's and the average of the y 's.
- **Worked:** for $A(1, 2)$ and $B(7, 10)$, $AB = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$ and $M = (4, 6)$.
- **Backwards:** if M is the midpoint of PQ , then $Q = (2x_M - x_P, 2y_M - y_P)$.

Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer): the line showing the squares being added, before the root is taken, is the method mark.

2 Practice

2.1 A is $(1, 2)$ and B is $(7, 10)$. Find the length of AB .

[2]

2.2 Find the midpoint of AB for the same two points. [2]

2.3 P is $(-3, 5)$ and Q is $(2, -7)$. Find the length of PQ . [2]

3 Exam-style questions

3.1 A is the point $(-2, 1)$ and B is the point $(4, 6)$.

(a) Find the coordinates of the midpoint of AB . [2]

(b) Find the length of AB , giving your answer in exact form. [3]

3.2 $M(3, -1)$ is the midpoint of PQ , and P is the point $(-1, 4)$. Find the coordinates of Q . [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

Each answer shows the steps, then the **result**.

2.1

- differences are 6 across and 8 up, so $AB = \sqrt{6^2 + 8^2} = \sqrt{36 + 64}$
 $= \sqrt{100} = \mathbf{10}$

2.2

- average each coordinate: $\left(\frac{1+7}{2}, \frac{2+10}{2}\right)$
 $= (\mathbf{4}, \mathbf{6})$

2.3

- differences are 5 and -12 , so $PQ = \sqrt{5^2 + (-12)^2} = \sqrt{25 + 144}$
 $= \sqrt{169} = \mathbf{13}$

3.1

(a)

- $\left(\frac{-2+4}{2}, \frac{1+6}{2}\right)$
 $= (\mathbf{1}, \mathbf{3.5})$

(b)

- differences are 6 and 5
- $AB = \sqrt{36 + 25}$
 $= \sqrt{\mathbf{61}}$
- 61 has no square factor, so $\sqrt{61}$ is already the exact form; a decimal would lose the mark

3.2

- the midpoint is the average, so $\frac{-1+x_Q}{2} = 3$ and $\frac{4+y_Q}{2} = -1$
- $x_Q = 6 - (-1) = 7$ and $y_Q = -2 - 4 = -6$

$$Q = (\mathbf{7}, \mathbf{-6})$$