

E2.5 Equations

Name: _____ Class: _____ Date: _____

Total: 29 marks

KEY WORDS equations 方程 • simultaneous 联立 • simultaneous equations 联立方程
• linear 一次 • quadratic 二次

Objective

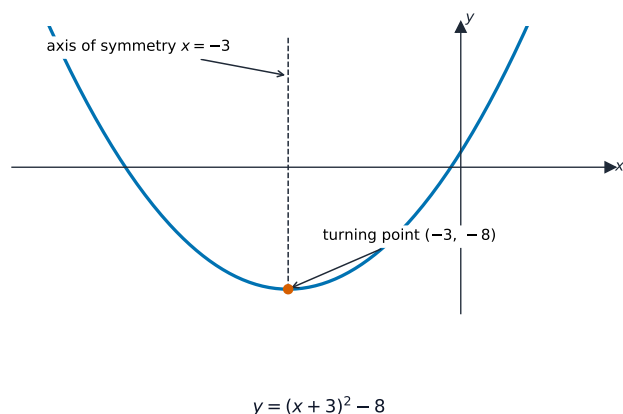
Build the skills to answer Extended exam questions on **equations** 方程 — **linear** 一次, **simultaneous** 联立, and **quadratic** 二次 equations (factorising and the formula).

You must be able to:

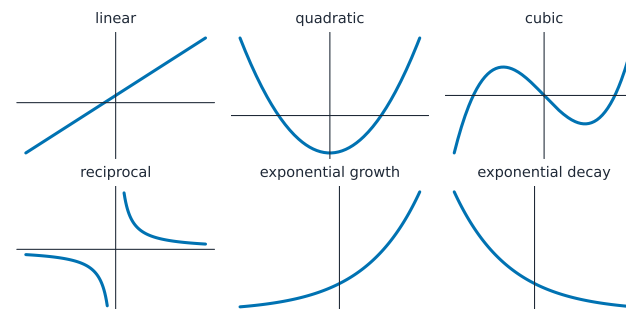
- solve a linear equation with brackets
- solve simultaneous linear equations
- solve a quadratic by factorising or the formula

1 Worked examples

■ Solving equations



Do the same to both sides until the unknown is alone



recognise $y = x^2$, $y = 1/x$, $y = 2^x$ from shape

An equation is solved graphically where curves cross

- **Linear:** $5 - 2x = 3(x + 7) \rightarrow -16 = 5x \rightarrow x = -\frac{16}{5}$.
- **Simultaneous:** $2x + y = 7$, $3x - y = 8 \rightarrow$ add: $5x = 15$, $x = 3$, $y = 1$.
- **Quadratic formula:** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Answer shapes: **M1** a correct method, **A1** the correct answer.

■ The three routes through a quadratic, and how to pick one

Try them in this order.

1. **Factorising.** Find two numbers that multiply to give the constant and add to give the coefficient of x . For $x^2 - 2x - 8 = 0$ they are -4 and 2 , so $(x - 4)(x + 2) = 0$ and $x = 4$ or $x = -2$. Fastest when the numbers are whole.
2. **The formula.** Use it whenever the question says “correct to 2 decimal places”, because that is a sign the roots are not whole numbers:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

3. **Completing the square.** Use it when the question asks for the turning point or for an exact (surd) answer.

Worked example (formula). Solve $2x^2 + 3x - 4 = 0$ to 2 decimal places. Here $a = 2$, $b = 3$, $c = -4$.

1. $b^2 - 4ac = 9 - 4(2)(-4) = 9 + 32 = 41$.
2. $x = \frac{-3 \pm \sqrt{41}}{4}$.
3. $\sqrt{41} = 6.403$, so $x = \frac{3.403}{4} = 0.85$ or $x = \frac{-9.403}{4} = -2.35$.

Show the substituted formula before evaluating: that line earns the method mark even if the arithmetic slips.

Simultaneous equations. With one linear and one quadratic, **substitute** the linear into the quadratic; with two linear equations, **eliminate** by adding or subtracting once the coefficients match. Always find both unknowns and pair them correctly.

2 Practice

2.1 Solve $3x - 7 = 11$. [2]

2.2 Solve $x^2 + 5x + 6 = 0$. [2]

2.3 Solve $4(x - 1) = 2x + 6$. [2]

2.4 Solve $x^2 - 7x + 12 = 0$ by factorising. [2]

2.5 Write down the values of a , b and c for $3x^2 - 5x - 2 = 0$. [1]

3 Exam-style questions

3.1 Solve the simultaneous equations $3x + 2y = 12$ and $x - 2y = 4$. [3]

3.2 Solve $x^2 - 2x - 8 = 0$. [2]

3.3 Solve $2x^2 + 3x - 4 = 0$, giving your answers correct to 2 decimal places. [3]

3.4 (a) Solve the simultaneous equations $y = x + 3$ and $x^2 + y^2 = 29$. [5]

(b) Solve $3x^2 - 7x + 2 = 0$ by factorising. [3]

(c) Solve $x^2 + 6x + 4 = 0$, giving your answers correct to 2 decimal places. [4]

Solutions

Marking: **M1** a correct method, **A1** the correct answer.

2.1 $3x = 18 \rightarrow x = 6$.

2.2 $(x+2)(x+3) = 0 \rightarrow x = -2$ or $x = -3$.

2.3 $4x - 4 = 2x + 6 \rightarrow 2x = 10 \rightarrow x = 5$.

2.4 $(x-3)(x-4) = 0$; $x = 3$ or $x = 4$.

2.5 $a = 3$, $b = -5$, $c = -2$.

3.1 add the two equations: $4x = 16 \rightarrow x = 4$; then $4 - 2y = 4 \rightarrow y = 0$.

3.2 $(x-4)(x+2) = 0 \rightarrow x = 4$ or $x = -2$.

3.3 $a = 2$, $b = 3$, $c = -4$: $x = \frac{-3 \pm \sqrt{9+32}}{4} = \frac{-3 \pm \sqrt{41}}{4} \rightarrow x = 0.85$ or $x = -2.35$.

3.4 (a) Substitute $y = x + 3$: $x^2 + (x+3)^2 = 29$; $2x^2 + 6x + 9 = 29$, so $2x^2 + 6x - 20 = 0$ and $x^2 + 3x - 10 = 0$; $(x+5)(x-2) = 0$; $x = -5$ or $x = 2$; the matching values are $y = -2$ and $y = 5$.

(b) $(3x-1)(x-2) = 0$; $x = \frac{1}{3}$ or $x = 2$.

(c) $a = 1$, $b = 6$, $c = 4$, so $x = \frac{-6 \pm \sqrt{36-16}}{2}$; $\sqrt{20} = 4.472$; $x = -0.76$ or $x = -5.24$.