

E2.8 Proportion

Name: _____ Class: _____ Date: _____ **Total: 17 marks**

KEY WORDS direct proportion 正比例 • inverse proportion 反比例
• constant of proportionality 比例常数 • proportional to 与 ... 成正比 • reciprocal 倒数

Objective

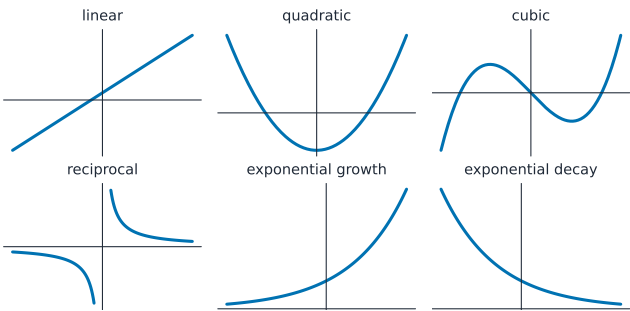
Build the skills to answer Extended exam questions on **proportion** 比例 — writing **direct proportion** 正比例 and **inverse proportion** 反比例 algebraically, finding the **constant of proportionality** 比例常数, and using the relationship to find a missing value.

You must be able to:

- translate a proportion statement into an equation with a constant k
- find k from one pair of values, then use the equation for any other pair
- handle proportion to a square, a cube or a square root

1 Worked examples

■ Write the equation, find k , then answer the question



recognise $y = x^2$, $y = 1/x$, $y = 2^x$ from shape

Direct proportion is a straight line through the origin; inverse proportion is a reciprocal curve

statement	equation
y is directly proportional to x	$y = kx$
y is proportional to x^2	$y = kx^2$
y is inversely proportional to x	$y = \frac{k}{x}$
y is inversely proportional to x^2	$y = \frac{k}{x^2}$
y is proportional to $\sqrt{x}$	$y = k\sqrt{x}$

- **Three steps, always:** write the equation with k , substitute the given pair to find k , then substitute again.
- **Inverse means divide.** "Inversely proportional to x " is $y = k/x$, so doubling x halves y .
- **Worked:** $y \propto x^2$ and $y = 45$ when $x = 3$, so $45 = 9k$ and $k = 5$. Then at $x = 5$, $y = 5 \times 25 = 125$.

Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer): the equation with k in it is the first method mark, and the value of k is the second.

2 Practice

2.1 y is directly proportional to x . When $x = 4$, $y = 18$. Find y when $x = 10$. [3]

2.2 y is inversely proportional to x . When $x = 5$, $y = 12$. Find y when $x = 4$. [3]

2.3 y is proportional to x^2 . When $x = 3$, $y = 45$. Find y when $x = 5$. [3]

3 Exam-style questions

3.1 y is inversely proportional to the square of x . When $x = 2$, $y = 9$.

(a) Find an expression for y in terms of x . [2]

(b) Find y when $x = 6$. [1]

(c) Find the positive value of x when $y = 4$. [2]

3.2 The time T hours taken to complete a job is inversely proportional to the number of workers n . Six workers take 10 hours. Find the time taken by four workers. [3]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

Each answer shows the steps, then the **result**.

2.1

- $y = kx$, and $18 = 4k$ so $k = 4.5$

$$y = 4.5 \times 10 = \mathbf{45}$$

2.2

- $y = \frac{k}{x}$, and $12 = \frac{k}{5}$ so $k = 60$

$$y = \frac{60}{4} = \mathbf{15}$$

2.3

- $y = kx^2$, and $45 = 9k$ so $k = 5$

$$y = 5 \times 25 = \mathbf{125}$$

3.1

(a)

- $y = \frac{k}{x^2}$, and $9 = \frac{k}{4}$ so $k = 36$

$$y = \frac{\mathbf{36}}{\mathbf{x^2}}$$

(b)

- $y = \frac{36}{36}$

$$= \mathbf{1}$$

(c)

- $4 = \frac{36}{x^2}$, so $x^2 = 9$

$$x = \mathbf{3}$$

3.2

- $T = \frac{k}{n}$, and $10 = \frac{k}{6}$ so $k = 60$

$$T = \frac{60}{4} = \mathbf{15} \text{ hours}$$

- fewer workers means MORE time, which is the check that the relationship was written as an inverse