

E2.7 Sequences

Name: _____ Class: _____ Date: _____ **Total: 11 marks**

KEY WORDS sequence 数列 • nth term 通项 • common difference 公差
• second difference 二阶差分 • exponential sequence 指数数列

Objective

Build the skills to answer Extended exam questions on **sequences** 数列 — continuing a sequence and finding the **nth term** 通项 of a **linear** 线性, **quadratic** 二次, **cubic** 三次 or **exponential** 指数 sequence.

You must be able to:

- find the n th term of a linear sequence from its common difference
- recognise a quadratic sequence from its constant second difference and find its n th term
- recognise a cubic or an exponential sequence and write down its n th term

1 Worked examples

■ The differences tell you which kind of sequence it is



A constant first difference means a linear sequence

- **Linear:** constant FIRST difference d , so the n th term is $dn + c$. For 5, 8, 11, 14, $d = 3$ and the n th term is $3n + 2$.
- **Quadratic:** constant SECOND difference. Half of it is the coefficient of n^2 , then subtract an^2 from each term and the remainder is linear. For 3, 8, 15, 24 the second difference is 2, so the n th term is $n^2 + 2n$.
- **Cubic:** the THIRD difference is constant. 1, 8, 27, 64 is n^3 .
- **Exponential:** each term is the previous one $\times r$. For 2, 6, 18, 54 the n th term is $2 \times 3^{n-1}$.

Answer shapes (marked by **method marks** — M1 for a correct method, A1 for the correct answer): writing the difference row down is the method mark, and it also tells you which method to use.

2 Practice

2.1 Find the n th term of the sequence 7, 12, 17, 22, ... [2]

2.2 Write down the next two terms of the sequence 4, 12, 36, 108, ... [2]

2.3 Find the n th term of the sequence 3, 12, 48, 192, ... [2]

3 Exam-style questions

3.1 The first five terms of a sequence are 2, 9, 20, 35, 54.

(a) Write down the next term. [1]

(b) Find an expression for the n th term. [3]

3.2 Write down the n th term of the sequence 1, 8, 27, 64, ... [1]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

Each answer shows the steps, then the **result**.

2.1

- the common difference is 5, so the n th term starts $5n$; at $n = 1$, $5 \times 1 = 5$ and the term is 7

$$= 5n + 2$$

2.2

- each term is $\times 3$

$$324 \text{ and } 972$$

2.3

- each term is $\times 4$ and the first term is 3

$$= 3 \times 4^{n-1}$$

3.1

(a)

- first differences 7, 11, 15, 19 rise by 4, so the next is 23

$$54 + 23 = 77$$

(b)

- the second difference is 4, so the n^2 coefficient is $4 \div 2 = 2$
- subtract $2n^2$, i.e. 2, 8, 18, 32, 50, from the terms: 0, 1, 2, 3, 4
- that remainder is $n - 1$

$$= 2n^2 + n - 1$$

3.2

- the terms are 1^3 , 2^3 , 3^3 , 4^3

$$= n^3$$