

E3.1 Coordinates

Name: _____ Class: _____ Date: _____

Total: 29 marks

KEY WORDS coordinates 坐标 • midpoint 中点 • length 长度 • origin 原点 • gradient 斜率

Objective

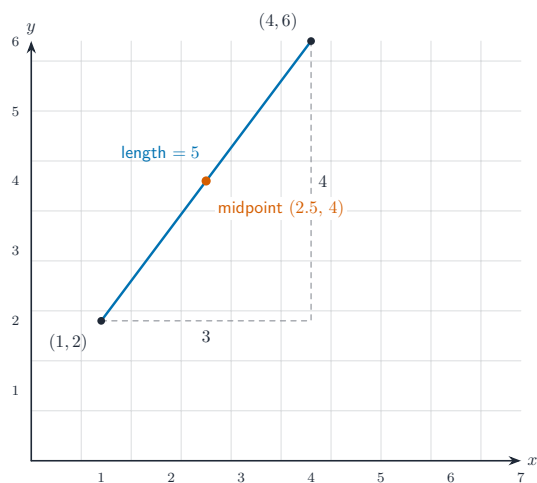
Build the skills to answer Extended exam questions on **coordinates** 坐标 — plotting points, and finding the **length** 长度 and **midpoint** 中点 of a line segment.

You must be able to:

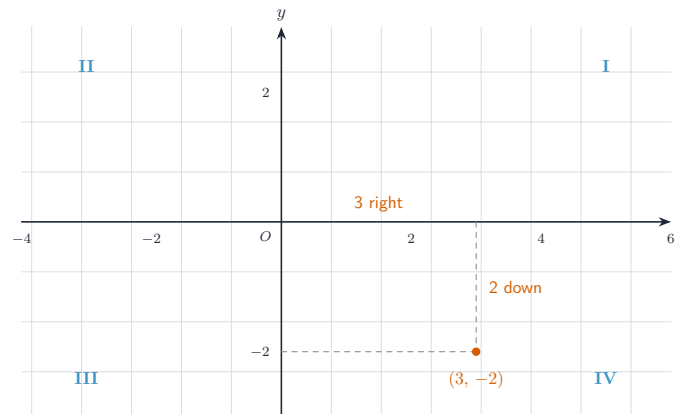
- plot and read coordinates (x, y)
- find the length of a segment using Pythagoras
- find the midpoint of a segment

1 Worked examples

■ Length and midpoint



(x, y) : first across, then up



Points are plotted on the coordinate plane

$$\text{length} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

- **Worked** (1, 2) to (4, 6): length = $\sqrt{3^2 + 4^2} = 5$; midpoint = (2.5, 4).

Answer shapes: M1 a correct method, A1 the correct answer.

■ The three coordinate formulas, and when each is wanted

For $A(x_1, y_1)$ and $B(x_2, y_2)$:

$$\text{midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right) \quad \text{length} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{gradient} = \frac{y_2 - y_1}{x_2 - x_1}$$

The midpoint **adds** and halves; the length and gradient **subtract**. Mixing the two is the commonest error, and it is easy to spot: a midpoint should lie between the two points.

Parallel and perpendicular. Parallel lines have the **same** gradient. Perpendicular lines have gradients whose product is -1 , so the perpendicular gradient is the **negative reciprocal**: if $m = \frac{2}{3}$ then the perpendicular gradient is $-\frac{3}{2}$.

The equation of a line is easiest from $y - y_1 = m(x - x_1)$ using any point on it; rearrange to $y = mx + c$ at the end if the question asks for that form.

Worked example. $A(1, 1)$ and $B(7, 9)$. Find the equation of the perpendicular bisector of AB .

1. Midpoint: $\left(\frac{1+7}{2}, \frac{1+9}{2} \right) = (4, 5)$.

2. Gradient of AB : $\frac{9-1}{7-1} = \frac{8}{6} = \frac{4}{3}$.

3. Perpendicular gradient: the negative reciprocal, $-\frac{3}{4}$.

4. Through the midpoint: $y - 5 = -\frac{3}{4}(x - 4)$.

5. Rearranged: $y = -\frac{3}{4}x + 8$.

The bisector must pass through the **midpoint** — using A or B instead gives a parallel line and loses every mark after step 3.

2 Practice

2.1 Find the midpoint of the segment joining $(2, 3)$ and $(6, 9)$. [2]

2.2 Find the length of the segment from $(0, 0)$ to $(3, 4)$. [2]

2.3 Write down the coordinates of the point 3 left and 2 up from the origin. [1]

2.4 Find the gradient of the line joining $(2, -1)$ and $(6, 7)$. [2]

2.5 State the gradient of a line perpendicular to $y = \frac{2}{5}x + 1$. [1]

2.6 Find the length of the line joining $(0, 0)$ and $(9, 12)$. [2]

3 Exam-style questions

3.1 $A = (1, 1)$ and $B = (7, 9)$.

(a) Find the midpoint of AB . [2]

(b) Find the length of AB . [2]

3.2 $M(3, 5)$ is the midpoint of PQ . Given $P = (1, 2)$, find the coordinates of Q . [2]

3.3 $P = (-2, 3)$ and $Q = (6, 9)$.

(a) Find the midpoint of PQ .

[2]

(b) Find the length of PQ .

[3]

(c) Find the gradient of PQ .

[2]

(d) Find the equation of the line through P and Q , giving your answer in the form $y = mx + c$.

[3]

(e) Find the equation of the perpendicular bisector of PQ .

[3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $\left(\frac{2+6}{2}, \frac{3+9}{2}\right) = (4, 6)$.

2.2 $\sqrt{3^2 + 4^2} = \sqrt{25} = 5$.

2.3 $(-3, 2)$.

2.4 $\frac{7 - (-1)}{6 - 2} = \frac{8}{4} = 2$.

2.5 $-\frac{5}{2}$.

2.6 $\sqrt{9^2 + 12^2} = \sqrt{81 + 144} = \sqrt{225} = 15$.

3.1 (a) $\left(\frac{1+7}{2}, \frac{1+9}{2}\right) = (4, 5)$. (b) $\sqrt{(7-1)^2 + (9-1)^2} = \sqrt{36 + 64} = \sqrt{100} = 10$.

3.2 if M is the midpoint, $Q = (2 \times 3 - 1, 2 \times 5 - 2) = (5, 8)$.

3.3 (a) $\left(\frac{-2+6}{2}, \frac{3+9}{2}\right) = (2, 6)$.

(b) $\sqrt{(6 - (-2))^2 + (9 - 3)^2} = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$.

(c) $\frac{9-3}{6 - (-2)} = \frac{6}{8} = \frac{3}{4}$.

(d) $y - 3 = \frac{3}{4}(x + 2)$; $y = \frac{3}{4}x + \frac{3}{2} + 3$; $y = \frac{3}{4}x + 4.5$.

(e) Perpendicular gradient $= -\frac{4}{3}$; through the midpoint $(2, 6)$: $y - 6 = -\frac{4}{3}(x - 2)$;
 $y = -\frac{4}{3}x + \frac{26}{3}$.