IGCSE MATHEMATICS • EXERCISE SHEET

E2.6 Inequalities

Name: _____ Class: _____ Date: _____ **Total: 29 marks**

KEY WORDS inequality 不等式 • number line 数轴 • region 区域 • term 项 • equation 方程

Objective

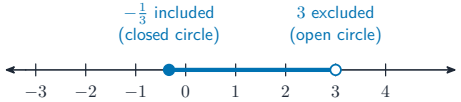
Build the skills to answer Extended exam questions on **inequalities** 不等式 — solving them, the rule for **multiplying or dividing by a negative** 乘除负数, and showing a solution on a **number line** 数轴.

You must be able to:

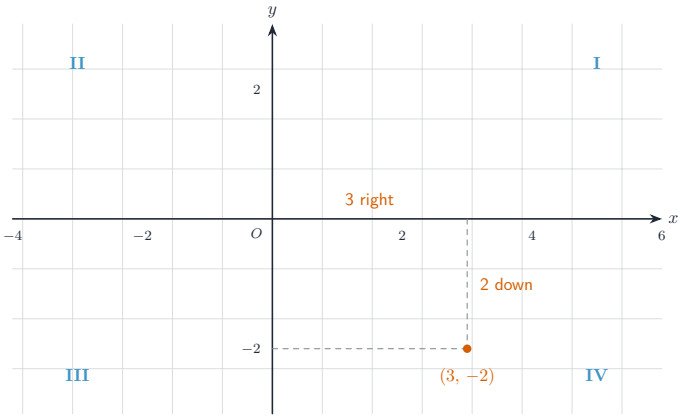
- solve a linear inequality (and reverse the sign when needed)
- solve a double inequality
- list the integers that satisfy an inequality

1 Worked examples

Solving inequalities



Solve like an equation, but reverse the sign if you ÷ or × by a negative



Two-variable inequalities shade regions of the plane

- **Worked:** $-3 \leqslant 3x - 2 < 7 \rightarrow$ add 2: $-1 \leqslant 3x < 9 \rightarrow \div 3$: $-\frac{1}{3} \leqslant x < 3$.
- **Reverse:** $-2x < 6 \rightarrow x > -3$ (sign flips when dividing by -2).
- **Number line:** closed circle for $\leqslant$ / $\geqslant$, open circle for $<$ / $>$.

Answer shapes: M1 a correct method, **A1** the correct answer.

Inequalities: the same steps as an equation, with one extra rule

Solve an inequality exactly as you would the matching equation — **except** that multiplying or dividing both sides by a **negative** number **reverses** the inequality sign.

$$-3x > 12 \;\Rightarrow\; x < -4$$

That single rule is the difference between full marks and none on half the questions in this topic. If you would rather not risk it, move the x term to the side where it is positive instead: $-3x > 12$ becomes $-12 > 3x$, so $-4 > x$, which is the same answer with no sign flip.

A double inequality is solved by doing the same operation to all three parts: $5 \leqslant 2x + 1 \leqslant 11$ gives $4 \leqslant 2x \leqslant 10$, then $2 \leqslant x \leqslant 5$.

Reading the answer as integers. For $-2 < x \leqslant 3$: the $<$ EXCLUDES -2 and the $\leqslant$ INCLUDES 3, so the integers are $-1, 0, 1, 2, 3$. Getting the endpoints right is usually a mark by itself.

On a number line, an open circle means the value is excluded ($<$ or $>$) and a filled circle means it is included ($\leqslant$ or $\geqslant$).

Worked example. Find the integers n for which $-9 < 3n - 3 \leqslant 6$.

1. Add 3 throughout: $-6 < 3n \leqslant 9$.
2. Divide by 3 throughout: $-2 < n \leqslant 3$.
3. The endpoints: -2 is excluded, 3 is included.
4. So $n = -1, 0, 1, 2, 3$.

2 Practice

2.1 Solve $2x + 1 > 9$. [2]

2.2 Solve $-3x \leqslant 12$. [2]

2.3 Solve $5x - 4 < 11$. [2]

2.4 Solve $-4x + 3 > 19$. [2]

2.5 Write down the inequality shown by an open circle at -1 and a filled circle at 4, joined by a line. [2]

3 Exam-style questions

3.1 Solve the inequality $5 \leqslant 2x + 1 \leqslant 11$. [3]

3.2 Write down the integers that satisfy $-2 < x \leqslant 3$. [2]

3.3 Solve $7 - 2x \geqslant 1$. [2]

3.4 (a) Solve the inequality $-11 < 4x + 5 \leqslant 13$. [3]

(b) Write down all the integers that satisfy your answer to part (a). [2]

(c) Solve $9 - 4x \geqslant 21$, and explain the step in which the inequality sign changes. [3]

(d) A rectangle has width x cm and length $(x + 5)$ cm. Its perimeter is at least 30 cm and less than 50 cm. Form an inequality in x and solve it. [4]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $2x > 8 \to x > 4$.

2.2 dividing by -3 reverses the sign: $x \geqslant -4$.

2.3 $5x < 15 \to x < 3$.

2.4 $-4x > 16$; dividing by -4 reverses the sign, so $x < -4$.

2.5 $-1 < x \leqslant 4$.

3.1 subtract 1: $4 \leqslant 2x \leqslant 10$; divide by 2: $2 \leqslant x \leqslant 5$.

3.2 the integers are $-1, 0, 1, 2, 3$.

3.3 $-2x \geqslant -6 \to$ dividing by -2 reverses the sign $\to x \leqslant 3$.

3.4 (a) Subtract 5 throughout: $-16 < 4x \leqslant 8$; divide by 4 throughout: $-4 < x \leqslant 2$.
(b) $-3, -2, -1, 0, 1, 2$.
(c) $-4x \geqslant 12$; dividing both sides by -4 reverses the inequality; $x \leqslant -3$.
(d) Perimeter $= 2(x) + 2(x+5) = 4x + 10$; $30 \leqslant 4x + 10 < 50$; $20 \leqslant 4x < 40$; $5 \leqslant x < 10$.